# ACTIVE CONTROLLER DESIGN FOR THE ANTI-SYNCHRONIZATION OF HYPERCHAOTIC QI AND HYPERCHAOTIC JHA SYSTEMS

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## ABSTRACT

*This paper derives new results for the anti-synchronization of identical hyperchaotic Qi systems (2008), identical hyperchaotic Jha systems (2007) and non-identical hyperchaotic Qi and hyperchaotic Jha systems. Active nonlinear control is the method adopted to achieve the anti-synchronization of the identical and different hyperchaotic Qi and Jha systems. Our stability results derived in this paper are established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the active nonlinear control method is effective and convenient to achieve anti-synchronization of the identical and different hyperchaotic Qi and hyperchaotic Jha systems. Numerical simulations are shown to validate and illustrate the effectiveness of the anti-synchronization results derived in this paper.*

## KEYWORDS

*Chaos, Hyperchaos, Anti-Synchronization, Active Control, Hyperchaotic Qi System, Hyperchaotic Jha System.*

## 1. INTRODUCTION

Chaotic systems are nonlinear dynamical systems that are highly sensitive to initial conditions. This sensitivity is popularly known as the butterfly effect [1]. Chaos is an interesting nonlinear phenomenon and has been extensively and intensively studied in the last two decades [1-23]. Chaos theory has been applied in many scientific disciplines such as Mathematics, Computer Science, Microbiology, Biology, Ecology, Economics, Population Dynamics and Robotics.

Hyperchaotic system is usually defined as a chaotic system with more than one positive Lyapunov exponent. The first hyperchaotic system was discovered by O.E. Rössler ([2], 1979). Since hyperchaotic system has the characteristics of high capacity, high security and high efficiency, it has the potential of broad applications in nonlinear circuits, secure communications, lasers, neural networks, biological systems and so on. Thus, the studies on hyperchaotic systems, viz. control, synchronization and circuit implementation are very challenging problems in the chaos literature [3].

In most of the chaos synchronization approaches, the master-slave or drive-response formalism has been used. If a particular chaotic system is called the master or drive system and another chaotic system is called the slave or response system, then the idea of anti-synchronization is to use the output of the master system to control the slave system so that the states of the slave

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system have the same amplitude but opposite signs as the states of the master system asymptotically.

In 1990, Pecora and Carroll [4] introduced a method to synchronize two identical chaotic systems and showed that it was possible for some chaotic systems to be completely synchronized. From then on, chaos synchronization has been widely explored in a variety of fields including physical [5], chemical [6], ecological [7] systems, secure communications [8-10], etc.

Since the seminal work by Pecora and Carroll [4], a variety of impressive approaches have been proposed for the synchronization of chaotic systems such as OGY method [11], active control method [12-15], adaptive control method [16-20], backstepping method [21-22], sampled-data feedback synchronization method [23], time-delay feedback method [24], sliding mode control method [25-27], etc.

In this paper, we derive new results for the anti-synchronization for identical and different hyperchaotic Qi and Jha systems using active nonlinear control. Explicitly, using active nonlinear control and Lyapunov stability theory, we achieve anti-synchronization for identical hyperchaotic Qi systems ([28], 2008), identical hyperchaotic Jha systems ([29], 2007) and non-identical hyperchaotic Qi and hyperchaotic Jha systems.

This paper has been organized as follows. In Section 2, we give the problem statement and our methodology. In Section 3, we give a description of the hyperchaotic Qi and Jha systems. In Section 4, we discuss the anti-synchronization of two identical hyperchaotic Qi systems. In Section 5, we discuss the anti-synchronization of two identical hyperchaotic Jha systems ([29], 2007). In Section 6, we discuss the anti-synchronization of non-identical hyperchaotic Qi and Jha systems. In Section 7, we summarize the main results of this paper.

## 2. PROBLEM STATEMENT AND OUR METHODOLOGY

Consider the chaotic system described by the dynamics

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in R^n$ is the state of the system, $A$ is the $n \times n$ matrix of the system parameters and $f : R^n \to R^n$ is the nonlinear part of the system. We consider the system (1) as the *master* or *drive* system.

As the *slave* or *response* system, we consider the following chaotic system described by the dynamics

$$\dot{y} = By + g(y) + u \tag{2}$$

where $y \in R^n$ is the state of the system, $B$ is the $n \times n$ matrix of the system parameters, $g : R^n \to R^n$ is the nonlinear part of the system and $u \in R^n$ is the active controller of the slave system.

If $A = B$ and $f = g$, then $x$ and $y$ are the states of two identical chaotic systems. If $A \neq B$ or $f \neq g$, then $x$ and $y$ are the states of two different chaotic systems.

In the active control method, we design a feedback controller $u$, which *anti-synchronizes* the states of the master system (1) and the slave system (2) for all initial conditions $x(0), y(0) \in R^n$.

If we define the *anti-synchronization error* as

$$e = y + x, \tag{3}$$

then the error dynamics is obtained as

$$\dot{e} = By + Ax + g(y) + f(x) + u \tag{4}$$

Thus, the anti-synchronization problem is essentially to find a feedback controller $u$ so as to stabilize the error dynamics (4) for all initial conditions $e(0) \in R^n$.

Hence, we find a feedback controller $u$ so that

$$\lim_{t \to \infty} \| e(t) \| = 0 \text{ for all } e(0) \in R^n \tag{5}$$

We take as a candidate Lyapunov function

$$V(e) = e^T P e, \tag{6}$$

where $P$ is a positive definite matrix.

Note that $V : R^n \to R$ is a positive definite function by construction.

We assume that the parameters of the master and slave system are known and that the states of both systems (1) and (2) are measurable.

If we find a feedback controller $u$ so that

$$\dot{V}(e) = -e^T Q e, \tag{7}$$

where $Q$ is a positive definite matrix, then $\dot{V} : R^n \to R$ is a negative definite function.

Thus, by Lyapunov stability theory [30], the error dynamics (4) is globally exponentially stable and hence the condition (5) will be satisfied. Hence, the states of the master system (1) and the slave system (2) will be globally and exponentially anti-synchronized.

## 3. SYSTEMS DESCRIPTION

In this section, we describe the hyperchaotic systems studied in this paper, *viz.* hyperchaotic Qi system ([28], 2008) and hyperchaotic Jha system ([29], 2007).

The hyperchaotic Qi system ([28], 2008) is described by the dynamics

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 &= b(x_1 + x_2) - x_1 x_3 \\ \dot{x}_3 &= -c x_3 - \varepsilon x_4 + x_1 x_2 \\ \dot{x}_4 &= -d x_4 + f x_3 + x_1 x_2\end{aligned} \tag{8}$$

where $x_1, x_2, x_3, x_4$ are the states and $a, b, c, d, \varepsilon, f$ are constant, positive parameters of the system.

The Qi system (8) exhibits a hyperchaotic attractor (see Figure 1), when the parameter values are taken as

$$a = 50, \quad b = 24, \quad c = 13, \quad d = 8, \quad \varepsilon = 33, \quad f = 30 \tag{9}$$

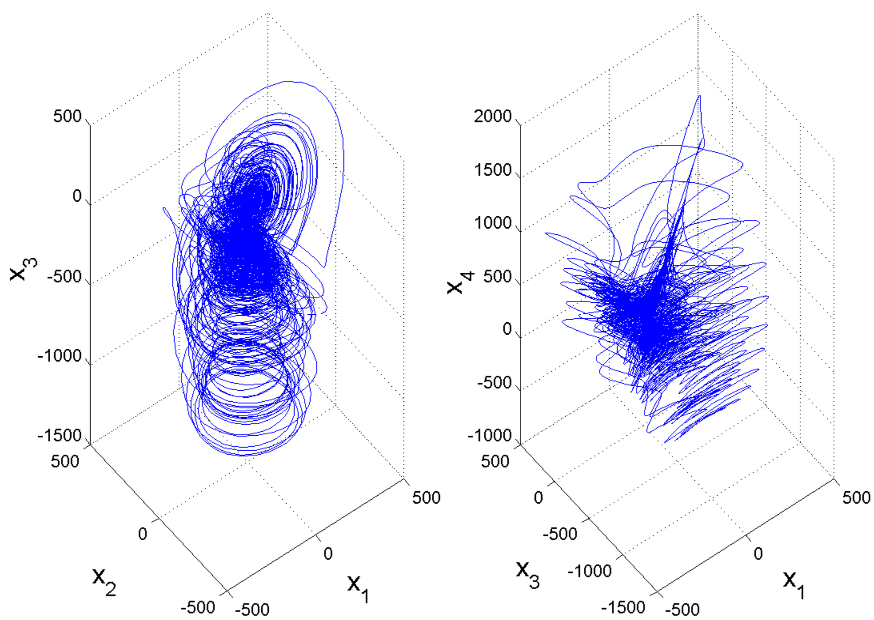


Figure 1. The Phase Portrait of the Hyperchaotic Qi System

The hyperchaotic Jha system ([29], 2007) is described by the dynamics

$$\begin{aligned}\dot{x}_1 &= \alpha(x_2 - x_1) + x_4 \\ \dot{x}_2 &= -x_1 x_3 + \beta x_1 - x_2 \\ \dot{x}_3 &= x_1 x_2 - \gamma x_3 \\ \dot{x}_4 &= -x_1 x_3 + \delta x_4\end{aligned} \tag{10}$$

where $x_1, x_2, x_3, x_4$ are the states and $\alpha, \beta, \gamma, \delta$ are constant, positive parameters of the system.

The Jha dynamics (10) exhibits a hyperchaotic attractor (see Figure 2), when the parameter values are taken as

$$\alpha = 10, \quad \beta = 28, \quad \gamma = 8/3, \quad \delta = 1.3 \tag{11}$$

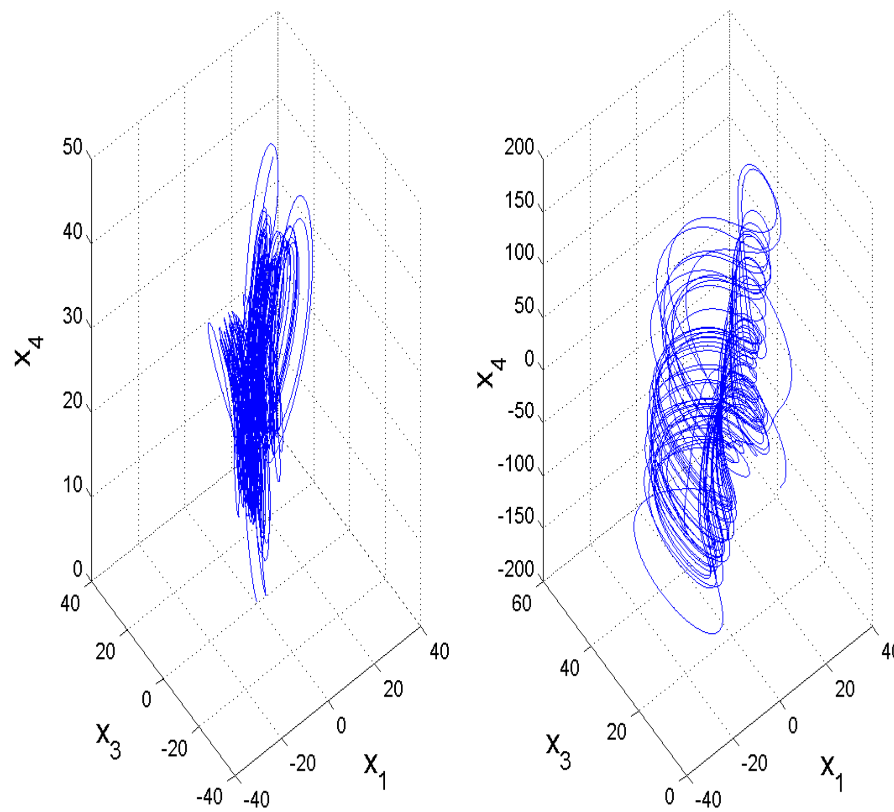


Figure 2. The Phase Portrait of the Hyperchaotic Jha System

## 4. ANTI-SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC QI SYSTEMS BY ACTIVE CONTROL

### 4.1 Theoretical Results

In this section, we apply the active nonlinear control method for the anti-synchronization of two identical hyperchaotic Qi systems (2008).

Thus, the master system is described by the hyperchaotic Qi dynamics

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 &= b(x_1 + x_2) - x_1 x_3 \\ \dot{x}_3 &= -c x_3 - \varepsilon x_4 + x_1 x_2 \\ \dot{x}_4 &= -d x_4 + f x_3 + x_1 x_2\end{aligned} \tag{12}$$

where $x_1, x_2, x_3, x_4$ are the state variables and $a, b, c, d, \varepsilon, f$ are positive parameters of the system.

The slave system is described by the controlled hyperchaotic Qi dynamics

$$\begin{aligned}\dot{y}_1 &= a(y_2 - y_1) + y_2 y_3 + u_1 \\ \dot{y}_2 &= b(y_1 + y_2) - y_1 y_3 + u_2 \\ \dot{y}_3 &= -c y_3 - \varepsilon y_4 + y_1 y_2 + u_3 \\ \dot{y}_4 &= -d y_4 + f y_3 + y_1 y_2 + u_4\end{aligned} \tag{13}$$

where $y_1, y_2, y_3, y_4$ are the state variables and $u_1, u_2, u_3, u_4$ are the active nonlinear controls to be designed.

The anti-synchronization error $e$ is defined by

$$e_i = y_i + x_i, \quad (i = 1, 2, 3, 4) \tag{14}$$

The error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= a(e_2 - e_1) + y_2 y_3 + x_2 x_3 + u_1 \\ \dot{e}_2 &= b(e_1 + e_2) - y_1 y_3 - x_1 x_3 + u_2 \\ \dot{e}_3 &= -c e_3 - \varepsilon e_4 + y_1 y_2 + x_1 x_2 + u_3 \\ \dot{e}_4 &= -d e_4 + f e_3 + y_1 y_2 + x_1 x_2 + u_4\end{aligned} \tag{15}$$

We choose the active nonlinear controller as

$$\begin{aligned}u_1 &= -a(e_2 - e_1) - y_2 y_3 - x_2 x_3 - k_1 e_1 \\ u_2 &= -b(e_1 + e_2) + y_1 y_3 + x_1 x_3 - k_2 e_2 \\ u_3 &= c e_3 + \varepsilon e_4 - y_1 y_2 - x_1 x_2 - k_3 e_3 \\ u_4 &= d e_4 - f e_3 - y_1 y_2 - x_1 x_2 - k_4 e_4\end{aligned} \tag{16}$$

where the gains $k_i, \ (i = 1, 2, 3, 4)$ are positive constants.

Substituting (16) into (15), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3 \\ \dot{e}_4 &= -k_4 e_4 \end{aligned} \tag{17}$$

Next, we prove the following result.

**Theorem 4.1.** The identical hyperchaotic Qi systems (12) and (13) are globally and exponentially anti-synchronized for all initial conditions with the active nonlinear controller defined by (16).

**Proof.** We consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{18}$$

which is a positive definite function on $R^4$.

Differentiating (18) along the trajectories of (17), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \tag{19}$$

which is a negative definite function on $R^4$.

Thus, by Lyapunov stability theory [30], the error dynamics (17) is globally exponentially stable.

Hence, the identical hyperchaotic Qi systems (12) and (13) are globally and exponentially anti-synchronized for all initial conditions with the active controller defined by (16).

This completes the proof. ■

## 4.2 Numerical Results

For simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-8}$ is used to solve the differential equations (12) and (13) with the active nonlinear controller (16).
The feedback gains used in the equation (16) are chosen as

$$k_1 = 5, \quad k_2 = 5, \quad k_3 = 5, \quad k_4 = 5$$

The parameters of the hyperchaotic Qi systems are chosen as

$$a = 50, \quad b = 24, \quad c = 13, \quad d = 8, \quad \varepsilon = 33, \quad f = 30$$

The initial conditions of the master system (12) are chosen as

$$x_1(0) = 8, \quad x_2(0) = 26, \quad x_3(0) = -20, \quad x_4(0) = -15$$

The initial conditions of the slave system (13) are chosen as

$$y_1(0) = 30, \quad y_2(0) = -5, \quad y_3(0) = 15, \quad y_4(0) = -22$$

Figure 3 shows the anti-synchronization of the identical hyperchaotic Qi systems.

Figure 4 shows the time-history of the anti-synchronization errors $e_1, e_2, e_3, e_4$.

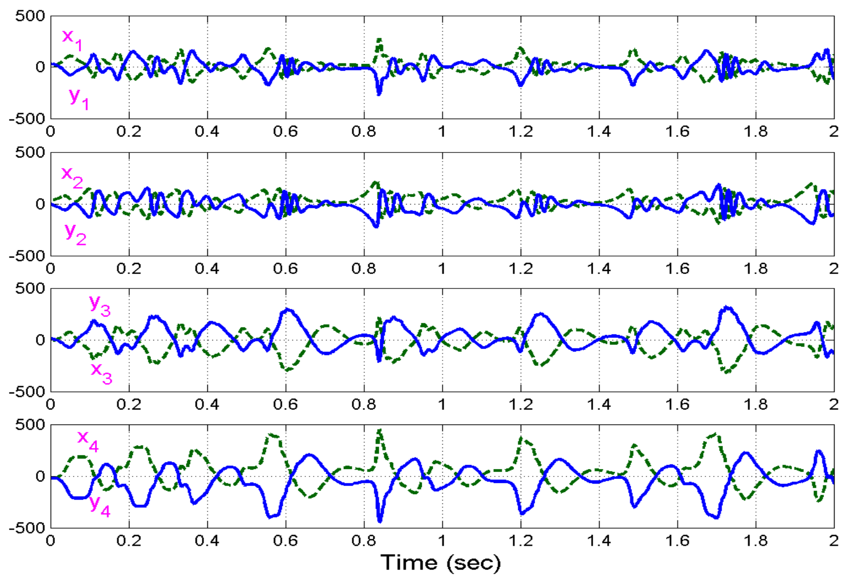


Figure 3. Anti-Synchronization of the Identical Hyperchaotic Qi Systems

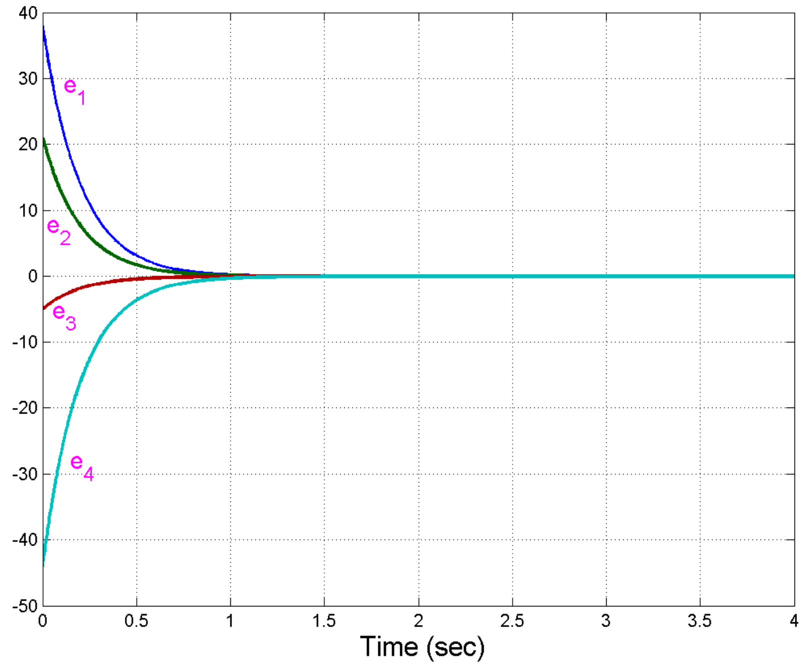


Figure 4. Time-History of the Anti-Synchronization Error

## 5. Anti- Synchronization of Identical Hyperchaotic Jha Systems by Active Control

### 5.1 Theoretical Results

In this section, we apply the active nonlinear control method for the anti-synchronization of two identical hyperchaotic Jha systems (2007). Thus, the master system is described by the hyperchaotic Jha dynamics

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) + x_4 \\ \dot{x}_2 &= -x_1 x_3 + \beta x_1 - x_2 \\ \dot{x}_3 &= x_1 x_2 - \gamma x_3 \\ \dot{x}_4 &= -x_1 x_3 + \delta x_4 \end{aligned} \tag{20}$$

where $x_1, x_2, x_3, x_4$ are the state variables and $\alpha, \beta, \gamma, \delta$ are positive parameters of the system.

The slave system is described by the controlled hyperchaotic Jha dynamics

$$\begin{aligned}\dot{y}_1 &= \alpha(y_2 - y_1) + y_4 + u_1 \\ \dot{y}_2 &= -y_1 y_3 + \beta y_1 - y_2 + u_2 \\ \dot{y}_3 &= y_1 y_2 - \gamma y_3 + u_3 \\ \dot{y}_4 &= -y_1 y_3 + \delta y_4 + u_4\end{aligned} \tag{21}$$

where $y_1, y_2, y_3, y_4$ are the state variables and $u_1, u_2, u_3, u_4$ are the active nonlinear controls to be designed.

The anti-synchronization error $e$ is defined by

$$e_i = y_i + x_i, \quad (i = 1, 2, 3, 4) \tag{22}$$

The error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= \alpha(e_2 - e_1) + e_4 + u_1 \\ \dot{e}_2 &= \beta e_1 - e_2 - y_1 y_3 - x_1 x_3 + u_2 \\ \dot{e}_3 &= -\gamma e_3 + y_1 y_2 + x_1 x_2 + u_3 \\ \dot{e}_4 &= \delta e_4 - y_1 y_3 - x_1 x_3 + u_4\end{aligned} \tag{23}$$

We choose the active nonlinear controller as

$$\begin{aligned}u_1 &= -\alpha(e_2 - e_1) - e_4 - k_1 e_1 \\ u_2 &= -\beta e_1 + e_2 + y_1 y_3 + x_1 x_3 - k_2 e_2 \\ u_3 &= \gamma e_3 - y_1 y_2 - x_1 x_2 - k_3 e_3 \\ u_4 &= -\delta e_4 + y_1 y_3 + x_1 x_3 - k_4 e_4\end{aligned} \tag{24}$$

where the gains $k_i, \ (i = 1, 2, 3, 4)$ are positive constants.

Substituting (24) into (23), the error dynamics simplifies to

$$\begin{aligned}\dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3 \\ \dot{e}_4 &= -k_4 e_4\end{aligned} \tag{25}$$

Next, we prove the following result.

**Theorem 5.1.** The identical hyperchaotic Jha systems (20) and (21) are globally and exponentially anti-synchronized for all initial conditions with the active nonlinear controller defined by (24).

**Proof.** We consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{26}$$

which is a positive definite function on $R^4$.

Differentiating (26) along the trajectories of (25), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \tag{27}$$

which is a negative definite function on $R^4$.

Thus, by Lyapunov stability theory [30], the error dynamics (25) is globally exponentially stable.

Hence, the identical hyperchaotic Jha systems (20) and (21) are globally and exponentially anti-synchronized for all initial conditions with the nonlinear controller defined by (24).

This completes the proof. ■

## 5.2 Numerical Results

For simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-8}$ is used to solve the differential equations (20) and (21) with the active nonlinear controller (24).
The feedback gains used in the equation (24) are chosen as

$$k_1 = 5, \quad k_2 = 5, \quad k_3 = 5, \quad k_4 = 5$$

The parameters of the hyperchaotic Jha systems are chosen as

$$\alpha = 10, \quad \beta = 28, \quad \gamma = 8/3, \quad \delta = 1.3$$

The initial conditions of the master system (20) are chosen as

$$x_1(0) = 8, \quad x_2(0) = 20, \quad x_3(0) = 18, \quad x_4(0) = 4$$

The initial conditions of the slave system (21) are chosen as

$$y_1(0) = -18, \quad y_2(0) = 24, \quad y_3(0) = -11, \quad y_4(0) = 22$$

Figure 5 shows the anti-synchronization of the identical hyperchaotic Jha systems.

Figure 6 shows the time-history of the anti-synchronization errors $e_1, e_2, e_3, e_4$.

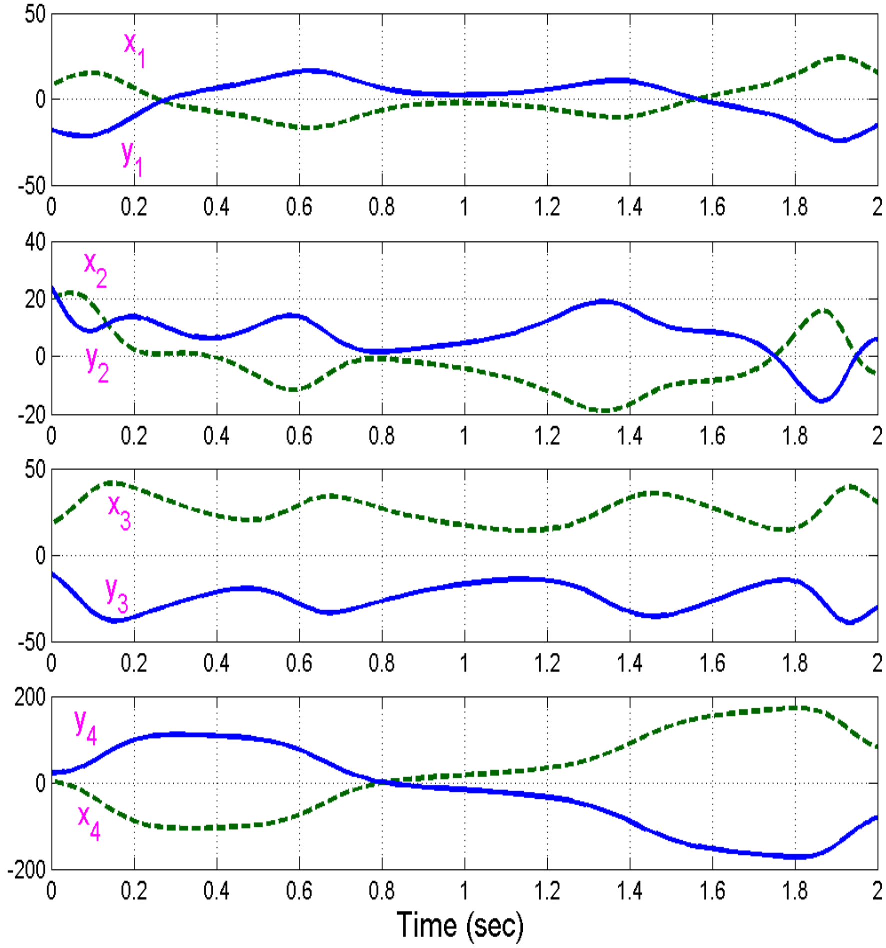


Figure 5. Anti-Synchronization of the Identical Hyperchaotic Jha Systems

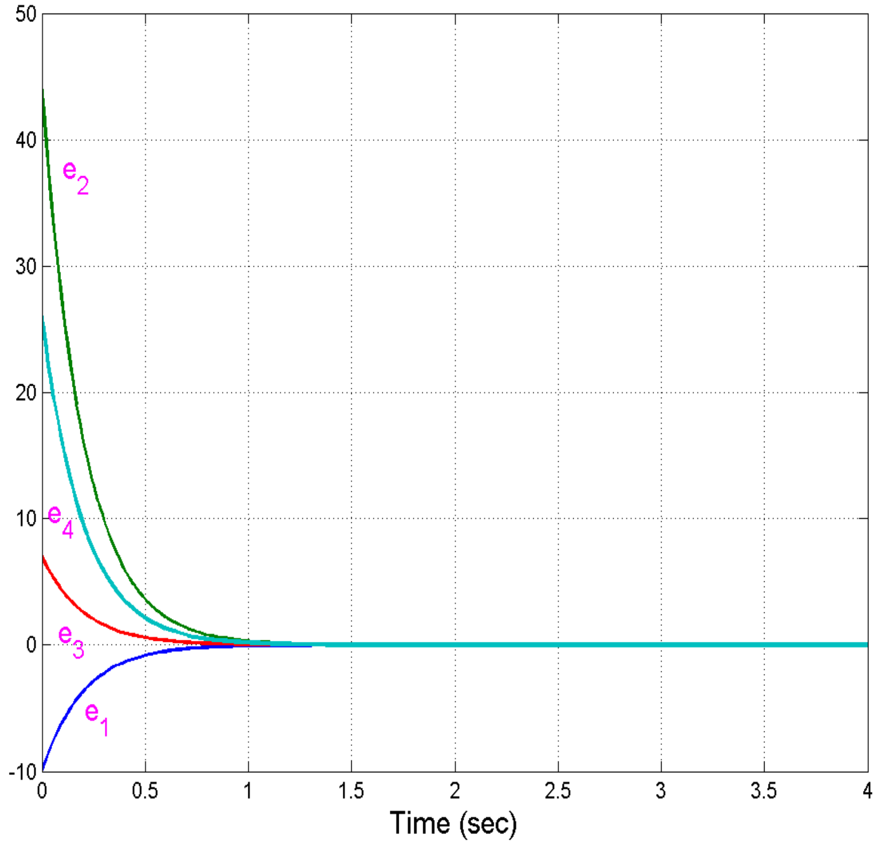


Figure 6. Time-History of the Anti-Synchronization Error

## 6. Anti- Synchronization of Non-Identical Hyperchaotic Qi and Hyperchaotic Jha Systems by Active Control

### 6.1 Theoretical Results

In this section, we apply the active nonlinear control method for the anti-synchronization of the non-identical hyperchaotic Qi system (2008) and hyperchaotic Jha system (2007).

Thus, the master system is described by the hyperchaotic Qi dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 &= b(x_1 + x_2) - x_1 x_3 \\ \dot{x}_3 &= -c x_3 - \varepsilon x_4 + x_1 x_2 \\ \dot{x}_4 &= -d x_4 + f x_3 + x_1 x_2 \end{aligned} \tag{28}$$

where $x_1, x_2, x_3, x_4$ are the state variables and $a, b, c, d, \varepsilon, f$ are positive parameters of the system.

The slave system is described by the controlled hyperchaotic Jha dynamics

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + y_4 + u_1 \\ \dot{y}_2 &= -y_1 y_3 + \beta y_1 - y_2 + u_2 \\ \dot{y}_3 &= y_1 y_2 - \gamma y_3 + u_3 \\ \dot{y}_4 &= -y_1 y_3 + \delta y_4 + u_4 \end{aligned} \tag{29}$$

where $y_1, y_2, y_3, y_4$ are the state variables, $\alpha, \beta, \gamma, \delta$ are positive parameters and $u_1, u_2, u_3, u_4$ are the active nonlinear controls to be designed.

The anti-synchronization error $e$ is defined by

$$e_i = y_i + x_i, \quad (i = 1, 2, 3, 4) \tag{30}$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= \alpha(y_2 - y_1) + y_4 + a(x_2 - x_1) + x_2 x_3 + u_1 \\ \dot{e}_2 &= \beta y_1 - y_2 + b(x_1 + x_2) - y_1 y_3 - x_1 x_3 + u_2 \\ \dot{e}_3 &= -\gamma y_3 - c x_3 - \varepsilon x_4 + y_1 y_2 + x_1 x_2 + u_3 \\ \dot{e}_4 &= \delta y_4 - d x_4 + f x_3 - y_1 y_3 + x_1 x_2 + u_4 \end{aligned} \tag{31}$$

We choose the active nonlinear controller as

$$\begin{aligned} u_1 &= -\alpha(y_2 - y_1) - y_4 - a(x_2 - x_1) - x_2 x_3 - k_1 e_1 \\ u_2 &= -\beta y_1 + y_2 - b(x_1 + x_2) + y_1 y_3 + x_1 x_3 - k_2 e_2 \\ u_3 &= \gamma y_3 + c x_3 + \varepsilon x_4 - y_1 y_2 - x_1 x_2 - k_3 e_3 \\ u_4 &= -\delta y_4 + d x_4 - f x_3 + y_1 y_3 - x_1 x_2 - k_4 e_4 \end{aligned} \tag{32}$$

where the gains $k_i, \ (i = 1, 2, 3, 4)$ are positive constants.

Substituting (32) into (31), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3 \\ \dot{e}_4 &= -k_4 e_4 \end{aligned} \tag{33}$$

Next, we prove the following result.

**Theorem 6.1.** The non-identical hyperchaotic Qi system (28) and hyperchaotic Jha system (29) are globally and exponentially anti-synchronized for all initial conditions with the active nonlinear controller defined by (32).

**Proof.** We consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{34}$$

which is a positive definite function on $R^4$.

Differentiating (34) along the trajectories of (33), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \tag{35}$$

which is a negative definite function on $R^4$.

Thus, by Lyapunov stability theory [30], the error dynamics (33) is globally exponentially stable.

Hence, the non- identical hyperchaotic Qi system (28) and hyperchaotic Jha system (29) are globally and exponentially anti-synchronized for all initial conditions with the active nonlinear controller defined by (32). This completes the proof. ■

## 6.2 Numerical Results

For simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-8}$ is used to solve the differential equations (28) and (29) with the active nonlinear controller (32).
The feedback gains used in the equation (32) are chosen as

$$k_1 = 5, \;\; k_2 = 5, \;\; k_3 = 5, \;\; k_4 = 5$$

The parameters of the hyperchaotic Qi systems are chosen as

$$a = 50, \;\; b = 24, \;\; c = 13, \;\; d = 8, \;\; \varepsilon = 33, \;\; f = 30$$

The parameters of the hyperchaotic Jha systems are chosen as

$$\alpha = 10, \;\; \beta = 28, \;\; \gamma = 8/3, \;\; \delta = 1.3$$

The initial conditions of the master system (28) are chosen as

$$x_1(0) = -25, \;\; x_2(0) = 15, \;\; x_3(0) = 18, \;\; x_4(0) = -6$$

The initial conditions of the slave system (29) are chosen as

$$y_1(0) = 8, \;\; y_2(0) = 24, \;\; y_3(0) = -7, \;\; y_4(0) = 33$$

Figure 7 shows the anti-synchronization of the hyperchaotic Qi and hyperchaotic Jha systems.

Figure 8 shows the time-history of the synchronization errors $e_1, e_2, e_3, e_4$.

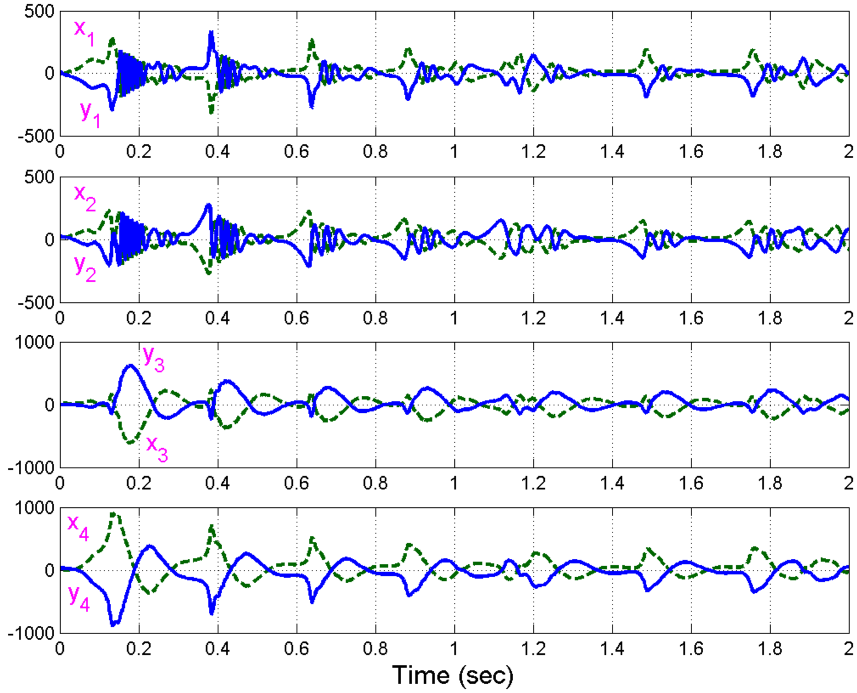


Figure 7. Anti-Synchronization of Hyperchaotic Qi and Hyperchaotic Jha Systems

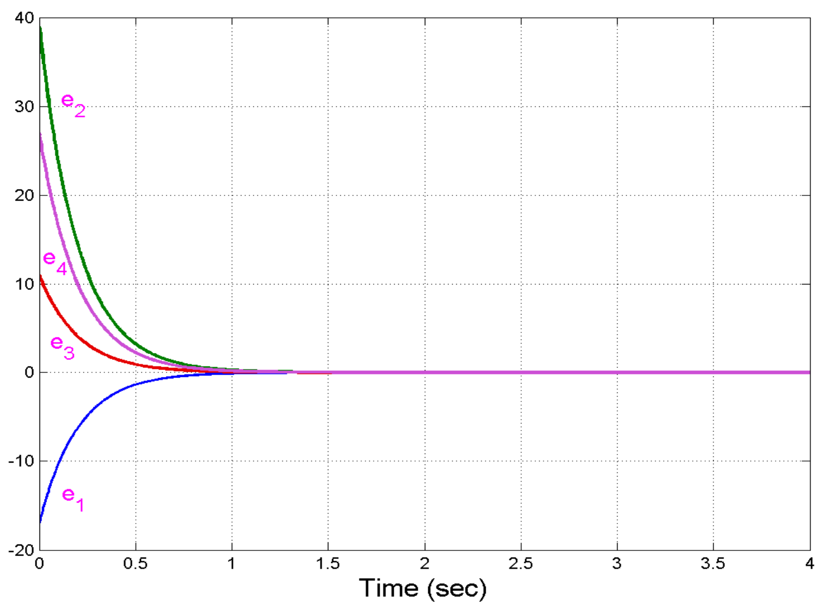


Figure 8. Time-History of the Anti-Synchronization Error

## 7. CONCLUSIONS

In this paper, we have used active nonlinear control method and Lyapunov stability theory to achieve anti-synchronization for the identical hyperchaotic Qi systems (2008), identical hyperchaotic Jha systems (2007) and non-identical hyperchaotic Qi and hyperchaotic Jha systems. Since the Lyapunov exponents are not required for these calculations, the active nonlinear control method is very effective and convenient to achieve anti-synchronization for the three master-slave pairs of hyperchaotic systems studied in this paper. Numerical simulations have been shown to illustrate the effectiveness of the anti-synchronization schemes derived in this paper for the hyperchaotic Qi and hyperchaotic Jha systems.

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