THE ACTIVE CONTROLLER DESIGN FOR ACHIEVING GENERALIZED PROJECTIVE SYNCHRONIZATION OF HYPERCHAOTIC LÜ AND HYPERCHAOTIC CAI SYSTEMS

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ABSTRACT

This paper discusses the design of active controllers for achieving generalized projective synchronization (GPS) of identical hyperchaotic Lü systems (Chen, Lu, Lü and Yu, 2006), identical hyperchaotic Cai systems (Wang and Cai, 2009) and non-identical hyperchaotic Lü and hyperchaotic Cai systems. The synchronization results (GPS) for the hyperchaotic systems have been derived using active control method and established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the active control method is very effective and convenient for achieving the GPS of the hyperchaotic systems addressed in this paper. Numerical simulations are provided to illustrate the effectiveness of the GPS synchronization results derived in this paper.

KEYWORDS

Active Control, Hyperchaos, Hyperchaotic Systems, Generalized Projective Synchronization, Hyperchaotic Lü System, Hyperchaotic Cai System.

1. INTRODUCTION

Chaotic systems are nonlinear dynamical systems which are highly sensitive to initial conditions. This sensitivity of chaotic systems is usually called as the *butterfly effect* [1]. Small differences in initial conditions (such as those due to rounding errors in numerical computation) yield widely diverging outcomes for chaotic systems.

Hyperchaotic system is usually defined as a chaotic system with more than one positive Lyapunov exponent. The first hyperchaotic system was discovered by O.E. Rössler ([2], 1979).

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Since hyperchaotic system has the characteristics of high capacity, high security and high efficiency, it has the potential of broad applications in nonlinear circuits, secure communications, lasers, neural networks, biological systems and so on. Thus, the studies on hyperchaotic systems, viz. control, synchronization and circuit implementation are very challenging problems in the chaos literature [3].

Chaos synchronization problem received great attention in the literature when Pecora and Carroll [4] published their results on chaos synchronization in 1990. From then on, chaos synchronization has been extensively and intensively studied in the last three decades [4-37]. Chaos theory has been explored in a variety of fields including physical systems [5], chemical systems [6], ecological systems [7], secure communications [8-10], etc.

Synchronization of chaotic systems is a phenomenon that may occur when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem.

In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of anti-synchronization is to use the output of the master system to control the slave system so that the states of the slave system have the same amplitude but opposite signs as the states of the master system asymptotically. In other words, the sum of the states of the master and slave systems are designed to converge to zero asymptotically, when anti-synchronization appears.

In the recent years, various schemes have been deployed for chaos synchronization such as PC method [4], OGY method [11], active control [12-15], adaptive control [16-20], backstepping design [21-23], sampled-data feedback [24], sliding mode control [25-28], etc.

In generalized projective synchronization (GPS) of chaotic systems [29-30], the chaotic systems can synchronize up to a constant scaling matrix. Complete synchronization [12-13], anti-synchronization [31-34], hybrid synchronization [35], projective synchronization [36] and generalized synchronization [37] are particular cases of generalized projective synchronization. GPS has important applications in areas like secure communications and secure data encryption.
In this paper, we deploy active control method so as to derive new results for the generalized projective synchronization (GPS) for identical and different hyperchaotic Lü and hyperchaotic Cai systems. Explicitly, using active nonlinear control and Lyapunov stability theory, we achieve generalized projective synchronization for identical hyperchaotic Lü systems (Chen, Lu, Lü and Yu, [38], 2006), identical hyperchaotic Cai systems (Wang and Cai, [39], 2009) and non-identical hyperchaotic Lü and hyperchaotic Cai systems.

This paper has been organized as follows. In Section 2, we give the problem statement and our methodology. In Section 3, we present a description of the hyperchaotic systems considered in this paper. In Section 4, we derive results for the GPS of two identical hyperchaotic Lü systems. In Section 5, we derive results for the GPS of two identical hyperchaotic Cai systems. In Section 6, we discuss the GPS of non-identical hyperchaotic Lü and hyperchaotic Cai systems. In Section 7, we summarize the main results derived in this paper.

2. PROBLEM STATEMENT AND OUR METHODOLOGY

Consider the chaotic system described by the dynamics

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in R^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : R^n \to R^n$ is the nonlinear part of the system. We consider the system (1) as the *master* or *drive* system.

As the *slave* or *response* system, we consider the chaotic system described by the dynamics

$$\dot{y} = By + g(y) + u \tag{2}$$

where $y \in R^n$ is the state of the system, B is the $n \times n$ matrix of the system parameters, $g : R^n \to R^n$ is the nonlinear part of the system and $u \in R^n$ is the controller of the slave system.

If $A = B$ and $f = g$, then x and y are the states of two identical chaotic systems. If $A \neq B$ or $f \neq g$, then x and y are the states of two different chaotic systems.

In the active control approach, we design a feedback controller u, which achieves the generalized projective synchronization (GPS) between the states of the master system (1) and the slave system (2) for all initial conditions $x(0), z(0) \in R^n$.

For the GPS of the systems (1) and (2), the *synchronization error* is defined as

$$e = y - Mx, \tag{3}$$

where

$$M = \begin{bmatrix} \alpha_1 & 0 & \cdots & 0 \\ 0 & \alpha_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_n \end{bmatrix} \tag{4}$$

In other words, we have

$$e_i = y_i - \alpha_i x_i, \quad (i = 1, 2, \ldots, n) \tag{5}$$

From (1)-(3), the error dynamics is easily obtained as

$$\dot{e} = By - MAx + g(y) - Mf(x) + u \tag{6}$$

The aim of GPS is to find a feedback controller u so that

$$\lim_{t \to \infty} \|e(t)\| = 0 \text{ for all } e(0) \in R^n. \tag{7}$$

Thus, the problem of generalized projective synchronization (GPS) between the master system (1) and slave system (2) can be translated into a problem of how to realize the asymptotic stabilization of the system (6). So, the objective is to design an active controller u for stabilizing the error dynamical system (6) at the origin.

We take as a candidate Lyapunov function

$$V(e) = e^T P e, \tag{8}$$

where P is a positive definite matrix.

Note that $V : R^n \to R$ is a positive definite function by construction.

We assume that the parameters of the master and slave system are known and that the states of both systems (1) and (2) are measurable.

If we find a feedback controller u so that

$$\dot{V}(e) = -e^T Q e, \tag{9}$$

where Q is a positive definite matrix, then $\dot{V} : R^n \to R$ is a negative definite function.

Thus, by Lyapunov stability theory [40], the error dynamics (6) is globally exponentially stable and hence the condition (7) will be satisfied. Hence, GPS is achieved between the states of the master system (1) and the slave system (2).

3. SYSTEMS DESCRIPTION

The hyperchaotic Lü system ([38], 2006) is described by the dynamics

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_4 \\
\dot{x}_2 &= c x_2 + x_1 x_3 \\
\dot{x}_3 &= -b x_3 + x_1 x_2 \\
\dot{x}_4 &= d x_4 + x_1 x_3
\end{aligned} \tag{10}$$

where x_1, x_2, x_3, x_4 are the states and a, b, c, d are constant, positive parameters of the system.

The Lü system (10) exhibits a hyperchaotic attractor when the parameter values are taken as

$$a = 36, \quad b = 3, \quad c = 20 \text{ and } d = 1.3$$

Figure 1 depicts the phase portrait of the hyperchaotic Lü system (10).

The hyperchaotic Cai system ([39], 2009) is described by the dynamics

$$\begin{aligned}
\dot{x}_1 &= p(x_2 - x_1) \\
\dot{x}_2 &= q x_1 + r x_2 + x_4 - x_1 x_3 \\
\dot{x}_3 &= -s x_3 + x_2^2 \\
\dot{x}_4 &= -\varepsilon x_1
\end{aligned} \tag{11}$$

where x_1, x_2, x_3, x_4 are the states and p, q, r, s, ε are constant, positive parameters of the system. The Cai dynamics (11) exhibits a hyperchaotic attractor when the parameter values are taken as

$$p = 27.5, \quad q = 3, \quad r = 19.3, \quad s = 2.9 \text{ and } \varepsilon = 3.3$$

Figure 2 depicts the phase portrait of the hyperchaotic Cai system (11).

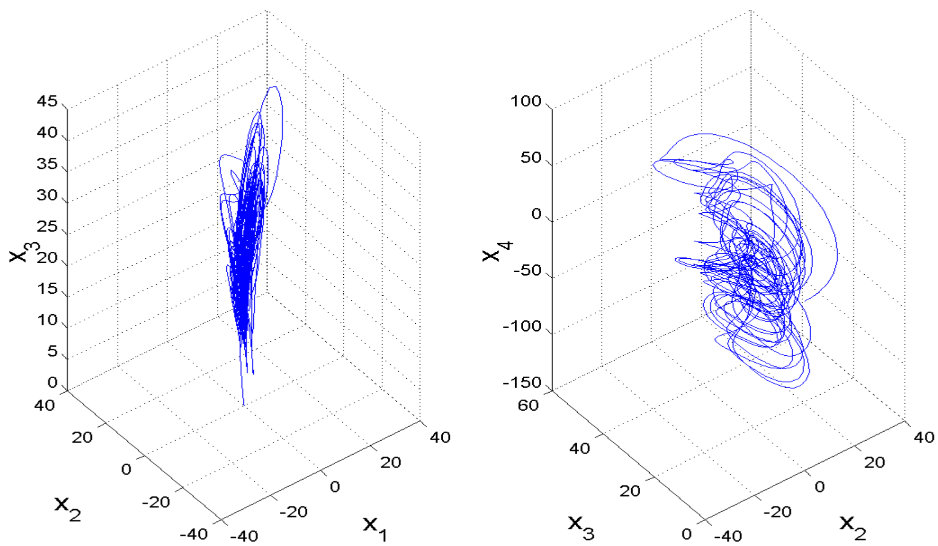

Figure 1. The Phase Portrait of the Hyperchaotic Lü System

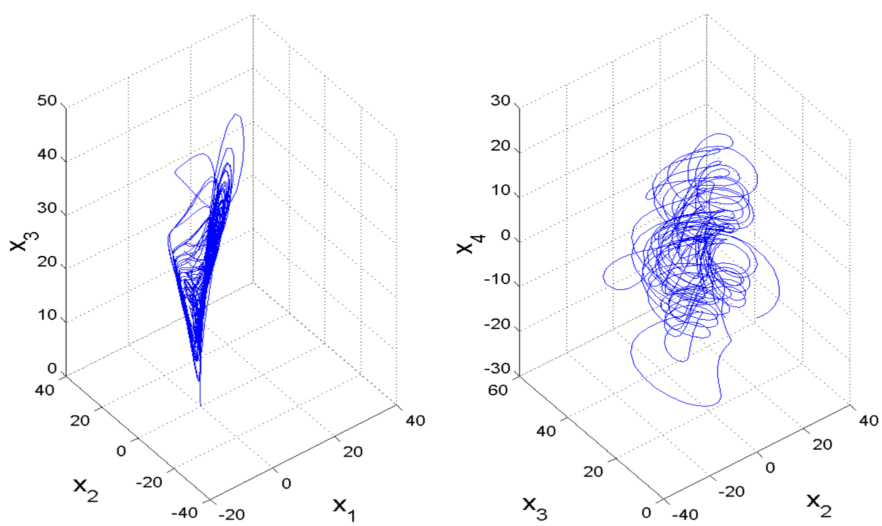

Figure 2. The Phase Portrait of the Hyperchaotic Cai System

4. GPS OF IDENTICAL HYPERCHAOTIC LÜ SYSTEMS

4.1 Theoretical Results

In this section, we apply the active nonlinear control method for the generalized projective synchronization (GPS) of two identical hyperchaotic Lü systems ([38], 2006).

Thus, the master system is described by the hyperchaotic Lü dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4 \\ \dot{x}_2 &= cx_2 - x_1 x_3 \\ \dot{x}_3 &= -bx_3 + x_1 x_2 \\ \dot{x}_4 &= dx_4 + x_1 x_3 \end{aligned} \tag{12}$$

where x_1, x_2, x_3, x_4 are the states and a, b, c, d are positive, constant parameters of the system.

The slave system is described by the controlled hyperchaotic Lü dynamics

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + y_4 + u_1 \\ \dot{y}_2 &= cy_2 - y_1 y_3 + u_2 \\ \dot{y}_3 &= -by_3 + y_1 y_2 + u_3 \\ \dot{y}_4 &= dy_4 + y_1 y_3 + u_4 \end{aligned} \tag{13}$$

where y_1, y_2, y_3, y_4 are the states and u_1, u_2, u_3, u_4 are the active controls to be designed.

For the GPS of the systems (12) and (13), the synchronization error e is defined by

$$e_i = y_i - \alpha_i x_i, \quad (i = 1, 2, 3, 4) \tag{14}$$

where the scales $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are real numbers.

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= -ae_1 + a(y_2 - \alpha_1 x_2) + y_4 - \alpha_1 x_4 + u_1 \\ \dot{e}_2 &= ce_2 - y_1 y_3 + \alpha_2 x_1 x_3 + u_2 \\ \dot{e}_3 &= -be_3 + y_1 y_2 - \alpha_3 x_1 x_2 + u_3 \\ \dot{e}_4 &= de_4 + y_1 y_3 - \alpha_4 x_1 x_3 + u_4 \end{aligned} \tag{15}$$

We choose the nonlinear controller as

$$\begin{aligned} u_1 &= ae_1 - a(y_2 - \alpha_1 x_2) - y_4 + \alpha_1 x_4 - k_1 e_1 \\ u_2 &= -ce_2 + y_1 y_3 - \alpha_2 x_1 x_3 - k_2 e_2 \\ u_3 &= be_3 - y_1 y_2 + \alpha_3 x_1 x_2 - k_3 e_3 \\ u_4 &= -de_4 - y_1 y_3 + \alpha_4 x_1 x_3 - k_4 e_4 \end{aligned} \tag{16}$$

where the gains k_1, k_2, k_3, k_4 are positive constants.

Substituting (16) into (15), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3 \\ \dot{e}_4 &= -k_4 e_3 \end{aligned} \tag{17}$$

Next, we prove the following result.

Theorem 1. The active feedback controller (16) achieves global chaos generalized projective synchronization (GPS) between the identical hyperchaotic Lü systems (12) and (13).

Proof. We consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{18}$$

which is a positive definite function on R^4.

Differentiating (18) along the trajectories of (17), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2, \tag{19}$$

which is a negative definite function on R^4.

Thus, by Lyapunov stability theory [40], the error dynamics (17) is globally exponentially stable. This completes the proof. ■

4.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the two systems of differential equations (12) and (13) with the active controller (16).
The parameters of the identical hyperchaotic Lü systems are chosen as

$$a = 36, \quad b = 3, \quad c = 20, \quad d = 1.3$$

The initial values for the master system (12) are taken as

$$x_1(0) = 24, \quad x_2(0) = -17, \quad x_3(0) = -12, \quad x_4(0) = 18$$

The initial values for the slave system (13) are taken as

$$y_1(0) = -11, \quad y_2(0) = 20, \quad y_3(0) = -5, \quad y_4(0) = 34$$

The GPS scales are taken as $\alpha_1 = 3.5$, $\alpha_2 = -2.9$, $\alpha_3 = 0.8$, and $\alpha_4 = -1.4$.

We take the state feedback gains as $k_i = 5$ for $i = 1, 2, 3, 4$.

Figure 3 shows the GPS synchronization of the identical hyperchaotic Lü systems. Figure 4 shows the time-history of the GPS errors e_1, e_2, e_3, e_4 for the identical hyperchaotic Lü systems.

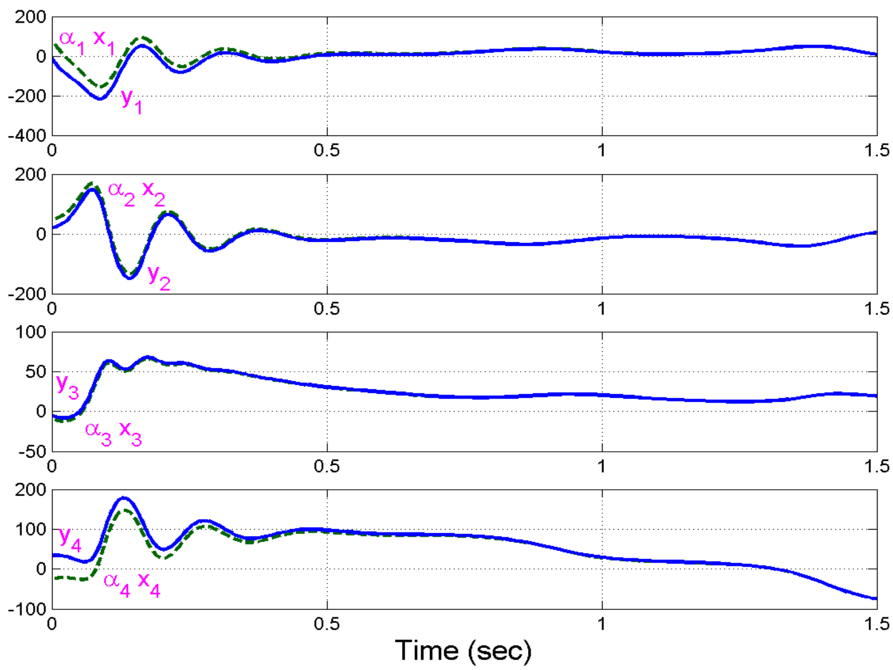

Figure 3. GPS Synchronization of the Identical Hyperchaotic Lü Systems

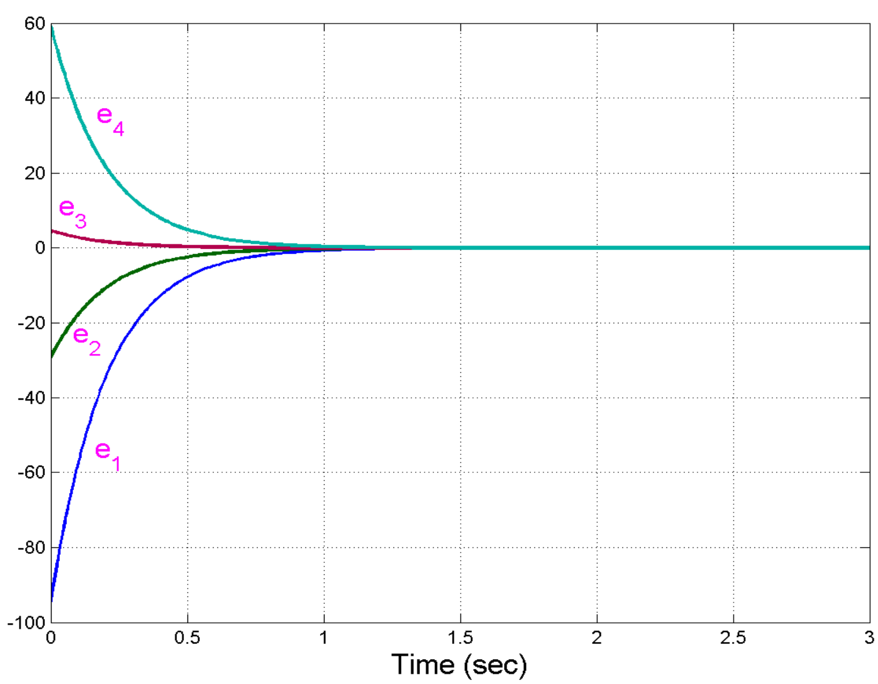

Figure 4. Time History of the GPS Synchronization Error

5. GPS OF IDENTICAL HYPERCHAOTIC CAI SYSTEMS

5.1 Theoretical Results

In this section, we apply the active nonlinear control method for the generalized projective synchronization (GPS) of two identical hyperchaotic Cai systems ([39], 2009).
Thus, the master system is described by the hyperchaotic Cai dynamics

$$\begin{aligned} \dot{x}_1 &= p(x_2 - x_1) \\ \dot{x}_2 &= qx_1 + rx_2 + x_4 - x_1x_3 \\ \dot{x}_3 &= -sx_3 + x_2^2 \\ \dot{x}_4 &= -\varepsilon x_1 \end{aligned} \tag{20}$$

where x_1, x_2, x_3, x_4 are the states and p, q, r, s, ε are positive, constant parameters of the system.

The slave system is described by the controlled hyperchaotic Cai dynamics

$$\begin{aligned} \dot{y}_1 &= p(y_2 - y_1) + u_1 \\ \dot{y}_2 &= qy_1 + ry_2 + y_4 - y_1y_3 + u_2 \\ \dot{y}_3 &= -sy_3 + y_2^2 + u_3 \\ \dot{y}_4 &= -\varepsilon y_1 + u_4 \end{aligned} \tag{21}$$

where y_1, y_2, y_3, y_4 are the states and u_1, u_2, u_3, u_4 are the active controls to be designed.

For the GPS of the systems (20) and (21), the synchronization error e is defined by

$$\begin{aligned} e_1 &= y_1 - \alpha_1 x_1 \\ e_2 &= y_2 - \alpha_2 x_2 \\ e_3 &= y_3 - \alpha_3 x_3 \\ e_4 &= y_4 - \alpha_4 x_4 \end{aligned} \tag{22}$$

where the scales $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are real numbers.

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= -pe_1 + p(y_2 - \alpha_1 x_2) + u_1 \\ \dot{e}_2 &= re_2 + q(y_1 - \alpha_2 x_1) + y_4 - \alpha_2 x_4 - y_1y_3 + \alpha_2 x_1 x_3 + u_2 \\ \dot{e}_3 &= -se_3 + y_2^2 - \alpha_3 x_2^2 + u_3 \\ \dot{e}_4 &= -\varepsilon(y_1 - \alpha_4 x_1) + u_4 \end{aligned} \tag{23}$$

We choose the nonlinear controller as

$$\begin{aligned} u_1 &= pe_1 - p(y_2 - \alpha_1 x_2) - k_1 e_1 \\ u_2 &= -re_2 - q(y_1 - \alpha_2 x_1) - y_4 + \alpha_2 x_4 + y_1 y_3 - \alpha_2 x_1 x_3 - k_2 e_2 \\ u_3 &= se_3 - y_2^2 + \alpha_3 x_2^2 - k_3 e_3 \\ u_4 &= \varepsilon(y_1 - \alpha_4 x_1) - k_4 e_4 \end{aligned} \tag{24}$$

where the gains k_1, k_2, k_3 are positive constants.

Substituting (24) into (23), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3 \\ \dot{e}_4 &= -k_4 e_4 \end{aligned} \tag{25}$$

Next, we prove the following result.

Theorem 2. The active feedback controller (24) achieves global chaos generalized projective synchronization (GPS) between the identical hyperchaotic Cai systems (20) and (21).

Proof. We consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{26}$$

which is a positive definite function on R^4.

Differentiating (26) along the trajectories of (25), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2, \tag{27}$$

which is a negative definite function on R^4.

Thus, by Lyapunov stability theory [40], the error dynamics (25) is globally exponentially stable. This completes the proof. ■

5.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the two systems of differential equations (20) and (21) with the active controller (24).

The parameters of the identical hyperchaotic Cai systems are chosen as

$$p = 27.5, \quad q = 3, \quad r = 19.3, \quad s = 2.9, \quad \varepsilon = 3.3$$

The initial values for the master system (20) are taken as

$$x_1(0) = 15, \quad x_2(0) = 26, \quad x_3(0) = -10, \quad x_4(0) = 8$$

The initial values for the slave system (21) are taken as

$$y_1(0) = -21, \quad y_2(0) = 17, \quad y_3(0) = -34, \quad y_4(0) = 12$$

The GPS scales are taken as

$$\alpha_1 = -1.8, \quad \alpha_2 = 0.6, \quad \alpha_3 = -3.8, \quad \alpha_4 = 2.7$$

We take the state feedback gains as

$$k_i = 5 \text{ for } i = 1, 2, 3, 4.$$

Figure 5 shows the GPS synchronization of the identical hyperchaotic Cai systems.

Figure 6 shows the time-history of the GPS synchronization errors for the identical hyperchaotic Cai systems.

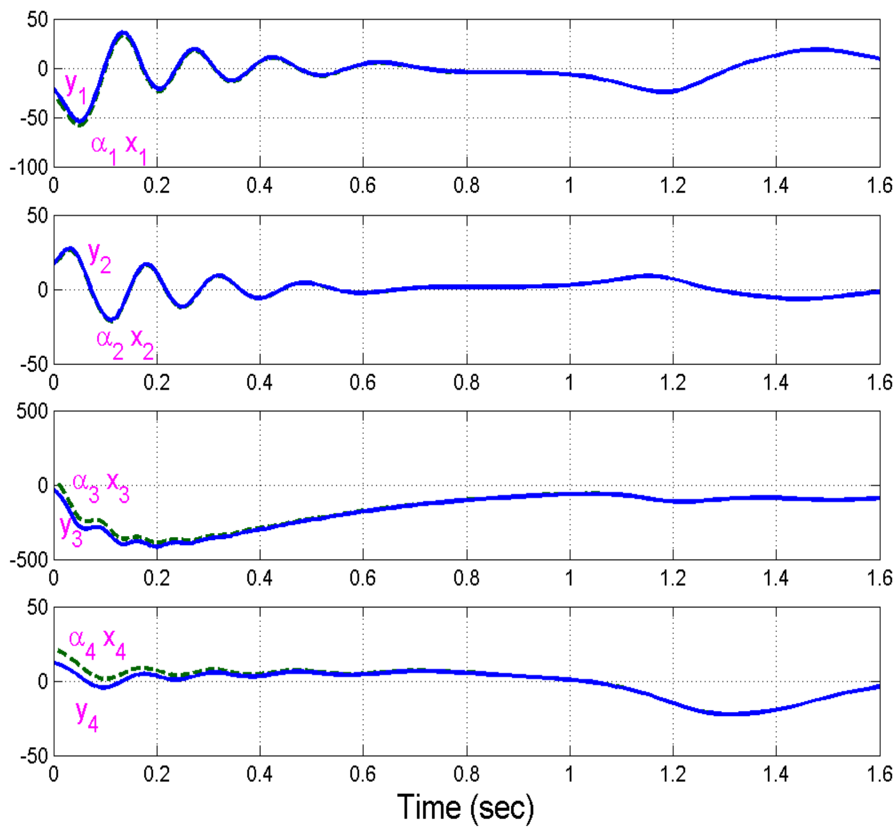

Figure 5. GPS Synchronization of the Identical Hyperchaotic Cai Systems

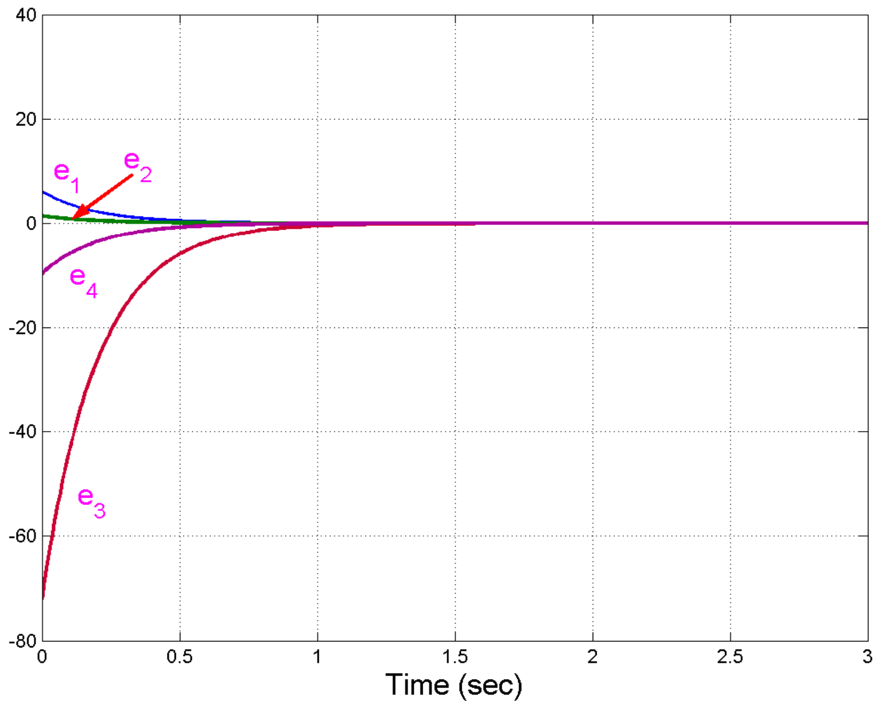

Figure 6. Time History of the GPS Synchronization Error

6. GPS OF HYPERCHAOTIC LÜ AND HYPERCHAOTIC CAI SYSTEMS

6.1 Theoretical Results

In this section, we apply the active nonlinear control method for the generalized projective synchronization (GPS) of hyperchaotic Lü and hyperchaotic Cai systems.

Thus, the master system is described by the hyperchaotic Lü dynamics

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + x_4 \\ \dot{x}_2 &= cx_2 - x_1 x_3 \\ \dot{x}_3 &= -bx_3 + x_1 x_2 \\ \dot{x}_4 &= dx_4 + x_1 x_3\end{aligned} \tag{28}$$

where x_1, x_2, x_3, x_4 are the states and a, b, c, d are constant, positive parameters of the system. The slave system is described by the controlled hyperchaotic Cai dynamics

$$\begin{aligned}\dot{y}_1 &= p(y_2 - y_1) + u_1 \\ \dot{y}_2 &= qy_1 + ry_2 + y_4 - y_1 y_3 + u_2 \\ \dot{y}_3 &= -sy_3 + y_2^2 + u_3 \\ \dot{y}_4 &= -\varepsilon y_1 + u_4\end{aligned} \tag{29}$$

where y_1, y_2, y_3, y_4 are the states, p, q, r, s, ε are positive, constant parameters of the system and u_1, u_2, u_3, u_4 are the active nonlinear controls to be designed.

For the GPS of the systems (28) and (29), the synchronization error e is defined by

$$e_i = y_i - \alpha_i x_i, \quad (i = 1, 2, 3, 4) \tag{30}$$

where the scales $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are real numbers.

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= p(y_2 - y_1) - \alpha_1\left[a(x_2 - x_1) + x_4\right] + u_1 \\ \dot{e}_2 &= qy_1 + ry_2 + y_4 - y_1 y_3 - \alpha_2\left[cx_2 - x_1 x_3\right] + u_2 \\ \dot{e}_3 &= -sy_3 + y_2^2 - \alpha_3\left[-bx_3 + x_1 x_2\right] + u_3 \\ \dot{e}_4 &= -\varepsilon y_1 - \alpha_4\left[dx_4 + x_1 x_3\right] + u_4 \end{aligned} \tag{31}$$

We choose the nonlinear controller as

$$\begin{aligned} u_1 &= -p(y_2 - y_1) + \alpha_1\left[a(x_2 - x_1) + x_4\right] - k_1 e_1 \\ u_2 &= -qy_1 - ry_2 - y_4 + y_1 y_3 + \alpha_2\left[cx_2 - x_1 x_3\right] - k_2 e_2 \\ u_3 &= sy_3 - y_2^2 + \alpha_3\left[-bx_3 + x_1 x_2\right] - k_3 e_3 \\ u_4 &= \varepsilon y_1 + \alpha_4\left[dx_4 + x_1 x_3\right] - k_4 e_4 \end{aligned} \tag{32}$$

where the gains k_1, k_2, k_3, k_4 are positive constants.
Substituting (32) into (31), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3 \\ \dot{e}_4 &= -k_4 e_4 \end{aligned} \tag{33}$$

Next, we prove the following result.

Theorem 3. The active feedback controller (32) achieves global chaos generalized projective synchronization (GPS) between the hyperchaotic Lü system (28) and hyperchaotic Cai system (29).

Proof. We consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{34}$$

which is a positive definite function on R^4.

Differentiating (26) along the trajectories of (33), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2, \tag{35}$$

which is a negative definite function on R^4.

Thus, by Lyapunov stability theory [40], the error dynamics (33) is globally exponentially stable. This completes the proof. ■

6.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the two systems of differential equations (28) and (29) with the active controller (32).
The parameters of the hyperchaotic Lü system are chosen as

$$a = 36, \quad b = 3, \quad c = 20, \quad d = 1.3$$

The parameters of the hyperchaotic Cai system are chosen as

$$p = 27.5, \quad q = 3, \quad r = 19.3, \quad s = 2.9, \quad \varepsilon = 3.3$$

The initial values for the master system (28) are taken as

$$x_1(0) = 12, \quad x_2(0) = 24, \quad x_3(0) = -39, \quad x_4(0) = -17$$

The initial values for the slave system (29) are taken as

$$y_1(0) = -11, \quad y_2(0) = 28, \quad y_3(0) = 7, \quad y_4(0) = 20$$

The GPS scales are taken as

$$\alpha_1 = 2.1, \quad \alpha_2 = -1.5, \quad \alpha_3 = -3.6, \quad \alpha_4 = 0.6$$

We take the state feedback gains as

$$k_i = 5 \text{ for } i = 1, 2, 3, 4$$

Figure 7 shows the GPS synchronization of the non-identical hyperchaotic Lü and hyperchaotic Cai systems.
Figure 8 shows the time-history of the GPS synchronization errors e_1, e_2, e_3, e_4 for the non-identical hyperchaotic Lü and hyperchaotic Cai systems.

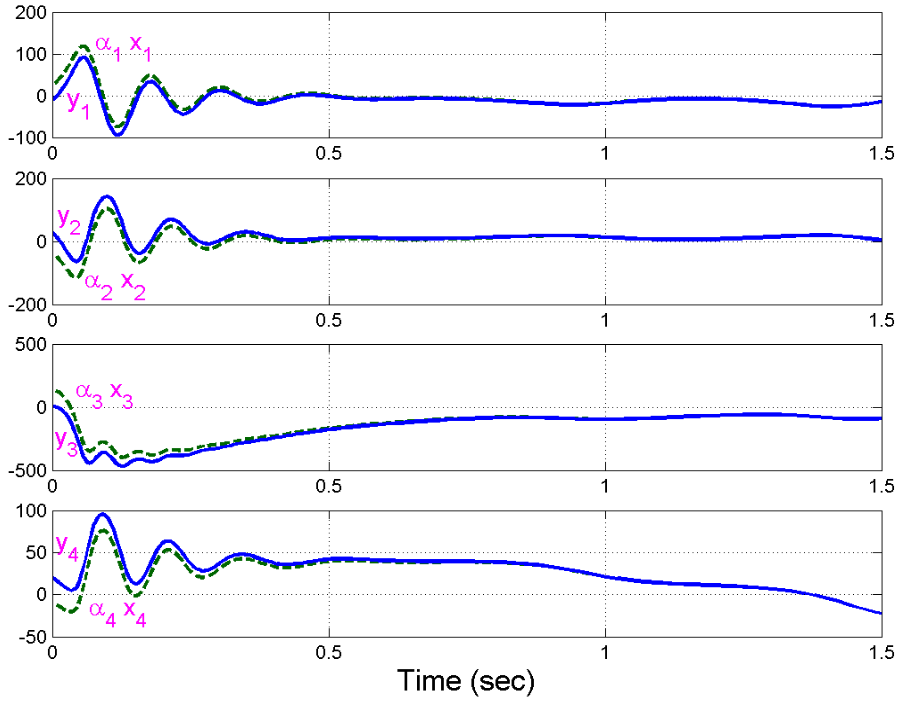

Figure 7. GPS Synchronization of the Hyperchaotic Lü and Hyperchaotic Cai Systems

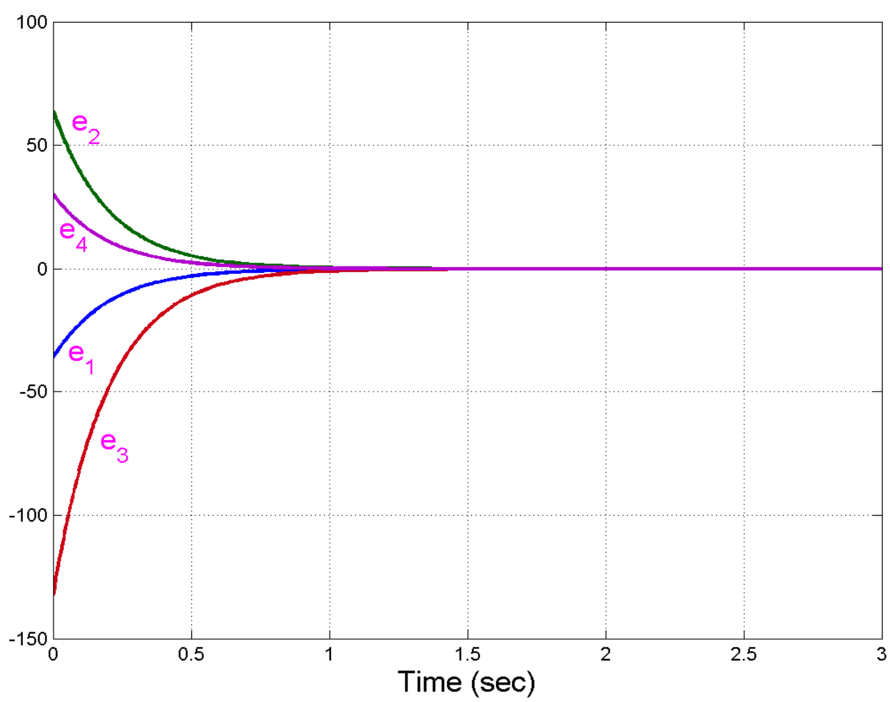

Figure 8. Time History of the GPS Synchronization Error

7. CONCLUSIONS

In this paper, we had derived active control laws for achieving generalized projective synchronization (GPS) of the following pairs of hyperchaotic systems:

(A) Identical Hyperchaotic Lü Systems (2006)
(B) Identical Hyperchaotic Cai systems (2009)
(C) Non-identical Hyperchaotic Lü and Hyperchaotic Cai systems

The synchronization results (GPS) derived in this paper for the hyperchaotic Lü and hyperchaotic Cai systems have been proved using Lyapunov stability theory. Since Lyapunov exponents are not required for these calculations, the proposed active control method is very effective and suitable for achieving GPS of the hyperchaotic systems addressed in this paper. Numerical simulations are shown to demonstrate the effectiveness of the GPS synchronization results derived in this paper for the hyperchaotic Lü and hyperchaotic Cai systems.

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