PERFORMANCE ASSESSMENT OF THE DESIGNED CONTROLLERS FOR THREE-TANK BENCHMARK SYSTEM

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ABSTRACT

Three-tank benchmark system is a nonlinear multivariable process that is a good prototype of chemical industrial processes. The open loop process is unstable inherently, thus a proper controller must be applied to stabilize the system as well as reaching good dynamic response. In this paper, several controllers have been proposed using pole-placement with integral control. Then, the best controller is selected considering deterministic and stochastic performances. Deterministic performance was evaluated using step test on closed loop. Minimum variance (MV) index was used for stochastic performance assessment. Multivariate time series (MTS) is cached from sampled outputs of the simulated system in closed loop. Then, max-likelihood method is used to fit an ARMAV model to the MTS. The obtained model is used for determining MV index. This approach can be used for determining the efficiency of loop after implementation.

KEYWORDS

Process control, Three-tank benchmark, pole-placement integral control, performance assessment, minimum variance index

1. INTRODUCTION

In modern chemical process industries, there has been an ever increasing push to optimize the controllers to help fulfill the ever increasing demands for high quality products. Most chemical plants are multivariable processes and have to be controlled in order to perform well.

Controlling a process can be classified into two main subjects, regulatory control and tracking control. In regulatory control the controller's objective is to regulate the value of system output on a constant value (usually zero). Tracking control's objective is to make the output follow a reference input. Regulatory controllers' performance is determined via good stability margin, not having steady state error, good disturbance and noise rejection, and low output's variance. In addition to regulatory controllers, tracking controllers should have low rise time, low overshoot, and low settling time for good performance.

In this paper three-tank benchmark system is considered to be controlled. The three-tank system laboratory model can be viewed as a prototype of many industrial applications in process industry, such as chemical and petrochemical plants, oil and gas systems. The typical control issue involved in the system is how to keep the desired liquid level in each tank. The principle scheme of the model is shown in Figure 1. The basic apparatus consists of three plexiglass tanks numbered from left to right as 1, 3 and 2, with cross sectional area of S.

DOI : 10.5121/ijics.2011.1201

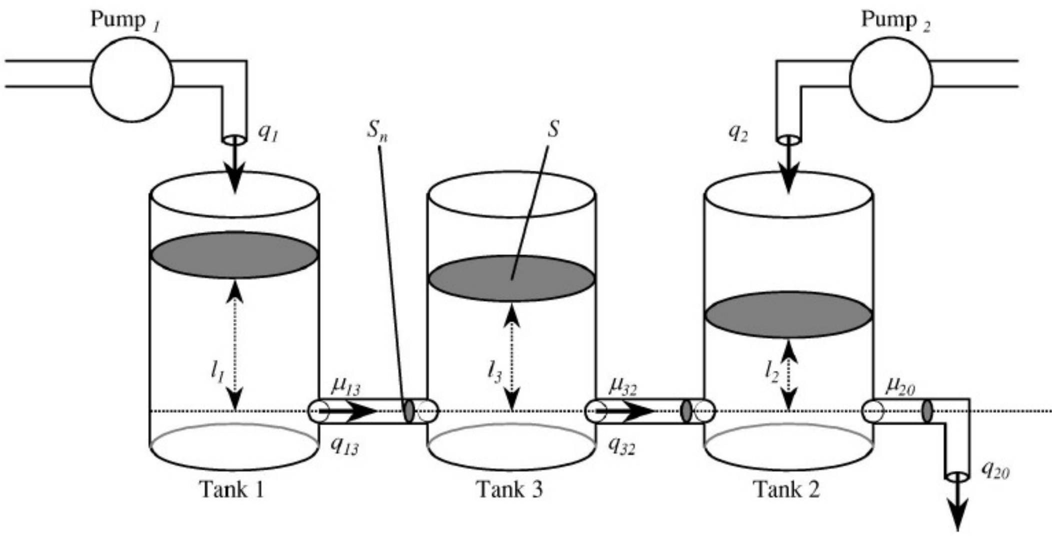


Figure 1. The principle scheme of the three-tank benchmark model

These are connected serially with each other by cylindrical pipes with cross sectional area of S_n. μ_{13} and μ_{32} are the outflow coefficients of the pipes between tanks 1 and 3, and tanks 3 and 2 respectively. μ_{20} also denotes the outflow coefficient from tank 2. Liquid, which is collected in a reservoir, is pumped into the tanks 1 and 2 to maintain their levels. The level in the tank 3 is a response that is uncontrollable but it affects the level in the two end tanks. Each tank is equipped with a static pressure sensor, which gives a voltage output proportional to the level of liquid in the tank [1,2].

l_{max} denotes the highest possible liquid level in tanks. In case the liquid level of 1 and 2 exceeds this value the corresponding pump will be switched off automatically. q_1 and q_2 are the flow rates of pump 1 and 2. Two variable speed pumps driven by DC motor are used in this apparatus. These pumps are designed to give an accurate flow per rotation. Thus, the flow rate provided by each pump is proportional to the voltage applied to its DC motor. q_{1max} and $q_{2,max}$ are the maximum flow rates of pump 1 and 2 respectively [1,2].

This article has two main phases. The first phase is designing, tuning and implementation of the control strategies. Choosing proper controller is one of the main concerns of control engineers. However, after some time in operation, changes in the characteristics of the material being used, modifications of operation strategy and changes in the status of the plant equipment (aging, wear, fouling, component modifications, etc.) may lead to the degradation of control performance. Problems can arise even in well-designed control loops for a variety of reasons, ranging from a need for re-tuning due to the changes mentioned, difficulties with the sensors, or actuator operation, which can occur in an unpredictable fashion. The supervision of the control loops and the early detection of performance deterioration is an important part of controlling problems. This task has traditionally been made by the plant personnel, i.e., maintenance and control staff. However, the necessity in the process industries during the last decades has led to a reduction of personnel [3]. This makes automation necessary. Therefore, our second phase in this work is to determine an index which can automatically evaluate how well the designed controllers in the first phase perform in the presence of unpredicted changes. Even, this index can be used to monitor the controller after implementation. These two phases, designing controller and performance assessment of loop are two main concerns of all systems, especially in industrial environments. Control performance monitoring and assessment is an important technology to keep highly efficient operation of automation systems in production plants. The main objective of control performance monitoring is to provide online automated procedures that evaluate the performance of the control system. This should help detect and avoid performance deterioration owing to variations in the process and operation. This is achieved by indicating how far a control system is operating from its optimum. The method used in this paper solves this problem without any need to a prior knowledge.

CPM includes two main monitoring classes. Usually, CPM is concerned with the assessment of the output variance due to unmeasured, stochastic disturbances, which are further assumed to be generated from a dynamic system driven by noise. For this reason, this class of CPM methods is referred to as stochastic performance monitoring. Whereas these methods provide an important aspect of the controller performance, they do not bring up any information about the traditionally concerned performance, such as step changes in set-point or disturbance variables, settling time, decay ratio and stability margin of the control system. This class of CPM techniques is known as deterministic performance monitoring [3]. In this paper, both deterministic and stochastic criteria have been used for deciding the best controller.

As mentioned, second phase of this article is to select the best controller among those that have been designed in the first phase. For this selection there is need for a criterion to test the controllers. In many applications, it is useful to combine stochastic and deterministic criteria. Rise time, over shoot, settling time, and steady state error are some candidates for deterministic criterion. In stochastic area, Minimum Variance (MV) at output is one of the main criteria. More than 60% industrial loops use MV criteria for deciding the efficiency of their loop. The widespread use of the variance as performance criterion is due to the fact that it typically represents the product-quality consistency. The reduction of variances of many quality variables not only implies improved product quality but also makes it possible to operate near the constraints to increase throughput, reduce energy consumption and save raw materials.

An index for MV is needed which reveals the capability of controllers and loops in decreasing variance in outputs. There are some methods in SISO system (one input-one output), but in MIMO systems (multi input-multi output) determining the minimum reachable variance (optimum performance) is hard task. In this study, the minimum variance in output is compared with the minimum reachable variance by modeling multivariate time series. This can help us to determine the performance of system without any knowledge of system and controller automatically. With this method, it can be determined how well controller and loop works and how well it performs in decreasing noise variance in output. A recent research [4] in Canada reveals that only 20% of the control loops worked well and decreased process variability. The reasons why the performance was poor are bad tuning (30%) and valve problems (30%). The remaining 20% of the controllers functioned poorly for a variety of reasons, such as sensor problems, bad choice of sampling rates and poor or non-existing anti-aliasing filters. Similar observations are given in this study, where it is claimed that 30% of installed process controllers operated in manual mode, 20% of the loops use default parameters set by the controller manufacturer (so-called "factory tuning"), and 30% of the loops showed poor performance because of equipment problems in valves and sensors. Therefore, it is important to decide how well loop is working without opening loop or stopping system routing works, this method can decide best controller along with.

2. TRACKING SYSTEM DESIGN

Using state feedback and integral control, the control system maintains tracking property and can reject constant disturbances. Consider the controllable system given below:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

In order to maintain controllability, the number of output variables that can track a reference input vector cannot exceed the number of control inputs. Though we can rewrite the output as follow:

$$y(t) = Cx(t) = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} \tag{2}$$

Where $y_1(t) \in R^h \, (h \leq m)$ is the vector of outputs that have to track the reference input vector *r(t)*. The controller task is to make $y_1(t)$ track the reference vector *r(t)* so that we have in steady state:

$$\lim_{t \to +\infty} y_1(t) = r(t) \tag{3}$$

For this purpose we define the integral state *q*(t) as follow:

$$\dot{q}(t) = r(t) - y_1(t) = r(t) - C_1 x(t) \tag{4}$$

Augmenting the integral state with the state equations of the systems we get:

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{q}(t) \end{bmatrix} &= \begin{bmatrix} A & 0 \\ -C_1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ I \end{bmatrix} r(t) \\ y(t) &= \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ q(t) \end{bmatrix} \end{aligned} \tag{5}$$

Now the designing problem is to determine the control signal u(t) by state feedback, so that it makes the augmented system stable. The necessary and sufficient condition for the existence of such control signal is the controllability of the augmented system. For complete state controllability, the following matrix must be of full rank:

$$\begin{bmatrix} B & A \\ 0 & -C_1 \end{bmatrix} \tag{6}$$

If this condition holds, then we just have to determine the state feedback gain so that the augmented system remains stable and have a good response [5]. From stability in steady state we will have:

$$\lim_{t \to +\infty} \dot{q}(t) = \lim_{t \to +\infty} r(t) - C_1 x(t) = o$$

So we have:

$$\lim_{t \to +\infty} y_1(t) = r(t) \tag{7}$$

Therefore, the tracking property is satisfied.

Writing the mass balance equations for the three-tank system we will have:

$$S\frac{dl_1(t)}{dt} = q_1(t) - q_{13}(t)$$

$$S\frac{dl_2(t)}{dt} = q_2(t) + q_{32}(t) - q_{20}(t) \tag{8}$$

$$S\frac{dl_3(t)}{dt} = q_{13}(t) - q_{32}(t)$$

Where q_{13}, q_{32} represent the flow rates between tanks 1 and 3, and tanks 3 and 2 respectively. q_{20} also represents the outflow rate from tank 2. These flow rates are unmeasured but can be determined using the Torriceli-rule [2]:

$$q_{13}(t) = \mu_{13} S_n \operatorname{sgn}[l_1(t) - l_3(t)] \times \sqrt{2g|l_1(t) - l_3(t)|}$$

$$q_{32}(t) = \mu_{32} S_n \operatorname{sgn}[l_3(t) - l_2(t)] \times \sqrt{2g|l_3(t) - l_2(t)|} \tag{9}$$

$$q_{20}(t) = \mu_{20} S_n \sqrt{2g l_2(t)}$$

The numerical values of the simulated plant are listed in Table 1.

Table 1. The numerical values of the simulated plant.

Variable	**Symbol**	**Value**
Tank cross sectional area	S	0.0154 m^2
Inter tank cross sectional area	S_n	5×10^{-5} m^2
Outflow coefficient	$\mu_{13}=\mu_{32}$ μ_{20}	0.5 0.6
Maximum flow rate	$q_{i,max}$	1.5×10^{-5} m^3s^{-1}
Maximum level	$l_{i,max}$	1.4 m

Using equations (8) and (9) we can have the nonlinear state space model of the system as follow:

$$\begin{aligned} \frac{d\bar{x}(t)}{dt} &= f(\bar{x}(t), \bar{u}(t)) \\ \bar{y}(t) &= \bar{x}(t) \end{aligned} \tag{10}$$

Where $\bar{y}(t)$=[l_1 l_2 l_3 $]^T$ is the output vector, and $\bar{u}(t)$=[q_1 q_2 $]^T$ is the input vector. State feedback is used to control the system inputs (q_1 and q_2) in closed loop such that the liquid levels in the corresponding tanks (l_1 and l_2) follow a reference vector $r(t)$. The third output of the process, level l_3 in the middle tank, is uncontrollable. The purpose is to control the system around an operating point ($\bar{u}_0$, $\bar{y}_0$), which is fixed to

$$\bar{u}_0 = [0.35\ 0.375]^T 10^{-4} m^3 s^{-1}$$

and

$$\bar{y}_0 = [0.4975\ 0.2977\ 0.3976]^T m. \tag{11}$$

The system is linearized around this operating point using a Taylor expansion. The linearized system is described by a linear state space model as

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= x(t)\end{aligned} \tag{12}$$

Where $y(t) = \bar{y}(t) - y_0(t) = \begin{bmatrix} l_1'(t) & l_2'(t) & l_3'(t) \end{bmatrix}^T$, $x(t) = \bar{x}(t) - x_0(t)$, $u(t) = \bar{u}(t) - u_0(t)$,

$$A = \begin{bmatrix} -0.0114 & 0 & 0.0114 \\ 0 & -0.025 & 0.0114 \\ 0.0066 & 0.0066 & -0.0132 \end{bmatrix}$$

$$B = \begin{bmatrix} 64.9351 & 0 \\ 0 & 64.9351 \\ 0 & 0 \end{bmatrix} \tag{13}$$

Liquid levels in tanks 1 and 2 have to track the reference input vector *r(t)*, so the integral states would be:

$$\begin{bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \end{bmatrix} = r(t) - Cx(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} l_1'(t) \\ l_2'(t) \end{bmatrix} \tag{14}$$

The state feedback gain is determined using pole-placement. Four controllers were designed with different sets of poles. The list of poles sets and the computed gain for each set are given in Table 2.

By computing these gains, the first phase of the article is done, that is to design controllers for the three-tank process. Assessing the performance of each controller is discussed in the next section. Stochastic performance of the designed controllers will be determined by minimum variance index.

Table 2. Calculated state feedback gains with pole-placement.

Pole set	**Gain value**
[-1.05 -1.05 -0.12 -0.065 -0.015]	$\begin{bmatrix} 172.2152 & 0.4750 & 1.5005 & -10.7656 & -0.1021 \\ 0.4324 & 179.3532 & 1.3117 & -0.0798 & -20.1976 \end{bmatrix} \times 1$
[-0.084+0.016i -0.084-0.016i -0.08 -0.05 -0.015]	$\begin{bmatrix} 21.63 & 3 & -5 & -0.95 & -0.32 \\ 2.9 & 19 & -4 & -0.3 & -0.91 \end{bmatrix} \times 10^{-4}$
[-1 -1 -0.02 -0.015 -0.065]	$\begin{bmatrix} 165.7143 & -0.3692 & 1.7311 & -3.2884 & 0.0561 \\ -0.3332 & 170.0025 & 1.4779 & 0.0308 & -10.7578 \end{bmatrix} \times 1$
[-1 -1 -0.2 -0.1 -0.05]	$\begin{bmatrix} 0.0179 & 0.0011 & -0.6227 & -0.0068 & -0.0052 \\ 0.0010 & 0.0175 & -0.5765 & -0.0049 & -0.0064 \end{bmatrix}$

3. PERFORMANCE ASSESSMENT (MINIMUM VARIANCE INDEX)

When a process is under control or is a candidate for process control, with the objective being to minimum variation in the output variables, one significant question arise. What is the potential reduction in the output variation that could be obtained or which control is the best in having less variance in output? Operating data collected from a process can be used to develop an autoregressive moving-average vector (MAV) time-series model that can provide quantitative information to answer the preceding questions. This method does not need opening the loop and can operate with outputs' data sampling.
The multivariate model can be used to determine an index which indicates how close the output variation is to its theoretical minimum and to aid in diagnosing sources of variation.

A simple measure of the performance of regulators in such a system would be a comparison of the observed output variation with a standard based on the ideal case. From linear stochastic control theory, the one-step prediction error variance of the outputs is this ideal case. Because the observed variation *can* be calculated directly, the problems to determine the one step prediction-error variance from operating data [6]. Kalman filtering is often used for this, Aasnaes and Kailath have pointed out that ARMAV time series models can also be used for prediction and system identification purposes [7]. Examples of time series application to prediction and control can be found in [8], [9] and others.

In this case, multivariate ARMAV *(n,m)* models of the form

$$X_t = \sum_{i=1}^{n} \phi_i X_{t-i} + a_t - \sum_{i=1}^{m} \theta_i a_{t-i} \qquad m < n \tag{15}$$

are identified for the p-dimensional vector of process variables given by $X_t=(X_{1t},X_{2t},\dots,X_{pt})^T$, $t = 1,2,\dots, N$, and the covariance or dispersion matrix for these measured variables is

$$\gamma_o = E\left[X_t X_t^T\right] \tag{16}$$

and the auto covariance of the p-dimension vector *a,* is

$$E\left[a_t a_{t-k}^T\right] = \delta_k \sigma_a . \tag{17}$$

i.e., *a* multivariate white noise process.

Since the process *a, can* be interpreted as the one-step prediction errors, and using the idea that theoretically this prediction error represents the minimum variance of the output measurements, an index of control effectiveness *can* be defined as

$$CE_i = \frac{\sigma_{ii}}{\gamma_{iio}} . \tag{18}$$

For a measured output X_{it}, (4) is the ratio of the minimum possible variance to the observed variance, where the subscripts refer to the rows and columns of σ_a and γ_o If the system, disturbances, and control strategy do not change, (4) should not change and will always be between zero and one, being identically one for the best possible stochastic control. In practice, the theoretical values are not available, but maximum-likelihood estimates of $\hat{\sigma}_{ii}$ and $\hat{\gamma}_{iio}$ *can* be obtained once an ARMAV model is identified, and these values substituted into (18). Founding best model for the multivariate time series is obtained by the method (max-likelihood) explained in [10].

This approach is justified when the multivariate time-series X_t evolves from a system where linear combinations of the prediction errors are contained- in the measurements. This criterion can be used for deciding which controller is close to theoretical minimum variance, besides it can reveal efficiency of the loop after works for years. Suppose there is a system that is working for times, this method can be used to determine how much its performance is near to the best reachable performance without opening or stopping the routing works of industrial loops. This criterion along with deterministic criteria can decide proper controller for having best performance.

3. SIMULATION RESULTS

To study deterministic performance of each controller designed in section 2, the nonlinear state space model of the three-tank process was simulated. Step test was used to determine the tracking performance of each controller in closed loop. Results are shown in Figure 2.

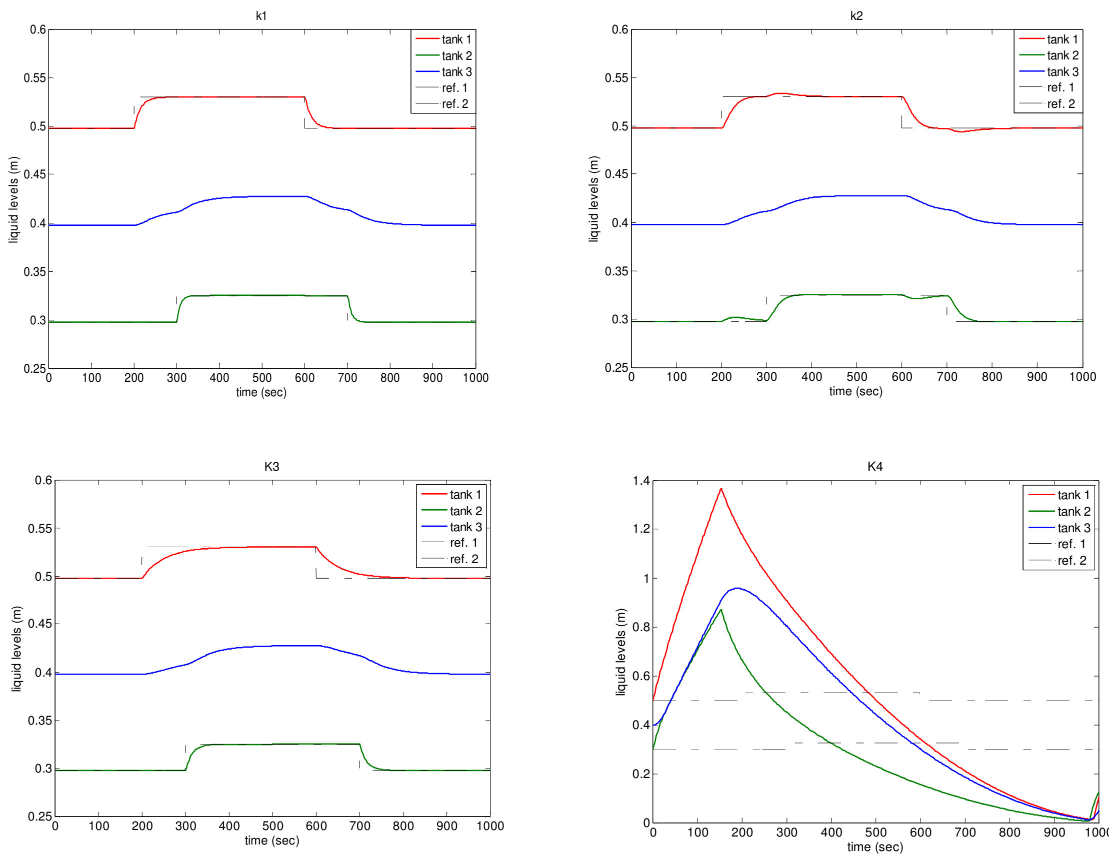


Figure 2. Tracking performance of designed controllers

According to Figure 2, we can see that controller 1 has good speed and low overshoot. Also the controlled outputs do not have interaction. Whereas, controller 2 could not eliminate outputs' interaction. However controller 3 has eliminated the interaction, it has lower speed than controllers 1 and 2. It is clear that controller 4 is not a proper controller. This is due to its high speed (its poles were placed far from imaginary axis), whereas the system has rather high delay, it will make controller's commands improper and the system's outputs oscillate severely.

For studying stochastic performance of controllers, a disturbance with transfer function (19) were applied to the pumps that supply the inputs of the process.

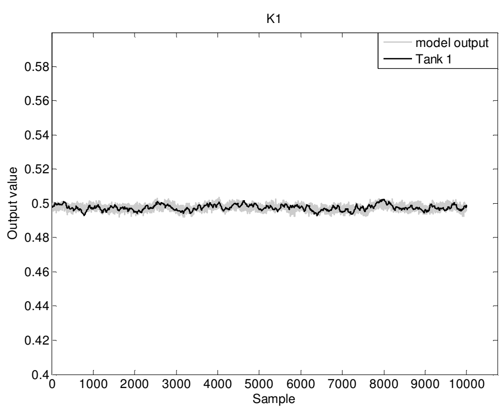
$$D(s) = \begin{bmatrix} \frac{1}{s+0.5} & \frac{1}{s+0.01} \\ \frac{1}{s+0.02} & \frac{1}{s+0.7} \end{bmatrix} N(s) \tag{19}$$

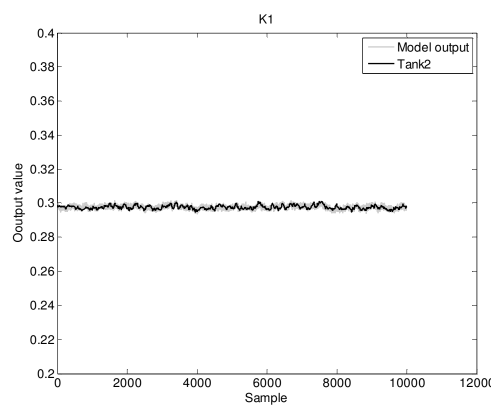
Where, *N(s)* is multivariate white noise.

As it can be seen in Figure 3, controller 2 that has rather good tracking performance (as it seen in Figure 2), clearly can't reject the disturbances and the outputs have rather high variances. Conversely, controllers 1 and 3 have good performances rejecting the disturbances. Controller 4 has poor performance similar to its deterministic performance.

Regarding section II, ARMAV models have been fitted to multivariate time series cached from outputs' samples of system. The data obtained from outputs' samples of system and ARMAV model outputs are drawn in Figure 3 for each controller. After determining the ARMAV model the minimum variance index is calculated by equation (18). Results are gathered in Table 3.

Table 3 shows minimum variance index for each controller. Second and third columns shows MV index for output 1 and 2, respectively. Last column shows MV index for closed loop. It can be seen that controller 1 has the best loop performance according to the MV index.





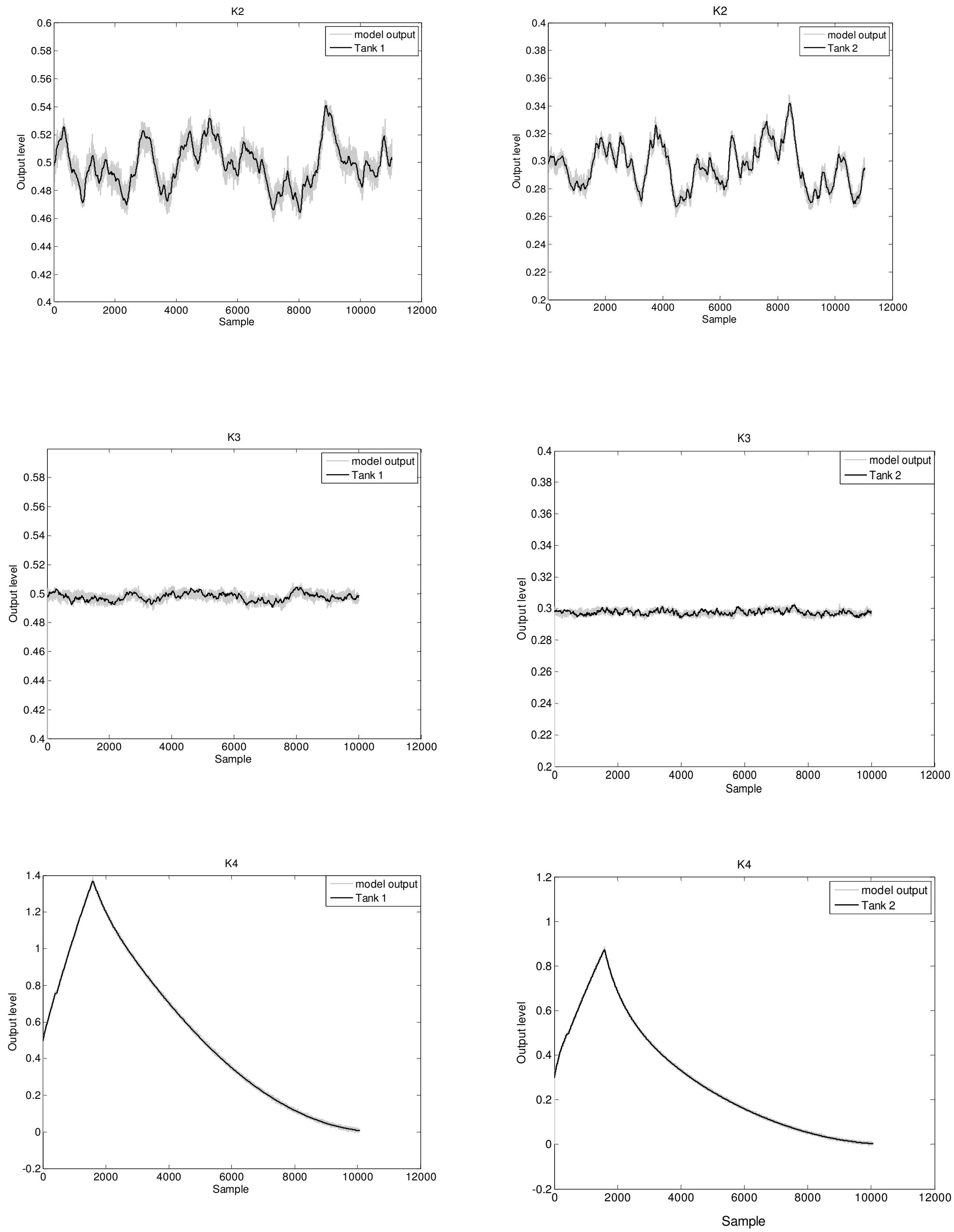


Figure 2. Time series of system and model outputs

Considering deterministic (Figure 2) and stochastic performances (Table 3), it can be concluded that controller 1 has both good deterministic and stochastic performances. Also controller 3 has rather good performances in both stochastic and deterministic aspects. Although controller 2 has rather good deterministic performance, its stochastic performance is poor. Controller 4 has both poor deterministic and stochastic performances.

Table 3. Minimum variance index for each controller.

Controller No.	MV index for output 1	MV index for output 2	Overall MV index
1	0.5738	0.4444	0.5091
2	0.0749	0.0271	0.0510
3	0.3632	0.4244	0.3938
4	0.00014	0.00014	0.00014

3. CONCLUSIONS

State feedback with integral control is used to control the three-tank benchmark system (as a good prototype of chemical industry processes). Several controllers were designed. Deterministic and stochastic performances of each controller were studied. Step test was used for determining the deterministic performance. Minimum variance index was used for stochastic performance assessment. Results show that controller 1 has the best performance among others.

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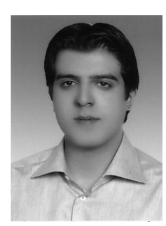

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