QUADROTOR ATTITUDE STABILIZATION USING TAKAGI-SUGENOMODEL

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ABSTRACT

In this paper a robust controller for attitude stabilization of a Quadrotor UAV is proposed. For this we design a Takagi-Sugeno (T-S) model for Quadrotor modelling, and then we use Linear Matrix Inequality (LMI), and PDC (Parallel Disturbance Compensation) technique to design a nonlinear state feedback controller with pole placement in a pre-specified region of the operating space.The stabilityof the whole closed-loop system is investigated using quadratic Lyapunovfunction.To demonstrateits usefulness, the proposed design methodology is applied to the problem ofQuadrotor attitude stabilization. Simulation results show that the proposedLMI-based design methodology yields good transient performance.In addition, it is observed that the proposed state feedback controller provides superior stability robustness againstparameter variationsand measurement noise.

KEYWORDS

Linear Matrix Inequality (LMI), measurement noise,Parameter Variations,Parallel Disturbance Compensation (PDC), Pole Placement, Quadrotor UAV, Takagi-Sugenomodel.

1. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) have been designed inthe military field since more than one half century. The mainobjective was to replace human pilot in a painful tasks andwhen the environment became hostile where the security ofpilots is not assured.These firsts designed UAV's date from the Second WorldWar; they have the dynamics and dimensions of plans and flew at very high altitudes [1].Quadrotor Helicopter is considered as one of the most popular UAV platform. This kind of helicopters are dynamically unstable, and therefore suitable control methods was used to make them stable, as back-stepping and sliding-mode techniques [2] [3].

In everyday life, the strategy how to solve a complex problem is called divide & conquer. The problem is divided into simpler parts, which are solved independently and together yields the solution to the whole problem. The same strategy can be used for modelling and control of non-linear systems, where the non-linear plant is substituted by locally valid set of linear sub models [4].The issue of stability and the synthesis of controllers for nonlinear systems described by continuous-time Takagi-Sugeno (T-S) models [5] have been considered actively. There has been also an increasing interest in the multiple model approach [6] [7] which also use the T-S systems to modelling.

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During the last years, many works have been carried out to investigate the stability analysis and the design of state feedback controller of T-S systems. Using a quadratic Lyapunov function and Parallel Disturbance Compensation (PDC) technique, sufficient conditions for the stability and stabilisability have been established [8] [9]. The stability depends on the existence of a common positive definite matrix guarantying the stability of all local subsystems. The PDC control is a nonlinear state feedback controller. The gain of this controller can be expressed as the solution of a linear matrix inequality (LMIs) set [10].

2. QUADROTOR DYNAMICAL MODEL

We can describe the vehicle as having four propellers in cross configuration. The two pairs of propellers (1, 3) and (2, 4) turn in opposite directions. By varying the rotor speeds, one can change the lift forces and create motion. Thus, increasing or decreasing the four propeller's speeds together generates vertical motion. Changing the 2 and 4 propeller's speed conversely produces roll rotation coupled with lateral motion. Pitch rotation and the corresponding lateral motion result from 1 and 3 propeller's speed conversely modified as described in Figure 1. Yaw rotation is more subtle, as it results from the difference in the counter-torque between each pair of propellers.

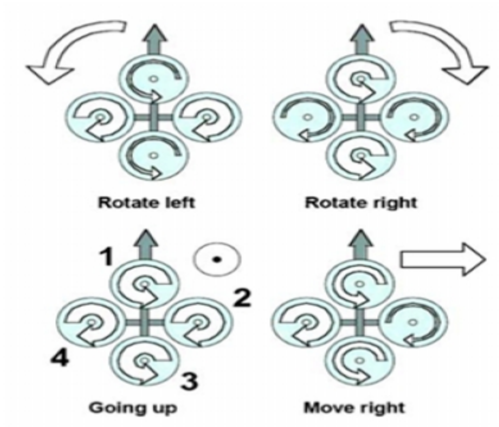

Figure 1.Quadrotor concept motion description

Quadrotor helicopter is one of the most complex flying systems that exist. This is due partly to the number of physical effects (Aerodynamic effects, gravity, gyroscopic, friction and inertial counter torques) acting on the system.

The first step before the control development is an adequate dynamic system modelling, especially for lightweight flying systems. Let us consider earth fixed frame R^b and body fixed frame R^m, as seen in Figure 2.

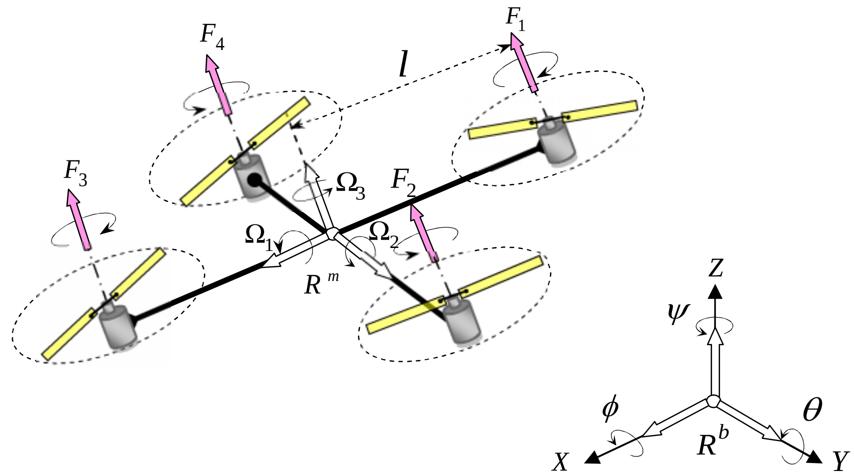

Figure 2.Quadrotor Architecture

The dynamics of the Quadrotor is described in the space bysix degrees of freedom according to the fixed inertial framerelated to the ground. This dynamics is related to thetranslational positions (x, y, z) and the attitude described bythe Euler angles (ϕ, θ, ψ). These six coordinates are theabsolute position of the centre of masse. The Euler angles are defined as follows:

- Roll angle ϕ: $-\pi/2 \leq \phi \leq \pi/2$;
- Pitch angle θ: $-\pi/2 \leq \theta \leq \pi/2$;
- Yaw angle ψ:$-\pi \leq \psi \leq \pi$.

The rotation transformation matrix R from the inertial fixedframe $R^b$to the body fixed frame $R^m$is given by:

$$R = \begin{bmatrix} c\psi c\theta & s\phi s\theta c\psi - s\psi c\phi & c\phi s\theta c\psi + s\psi s\phi \\ s\psi c\theta & s\phi s\theta s\psi + c\psi c\theta & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \quad (1)$$

With $s(.)$and $c(.)$ represent $\sin(.)$ and $\cos(.)$respectively.

To derive the dynamic model of the Quadrotor, the Newton Euler formalism will be used on both translation and rotation motions.In this work we mainly focus our interest to the attitude dynamics and we consider the reduced dynamical model as follows [11]:

$$\begin{cases} \ddot{\phi} = \dot{\theta}\dot{\psi}\dfrac{(I_y - I_z)}{I_x} - \dfrac{I_r}{I_x}\Omega_r\dot{\theta} - \dfrac{K_{fax}}{I_x}\dot{\phi}^2 + \dfrac{1}{I_x}u_1 \\ \ddot{\theta} = \dot{\phi}\dot{\psi}\dfrac{(I_z - I_x)}{I_y} + \dfrac{I_r}{I_y}\Omega_r\dot{\phi} - \dfrac{K_{fay}}{I_y}\dot{\theta}^2 + \dfrac{1}{I_y}u_2 \\ \ddot{\psi} = \dot{\theta}\dot{\phi}\dfrac{(I_x - I_y)}{I_z} - \dfrac{K_{faz}}{I_z}\dot{\psi}^2 + \dfrac{1}{I_z}u_3 \end{cases} \quad (2)$$

The inputs of the system are u_1, u_2, u_3 and Ω_r as a disturbance, obtaining:

$$\begin{cases} u_1 = bl\left(\omega_4^2 - \omega_2^2\right) \\ u_2 = bl\left(\omega_3^2 - \omega_1^2\right) \\ u_3 = d\left(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2\right) \\ \Omega_r = \omega_1 - \omega_2 + \omega_3 - \omega_4 \end{cases} \tag{3}$$

3. QUADROTOR TAKAGI-SUGENO MODEL

3.1. Takagi-Sugeno model

A T-S model is based on the interpolation between several LTI (linear time invariant) local models as follow:

$$\dot{x}_m(t) = \sum_{i=1}^{r} \mu_i\left(\xi(t)\right)\left(A_i x_i(t) + B_i u(t)\right) \tag{4}$$

Where r is the number of sub-models, $x_m(t) \in \mathbb{R}^p$ is the state vector, $u(t) \in \mathbb{R}^h$ is the input vector $A_i \in \mathbb{R}^{p\times p}$, $B_i \in \mathbb{R}^{p\times h}$, and $\xi(t) \in \mathbb{R}^q$ is the decision variable vector.

The variable $\xi(t)$ may represent measurable states and/or inputs and the form of this variable may leads to different class of systems: if $\xi(t)$ is known functions than the model (4) represents a nonlinear system and if there are unknown we consider that this leads to linear differential inclusion (LDI). This variable can also be a function of the measurable outputs of the system.
The normalized activation function $\mu_i(\xi(t))$ in relation with the ith sub-model is such that:

$$\begin{cases} \sum_{i=1}^{r} \mu_i\left(\xi(t)\right) = 1 \\ 0 \le \mu_i\left(\xi(t)\right) \le 1 \end{cases} \tag{5}$$

According to the zone where evolves the system, this function indicates the more or less important contribution of the local model corresponding in the global model (T-S model).

The global output of T-S model is interpolated as follows:

$$y_m(t) = \sum_{i=1}^{r} \mu_i\left(\xi(t)\right)\left(C_i x_i(t) + D_i u(t)\right) \tag{6}$$

Where $y_m(t) \in \mathbb{R}^l$ is the output vector and $C_i \in \mathbb{R}^{l\times p}$, $D_i \in \mathbb{R}^{l\times h}$. More detail about this type of representation can be found in [5].

3.2. Quadrotor Takagi-Sugeno model

The behaviour of a nonlinear system near an operating point (x_i, u_i), can be described by a linear time-invariant system (LTI). Using Taylor series about (x_i, u_i) and keeping only the linear terms yields:

$$\dot{x}(t) = A_i\left(x(t) - x_i\right) + B_i\left(u(t) - u_i\right) + f\left(x_i, u_i\right) \tag{7}$$

Which can written as

$$\dot{x}(t) = A_i x(t) + B_i u(t) + d_i \tag{8}$$

With:

$$A_i = \left.\frac{\partial f(x,u)}{\partial x}\right|_{\substack{x = x_i \\ u = u_i}},\ B_i = \left.\frac{\partial f(x,u)}{\partial u}\right|_{\substack{x = x_i \\ u = u_i}},\ f(x,u) = \dot{x}(t),\ d_i = f(x_i, u_i) - A_i x_i - B_i u_i$$

After calculation we obtained:

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & a_1 & 0 & a_2\dot{\psi} + a_3 & 0 & a_2\dot{\theta} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & a_5 + a_4\dot{\psi} & 0 & a_6 & 0 & a_4\dot{\phi} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & a_7\dot{\theta} & 0 & a_7\dot{\phi} & 0 & a_8 \end{bmatrix},\quad B_i = \begin{bmatrix} 0 & 0 & 0 \\ b_1 & b_2 & b_3 \\ 0 & 0 & 0 \\ b_4 & b_5 & b_6 \\ 0 & 0 & 0 \\ 0 & 0 & b_7 \end{bmatrix} \tag{9}$$

Combined local affine models (8)using Gaussian activation function we describe the dynamic model of the Quadrotor by a T-S model:

$$\begin{cases} x_m(t) = \sum_{i=1}^{3} \mu_i\left(\xi(t)\right)\left(A_i x_m(t) + B_i u(t) + d_i\right) \\ y_m(t) = C x_m(t) \end{cases} \tag{10}$$

With:

$$\mu_i\left(\xi(t)\right) = \frac{\omega_i\left(\xi(t)\right)}{\sum_{j=1}^{3} \omega_i\left(\xi(t)\right)},\ \omega_i\left(\xi(t)\right) = \prod_{j=1}^{3} \exp\left(-\frac{\left(\xi_j(t) - c_{i,j}\right)^2}{2\sigma_{i,j}^2}\right)$$

-The vector of decision variables $\xi(t) = \begin{bmatrix}\dot{\phi} & \dot{\theta} & \dot{\psi}\end{bmatrix}^T$

- The parameters of activations functions $(c_{i,j}, \sigma_{i,j})$ are given as:

- The centres$c_{i,j}$are defined according to the operation point.
- The Dispersions$\sigma_{i,j}$are defined by optimization of a criterion,which represent the quadratic error between Takagi-Sugeno model outputs and nonlinear system outputs, using Particle Swarm Optimisation algorithm(PSO) [12][13].

-The operating points are chosen to cover maximum space of the operating space, with small number of local models. The attitude of Quadrotor (roll, pitch, and yaw) has a limited bound

$(-\pi/2 \leq \phi \leq \pi/2, -\pi/2 \leq \theta \leq \pi/2, -\pi \leq \psi \leq \pi)$, for this reason we use three local models to cover this space.Linear local model are defined in this table as follow:

Table 1. Operation Points Parameters.

N° O.P	Parameters	d_i
1	$\dot{\phi} = \dot{\theta} = \dot{\psi} = -0.523\ rad/s$	$[0 \quad 0.1964 \quad 0 \quad -0.1964 \quad 0 \quad 0]^T$
2	$\dot{\phi} = \dot{\theta} = \dot{\psi} = 0\ rad/s$	$[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$
3	$\dot{\phi} = \dot{\theta} = \dot{\psi} = 0.523\ rad/s$	$[0 \quad 0.2771 \quad 0 \quad -0.2771 \quad 0 \quad 0]^T$

3.3. Quadrotor Takagi-Sugeno model validation

The input signals (rotors velocities) most appropriated for the local models network validation, and exit all dynamic of the system in this case is the Pseudo-Random Binary Signal (SBPA) due to different causes:the SBPA signal has a null mean and a variance that close to one, which allows the excitation of very good frequency range (dynamics system) without moving away too much the system from the operating point. It is periodic deterministic signal white-noise-like properties very adapted for identification and validation tasks.

A typical value of the amplitude of the SBPAis from 0.5% to 5% from the value of the operating point to which the SBPA is applied, in this case the amplitude of the SBPA is given as

$$A_{SBPA} = \omega_{eq} \pm 0.005 * \omega_{eq}\ ,\ \ \omega_{eq} = \sqrt{\frac{mg}{4b}}\ .$$

To validate the synthesized Takagi-Sugeno model a SBPA (input signal) is used, for Quadrotor nonlinear system and the T-S model. We simulate the two systems in parallel and we compare the resulting curves.

Figure 3 present the input signals of Quadrotor, which are SBPA signals with variable amplitude. This SBPA excite all dynamic of the system.

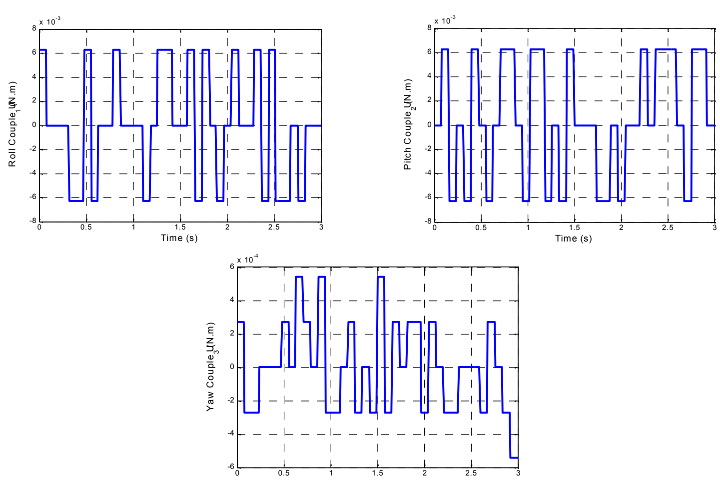

Figure 3. Validation input signals

Figure 4 present the attitude of Quadrotor and corresponding output of T-S model. We show the resemblance between the output of T-S model and Quadrotor nonlinear system. These results prove the quality of the approximation of a nonlinear system by a T-S model.

Figure 5 present attitude acceleration errors, which are close to a white-noise with null mean and a variance that close to one. Saw the designing T-S model give good approximation of the Quadrotor nonlinear system for a specific region of the operating space.

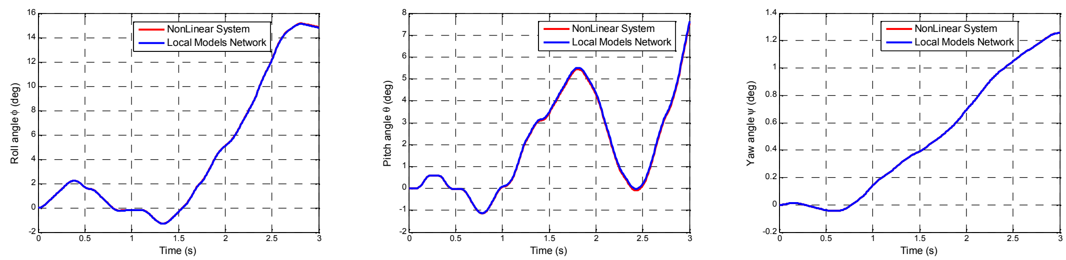

Figure 4. Takagi-Sugeno model and Quadrotor's outputs

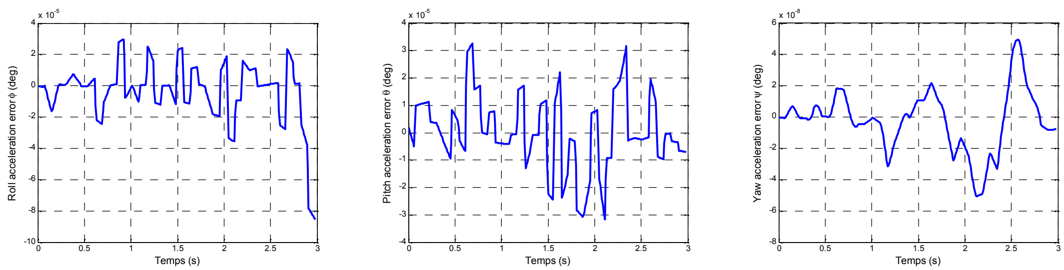

Figure 5. Attitude acceleration errors

4. Controller Design

4.1. State feedback controller

The concept of PDC, following the terminology [8], is utilized to design state-feedback controller on the basis of the T-S model (10). Linear control theory can be used to design the control law, because T-S model is described by linear state equations. The controller law is a convex linear combination of the local controller associated with the corresponding local sub-model. It can present as:

$$U(t) = -\sum_{i=1}^{r} \mu_i(\xi(t)) u_i(t) = -\sum_{i=1}^{r} \mu_i(\xi(t)) K_i x(t) \tag{11}$$

With: K_i is r vector of feedback gains.

It should be noted that the designed controller shares the same models sets with T-S models, and resulting controller (11) is nonlinear in general since the coefficient of the controller depends nonlinearly on the system input and output via the weighting functions. Substituting (11) into (10), the closed-loop T-S model can be represented by:

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(t))\mu_j(\xi(t))(A_i - B_i K_j)x(t) \tag{12}$$

The constant d_i was neglected in this formulate, because the control law can compensate the effect of this bias term.

4.1. Stabilisation using PDC

A sufficient quadratic stability condition derived by Tanaka and Sugeno[14] for ensuring stability of (12) is given as follows:

Theorem 1: The closed-loop T-S model (12) is quadratic-ally stable for some stable feedback K_i (via PDC scheme) if there exists a common positive definite matrix P such that:

$$\begin{aligned} & G_{ii}^T P + PG_{ii} < 0 \quad \forall i \in I_r \\ & \left(\frac{G_{ij}+G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij}+G_{ji}}{2}\right) < 0, \quad \forall (i,j) \in I_r^2, i < j \end{aligned} \tag{13}$$

With: $G_{ii} = A_i - B_i K_j$, $\mu_i(\xi(t))\mu_j(\xi(t)) \neq 0$.

Which is an LMI in P when K_i are predetermined. However, our objective is to design the gain matrix K_i such that conditions (13) are satisfied. That is, K_i are not pre-determined matrices any longer, but matrix variables. This is the quadratic stability problem and can be recast as an LMI feasibility problem. With linear fractional transformation $X = P^{-1}$ and $N_i = K_i X$, we may rewrite (13) as an LMI problem in N_i, X and S_{ij} [15]:

$$\begin{aligned} & X > 0 \\ & XA_i^T + A_i X - N_i^T B_i^T - B_i N_i + S_{ii} < 0, \quad \forall i \in I_r \\ & XA_i^T + A_i X + XA_j^T + A_j X - N_j^T B_i^T - B_i N_j \\ & -N_i^T B_j^T - B_j N_i + 2S_{ij} \le 0, \quad \forall (i,j) \in I_r^2, i < j \\ & \begin{pmatrix} S_{11} & \cdots & S_{1n} \\ \vdots & \ddots & \vdots \\ S_{1n} & \cdots & S_{nn} \end{pmatrix} > 0 \end{aligned} \tag{14}$$

With: $S_{ij} = XQ_{ij}X$, $\forall i \in \{1, \dots, r\}$, Q_{ij} are symmetric matrix.

4.2. LMI formulation for Pole placement

In order to achieve some desired transientperformance, a pole placement should beconsidered. For many problems, exact poleassignment may not be necessary; it suffices tolocate the pole of the closed loop system in a sub-regionof the complex left half plane. This sectiondiscusses a pole assignment in LMI regions. For this purpose, we introduce the following LMI-based representation of stability regions [16] [17].

Definition: A subset *D* of the complex planeis called an LMI region if there exist a symmetricmatrix $\alpha = (\alpha_{ij}) \epsilon \mathbb{R}^{p.p}$ and a matrix $\beta = (\beta_{ij}) \epsilon \mathbb{R}^{p.p}$ such that:

$$D = \{ z \in \sqcup : f_D(z) < 0 \} \tag{15}$$

Where: $f_D(z) = (\alpha_{ij} + \beta_{ij} z + \beta_{ij} \bar{z}), \forall i, j \epsilon \{1, \dots, p\}$

Theorem 2: A matrix A is D-stable if and onlyif there exists a symmetric positive definite matrix X such that:

$$M_D(X, A) < 0 \tag{16}$$

Where: $M_D(X, A) = \alpha \otimes X + \beta \otimes (AX) + \beta^T \otimes (AX)^T$

For example, a circle region D centred at $(-q, 0)$ with radius $\rho > 0$ can be obtained by taking thematrices and as follows:

$$\alpha = \begin{pmatrix} -\rho & q \\ q & -\rho \end{pmatrix}, \text{ and } \beta = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

What makes it possible to obtain the expression ofthe characteristic function:

$$f_D(z) = \begin{pmatrix} -\rho & z^* + q \\ z + q & -\rho \end{pmatrix} \tag{17}$$

As it is shown in figure 6, this region which includecircular region, allows fixing a lower bound on boththe exponential decay rate: $\rho - q$ and the dampingratio: $\zeta_{min} = \sqrt{1 - (\rho^2 / q^2)} (\rho < q)$ of the closed-loopresponse, and thus is very common in practical control design.

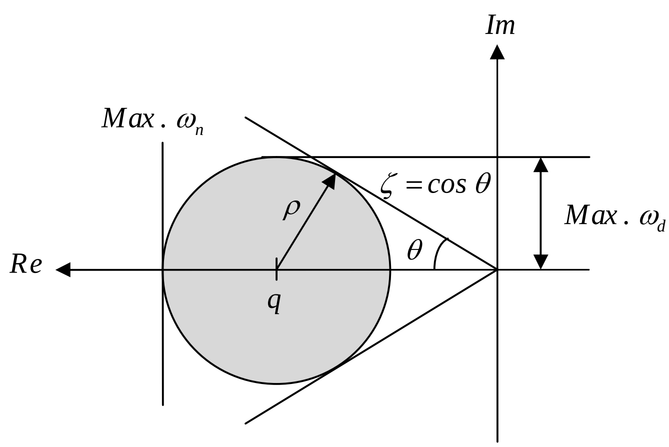

Figure 6. Circular region (D) for pole location

Since the prescribed LMI region (17) will be addedas supplementary constraints to these of thetheorem 1, it should be noted that itonly suffices to locate the poles of the dominantterm in the prescribe LMI regions, i.e. the case of $i = j$. It follows that the closed loop T-S model (12) is D-stable if there exists asymmetric matrix X such that [18]:

$$\begin{pmatrix} -\rho X & qX + X\left(A_i + B_i K_j\right)^T \\ qX + \left(A_i + B_i K_j\right)X & -\rho X \end{pmatrix} \tag{18}$$

With the same change of variables $N_i = K_i X$ leads to thefollowing LMI formulation:

$$\begin{pmatrix} -\rho X & qX + XA_i^T + B_i^T N_i^T \\ qX + XA + B_i N_i & -\rho X \end{pmatrix}, \ i = j \tag{19}$$

By combining Theorems 1 and 2 leads to the following LMI formulation of two objectives state-feedback synthesis problem, such that the resulting controller meets both the global stability and the desired transient performance simultaneously. The closed loop T-S model (12) is stabilizable in the specified region *D* ifthere exists asymmetric matrix Xsuch that [16]:

$$\begin{aligned} & X > 0 \\ & XA_i^T + A_i X - N_i^T B_i^T - B_i N_i + S_{ii} < 0, \quad \forall i \in I_r \\ & XA_i^T + A_i X + XA_j^T + A_j X - N_j^T B_i^T - B_i N_j \\ & -N_i^T B_j^T - B_j N_i + 2S_{ij} \le 0, \quad \forall (i,j) \in I_r^2, i < j \\ & \begin{pmatrix} -\rho X & qX + XA_i^T + N_i^T B_i^T \\ qX + A_i X + B_i N_i & -\rho X \end{pmatrix} < 0, \quad \forall i \in I_r \\ & \begin{pmatrix} S_{11} & \cdots & S_{1n} \\ \vdots & \ddots & \vdots \\ S_{1n} & \cdots & S_{nn} \end{pmatrix} > 0 \end{aligned} \tag{20}$$

With: $S_{ij} = XQ_{ij}X, K_i = N_i X, \ \forall i \in \{1, \dots, r\}$

4.3. State feedback gains calculation

Using Theorem 1 and 2; can design a nonlinear state feedback controller that guarantees global stability while provides desired transient behaviour by constraint the closed-loop poles in*D*. The stability region*D*is a circle of centre$(q, 0)$and radiusρ and the LMI synthesis is performed for a set of values$(q, \rho) = (4,1)$.

Then the LMI region has the following characteristic function:

$$f_D(z) = \begin{pmatrix} -1 & z^* + 4 \\ z + 4 & -1 \end{pmatrix} \tag{21}$$

This circle region puts a lower bound on both exponential decay rate$q - \rho = 3\,rad/s$and damping ratio$\zeta = \sqrt{1 - (\rho^2/q^2)} = 0.97$of the closed-loop response.By solving LMI feasibility problem (20), we can obtain a positive symmetric matrix X (by interior-point method in Matlab LMI-toolbox), and stat feedback Matrix K_i.

4.4. Simulation results

The controller described above was simulated for the nonlinear Quadrotor system. Simulations are made for initial values equal to$(\phi_0, \theta_0, \psi_0) = (20, 40, 60)deg$ for roll angle,$(-20, -40, -60)deg$ for pitch angle, and $(40, 80, 120)deg$ for yaw angle, and equal to zeros for tracking simulation. The values of the model parameters used for simulations are the following:
$m = 0.486kg, l = 0.225m, g = 9.81\,m/s^2, d = 3.23 \times 10^{-7}\,N.m/(rad.s)\,, b = 2.98 \times 10^{-5}\,N/(rad.s)\,, I_x = I_y = 3.82 \times 10^{-3}kg.m^2, I_z = 7.65 \times 10^{-3}kg.m^2, K_{fax} = K_{fay} = 5.567 \times 10^{-4}, K_{faz} = 6.354 \times 10^{-4}$.

The results of state feedback controller are shown in Figure 7 which indicates the output of Quadrotor nonlinear system (Quadrotor attitude, Roll, Pitch, and Yaw), and the corresponding control inputs in figure 8.

From the resultsof figure 7, 8; it can be noticed that state the feedback controller provides good transient performance(stabilization time, overtaking…), while, controller give stable response regardless of any initial displacement.The control inputs are smoother and realizable.

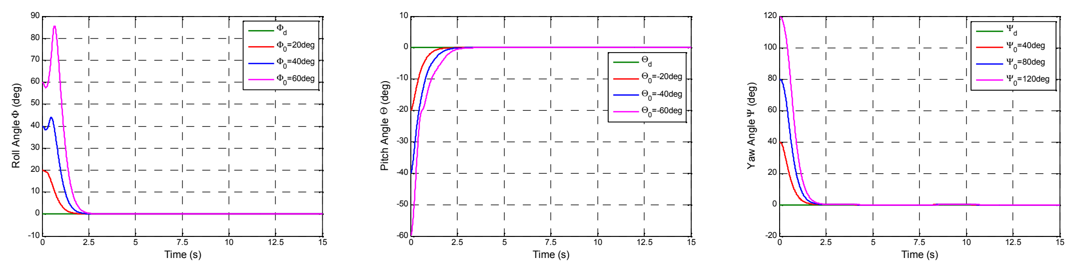

Figure 7. Quadrotor Attitude (ϕ, θ, ψ) for state feedback controller

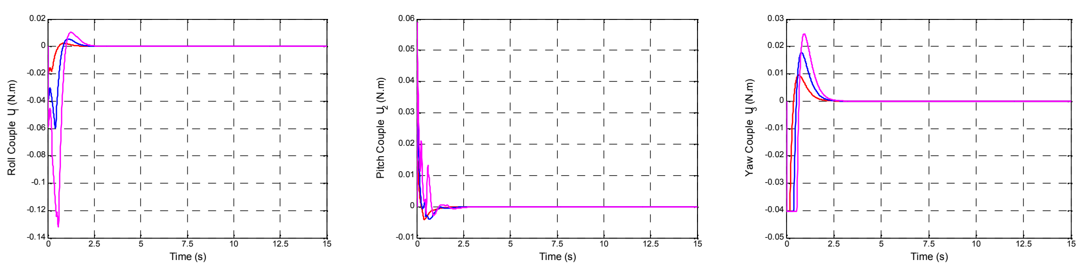

Figure 8. Quadrotor control inputs (u_1, u_2, u_3) for state feedback controller

To check the robustness of the proposed controls two tests are used; the first again measurement noise, and the second: again parameter variations.

- Measurement noise of a normal distribution, a covariance equal to 1, a zero mean, and amplitude close to 0.05 are added to the measured variables as shown in figures 10, 11.
- A change of 100% for (I_x, I_y, I_z) parameters and 40% variation of $b_i$parameters are performed between 20 to 40 seconds (figures 12, 13).

Figure 10 represent measurement noisetest, for Quadrotor attitude tracking (Roll, Pitch, and Yaw), with a sinusoidal trajectory, we can clearly see a good tracking of desired trajectories, which confirms the robustness of the proposed controlleragainmeasures noise. Figure 11 represent inputs control system.

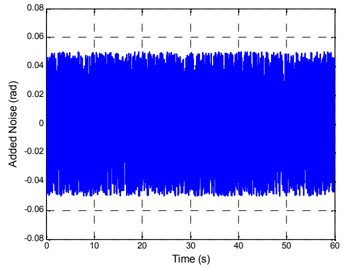

Figure 9. Measures noise added

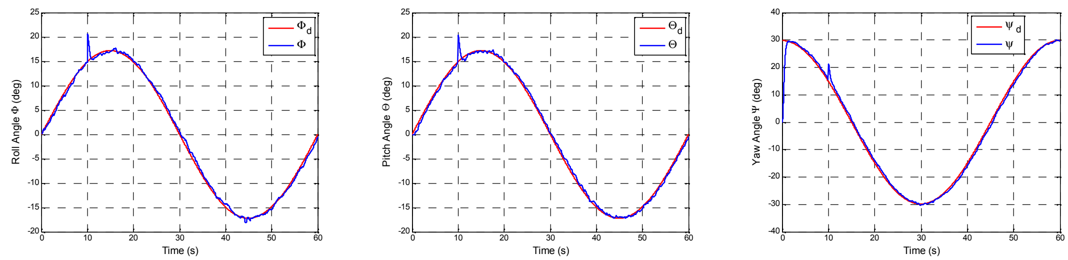

Figure 10. Quadrotor Attitude (ϕ, θ, ψ) for measurement noise test

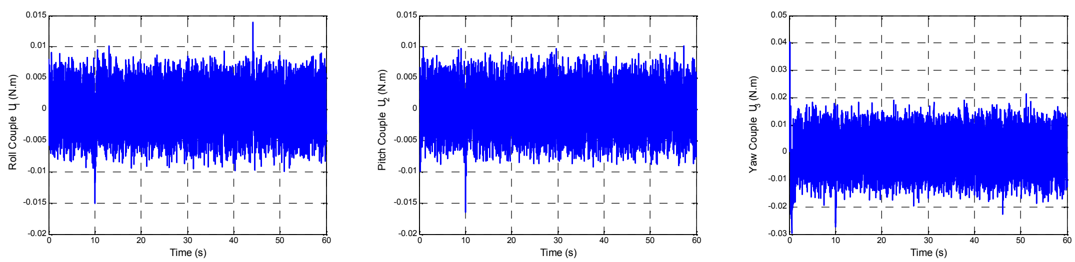

Figure 11. Quadrotor control inputs (u_1, u_2, u_3) for measurement noise test

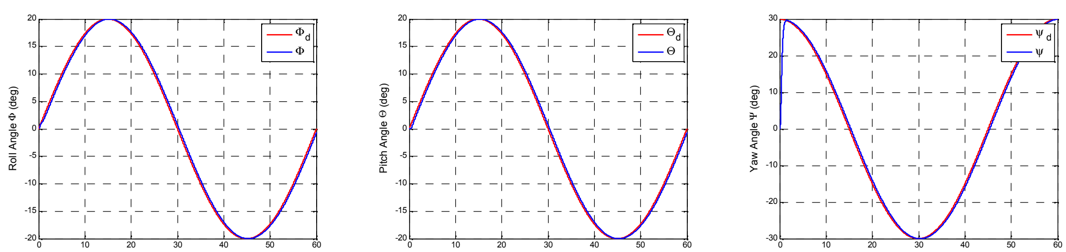

Figure 12. Quadrotor Attitude (ϕ, θ, ψ) for parameter variation test

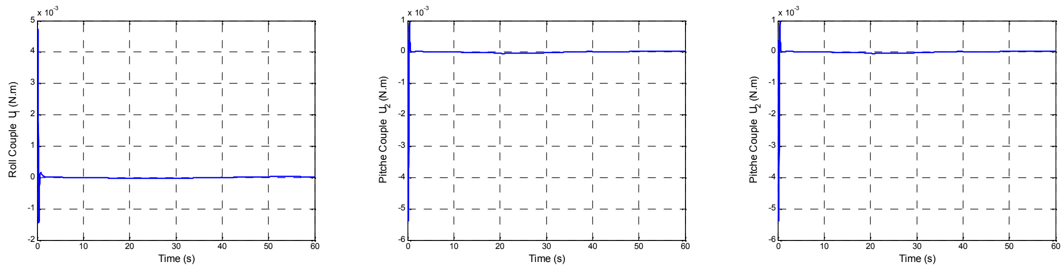

Figure 13. Quadrotor control inputs (u_1, u_2, u_3) for parameter variation test

Figure 12 represent parameter variationstest, for Quadrotor attitude tracking (Roll, Pitch, and Yaw), with a sinusoidal trajectory, we can clearly see a good tracking of desired trajectories, which confirms the robustness of the proposed controlleragainparameter variation. Figure 13 represent inputs control system.

5. CONCLUSIONS

In this paperthe problem of Quadrotor attitude stabilization isresolved using T-S modeland state feedback controller. A T-S model is designed for Quadrotor modelling. We use a Quadratic Lyapunov function to prove the stability of closed loop system.The designed methodologyis based on ParallelDistributed Compensation technique and pole placement in LMI region.Simulation results showed that the proposedcontroller providesstable response regardless of any initial conditions.In addition, it is observed that theproposed state feedback controller providessuperior stability robustness against parameter variations and measurement noise.

APPENDIX

Parameters of A_i and B_i matrix

$$\begin{cases} a_1 = -\dfrac{K_{fax}}{I_x} \\ a_2 = \dfrac{(I_y - I_z)}{I_x} \\ a_3 = -\dfrac{J_r}{I_x}\Omega_r \\ a_4 = \dfrac{(I_z - I_x)}{I_y} \end{cases}, \quad \begin{cases} a_5 = \dfrac{J_r}{I_y}\Omega_r \\ a_6 = -\dfrac{K_{fay}}{I_y} \\ a_7 = \dfrac{(I_x - I_y)}{I_z} \\ a_8 = -\dfrac{K_{faz}}{I_z} \end{cases}, \quad \begin{cases} b_1 = -\dfrac{J_r}{I_x}\dot{\theta}C_1 + \dfrac{1}{I_x} \\ b_2 = -\dfrac{J_r}{I_x}\dot{\theta}C_2 \\ b_3 = -\dfrac{J_r}{I_x}\dot{\theta}C_3 \\ b_4 = \dfrac{J_r}{I_y}\dot{\phi}C_1 \end{cases}, \quad \begin{cases} b_5 = -\dfrac{J_r}{I_y}\dot{\phi}C_2 + \dfrac{1}{I_y} \\ b_6 = -\dfrac{J_r}{I_y}\dot{\phi}C_3 \\ b_7 = \dfrac{1}{I_z} \end{cases}$$

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