SLIDING CONTROLLER DESIGN FOR THE HYBRID CHAOS SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC XU SYSTEMS

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ABSTRACT

This paper derives new results for the sliding controller design for the hybrid synchronization of identical hyperchaotic Xu systems (Xu, Cai and Zheng, 2009). In hybrid synchronization of a pair of chaotic systems consisting of master and slave systems, the odd states of the systems are completely synchronized, while the even states are anti-synchronized so that complete synchronization (CS) and anti-synchronization (AS) coexist in the chaos synchronization. The SMC-based stability results derived in this paper for the hybrid synchronization of identical hyperchaotic Xu systems are established using Lyapunov stability theory. Numerical simulations are depicted to illustrate and validate the hybrid synchronization schemes for the identical hyperchaotic Xu systems.

KEYWORDS

Sliding Mode Control, Hybrid Synchronization, Hyperchaotic Systems, Hyperchaotic Xu System.

1. INTRODUCTION

Chaotic systems are nonlinear dynamical systems that are highly sensitive to initial conditions and exhibit random behaviour in deterministic motion. The sensitive nature of chaotic systems is commonly called as the *butterfly effect* [1]. Synchronization of chaotic systems is a phenomenon which may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems having nearly the same initial conditions, synchronizing two chaotic systems is indeed a very challenging problem in the literature.

Hyperchaotic system is usually defined as a chaotic system with more than one positive Lyapunov exponent. The first hyperchaotic system was discovered by O.E. Rössler (1979). Since hyperchaotic system has the characteristics of high capacity, high security and high efficiency, it has the potential of broad applications in nonlinear circuits, secure communications, lasers, neural networks, biological systems and so on.

Thus, the studies on hyperchaotic systems, viz. control, synchronization and circuit implementation are very challenging problems in the chaos literature.

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In most of the chaos synchronization approaches, the *master-slave* or *drive-response* formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the anti-synchronization is to use the output of the master system to control the slave system so that the states of the slave system have the same amplitude but opposite signs as the states of the master system asymptotically.
Since the pioneering work by Pecora and Carroll ([2], 1990), chaos synchronization problem has been studied extensively and intensively in the literature [2-17]. Chaos theory has been applied to a variety of fields such as physical systems [3], chemical systems [4], ecological systems [5], secure communications [6-8], etc.

In the last two decades, various schemes have been successfully applied for chaos synchronization such as PC method [2], OGY method [9], active control method [10-14], adaptive control method [15-20], time-delay feedback method [21], backstepping design method [22], sampled-data feedback method [23], etc.

In hybrid synchronization of master and slave systems, the odd states are completely synchronized (CS) and the even states are anti-synchronized (AS). The co-existence of CS and AS in the synchronization enhances the security of secure communication systems.

In this paper, we derive new results based on the sliding mode control [24-26] for the hybrid synchronization of identical hyperchaotic Xu systems ([27], Xu, Cai and Zheng, 2009). In robust control systems, the sliding mode control method is often adopted due to its inherent advantages of easy realization, fast response and good transient performance as well as its insensitivity to parameter uncertainties and external disturbances.

This paper has been organized as follows. In Section 2, we describe the problem statement and our methodology using sliding mode control (SMC). In Section 3, we discuss the hybrid synchronization of identical hyperchaotic Xu systems. In Section 4, we summarize the main results obtained in this paper.

2. PROBLEM STATEMENT AND OUR METHODOLOGY USING SMC

Consider the chaotic system described by

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in R^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : R^n \to R^n$ is the nonlinear part of the system.

We consider the system (1) as the *master* or *drive* system.

As the *slave* or *response* system, we consider the following chaotic system described by the dynamics

$$\dot{y} = Ay + f(y) + u \tag{2}$$

where $y \in R^n$ is the state of the system and $u \in R^m$ is the controller to be designed.
We define the *hybrid synchronization error* as

$$e_i = \begin{cases} y_i - x_i & \text{if } i \text{ is odd} \\ y_i + x_i & \text{if } i \text{ is even} \end{cases} \tag{3}$$

Then the error dynamics is obtained as

$$\dot{e} = Ae + \eta(x, y) + u \tag{4}$$

The objective of the anti-synchronization problem is to find a controller u such that

$$\lim_{t\to\infty} \left\| e(t) \right\| = 0 \quad \text{for all } e(0) \in R^n. \tag{5}$$

To solve this problem, we first define the control u as

$$u = -\eta(x, y) + Bv \tag{6}$$

where B is a constant gain vector selected such that (A, B) is controllable.
Substituting (6) into (4), the error dynamics simplifies to

$$\dot{e} = Ae + Bv \tag{7}$$

which is a linear time-invariant control system with single input v.
Thus, the original anti-synchronization problem can be replaced by an equivalent problem of stabilizing the zero solution $e = 0$ of the system (7) by a suitable choice of the sliding mode control. In the sliding mode control, we define the variable

$$s(e) = Ce = c_1e_1 + c_2e_2 + \cdots + c_ne_n \tag{8}$$

where $C = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}$ is a constant vector to be determined.
In the sliding mode control, we constrain the motion of the system (7) to the sliding manifold defined by

$$S = \left\{ e \in R^n \mid s(e) = 0 \right\}$$

which is required to be invariant under the flow of the error dynamics (7).
When in sliding manifold S, the system (7) satisfies the following conditions:

$$s(e) = 0 \tag{9}$$

which is the defining equation for the manifold S and

$$\dot{s}(e) = 0 \tag{10}$$

which is the necessary condition for the state trajectory $e(t)$ of (7) to stay on the sliding manifold S.

Using (7) and (8), the equation (10) can be rewritten as

$$\dot{s}(e) = C\left[Ae + Bv\right] = 0 \tag{11}$$

Solving (11) for v, we obtain the equivalent control law

$$v_{\text{eq}}(t) = -(CB)^{-1}CA\ e(t) \tag{12}$$

where C is chosen such that $CB \neq 0$.
Substituting (12) into the error dynamics (7), we obtain the closed-loop dynamics as

$$\dot{e} = \left[I - B(CB)^{-1}C\right]Ae \tag{13}$$

The row vector C is selected such that the system matrix of the controlled dynamics $\left[I - B(CB)^{-1}C\right]A$ is Hurwitz, *i.e.* it has all eigenvalues with negative real parts. Then the controlled system (13) is globally asymptotically stable.
To design the sliding mode controller for (7), we apply the constant plus proportional rate reaching law

$$\dot{s} = -q\,\text{sgn}(s) - k\ s \tag{14}$$

where $\text{sgn}(\cdot)$ denotes the sign function and the gains $q > 0,\ \ k > 0$ are determined such that the sliding condition is satisfied and sliding motion will occur.
From equations (11) and (14), we can obtain the control $v(t)$ as

$$v(t) = -(CB)^{-1}\left[C(kI + A)e + q\,\text{sgn}(s)\right] \tag{15}$$

which yields

$$v(t) = \begin{cases} -(CB)^{-1}\left[C(kI + A)e + q\right], & \text{if } s(e) > 0 \\ -(CB)^{-1}\left[C(kI + A)e - q\right], & \text{if } s(e) < 0 \end{cases} \tag{16}$$

Theorem 2.1. *The master system (1) and the slave system (2) are globally and asymptotically hybrid synchronized for all initial conditions* $x(0), y(0) \in R^n$ *by the feedback control law*

$$u(t) = -\eta(x, y) + Bv(t) \tag{17}$$

where $v(t)$ *is defined by (15) and* B *is a column vector such that* (A, B) *is controllable. Also, the sliding mode gains* k, q *are positive.*

Proof. First, we note that substituting (17) and (15) into the error dynamics (4), we obtain the closed-loop error dynamics as

$$\dot{e} = Ae - B(CB)^{-1}\left[C(kI + A)e + q\,\text{sgn}(s)\right] \tag{18}$$

To prove that the error dynamics (18) is globally asymptotically stable, we consider the candidate Lyapunov function defined by the equation

$$V(e) = \frac{1}{2} s^2(e) \tag{19}$$

which is a positive definite function on R^n.

Differentiating V along the trajectories of (18) or the equivalent dynamics (14), we get

$$\dot{V}(e) = s(e)\dot{s}(e) = -ks^2 - q\,\text{sgn}(s)s \tag{20}$$

which is a negative definite function on R^n.

This calculation shows that V is a globally defined, positive definite, Lyapunov function for the error dynamics (18), which has a globally defined, negative definite time derivative $\dot{V}$.

Thus, by Lyapunov stability theory [28], it is immediate that the error dynamics (18) is globally asymptotically stable for all initial conditions $e(0) \in R^n$.

This means that for all initial conditions $e(0) \in R^n$, we have

$$\lim_{t \to \infty} \| e(t) \| = 0$$

Hence, it follows that the master system (1) and the slave system (2) are globally and asymptotically hybrid synchronized for all initial conditions $x(0), y(0) \in R^n$.

3. HYBRID SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC XU SYSTEMS VIA SLIDING MODE CONTROL

3.1 Theoretical Results

In this section, we apply the sliding mode control results derived in Section 2 for the hybrid synchronization of identical hyperchaotic Xu systems ([27], Xu *et al.* 2009).
Thus, the master system is described by the Xu dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4 \\ \dot{x}_2 &= bx_1 + fx_1x_3 \\ \dot{x}_3 &= -cx_3 - \varepsilon x_1x_2 \\ \dot{x}_4 &= -dx_4 + x_1x_3 \end{aligned} \tag{21}$$

where x_1, x_2, x_3, x_4 are state variables and $a, b, c, d, \varepsilon, f$ are positive, constant parameters of the system.

The Xu system (21) is *hyperchaotic* when the parameters are chosen as

$$a = 10, \quad b = 40, \; c = 2.5, \quad d = 2, \quad \varepsilon = 1 \;\text{ and }\; f = 16$$

Figure 1 illustrates the phase portrait of the hyperchaotic Xu system.

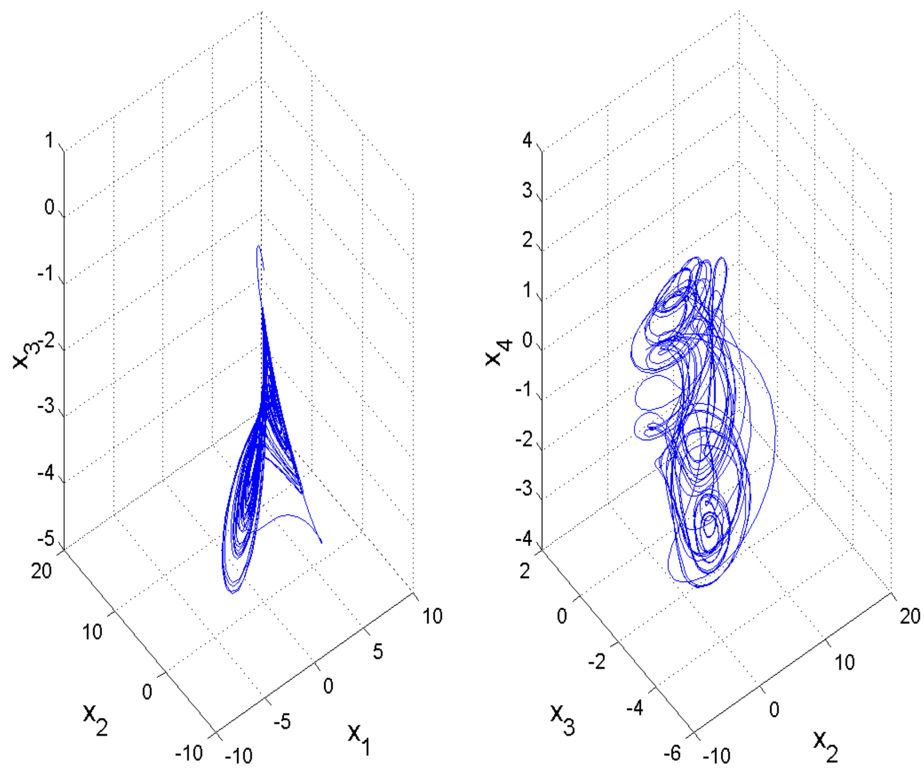

Figure 1. Phase Portrait of the Hyperchaotic Xu System

The slave system is described by the controlled hyperchaotic Xu dynamics

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + y_4 + u_1 \\
\dot{y}_2 &= by_1 + fy_1y_3 + u_2 \\
\dot{y}_3 &= -cy_3 - \varepsilon y_1y_2 + u_3 \\
\dot{y}_4 &= -dy_4 + y_1y_3 + u_4
\end{aligned} \tag{22}$$

where y_1, y_2, y_3, y_4 are state variables and u_1, u_2, u_3, u_4 are the controllers to be designed.

The hybrid synchronization error is defined by

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 + x_2 \\ e_3 &= y_3 - x_3 \\ e_4 &= y_4 + x_4 \end{aligned} \tag{23}$$

The error dynamics is easily obtained as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + e_4 - 2ax_2 - 2x_4 + u_1 \\ \dot{e}_2 &= be_1 + 2bx_1 + f(y_1 y_3 + x_1 x_3) + u_2 \\ \dot{e}_3 &= -ce_3 - \varepsilon y_1 y_2 + \varepsilon x_1 x_2 + u_3 \\ \dot{e}_4 &= -de_4 + y_1 y_3 + x_1 x_3 + u_4 \end{aligned} \tag{24}$$

We write the error dynamics (24) in the matrix notation as

$$\dot{e} = Ae + \eta(x, y) + u \tag{25}$$

where

$$A = \begin{bmatrix} -a & a & 0 & 1 \\ b & 0 & 0 & 0 \\ 0 & 0 & -c & 0 \\ 0 & 0 & 0 & -d \end{bmatrix}, \quad \eta(x, y) = \begin{bmatrix} -2ax_2 - 2x_4 \\ 2bx_1 + f(y_1 y_3 + x_1 x_3) \\ -\varepsilon(y_1 y_2 - x_1 x_2) \\ y_1 y_3 + x_1 x_3 \end{bmatrix} \text{ and } \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{26}$$

The sliding mode controller design is carried out as detailed in Section 2.

First, we set u as

$$u = -\eta(x, y) + Bv \tag{27}$$

where B is chosen such that (A, B) is controllable.
We take B as

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{28}$$

In the hyperchaotic case, the parameter values are taken as

$$a = 10, \quad b = 40, \quad c = 2.5, \quad d = 2, \quad \varepsilon = 1 \quad \text{and} \quad f = 16$$

The sliding mode variable is selected as

$$s = Ce = \begin{bmatrix} 9 & 1 & 1 & -9 \end{bmatrix} e = 9e_1 + e_2 + e_3 - 9e_4 \tag{29}$$

which makes the sliding mode state equation asymptotically stable.

We choose the sliding mode gains as $k = 6$ and $q = 0.2.$

We note that a large value of k can cause chattering and an appropriate value of q is chosen to speed up the time taken to reach the sliding manifold as well as to reduce the system chattering.

From Eq. (15), we can obtain $v(t)$ as

$$v(t) = -2e_1 - 48e_2 - 1.75e_3 + 13.5e_4 - 0.1\,\text{sgn}(s) \tag{30}$$

Thus, the required sliding mode controller is obtained as

$$u = -\eta(x, y) + Bv \tag{31}$$

where $\eta(x, y), B$ and $v(t)$ are defined as in the equations (26), (28) and (30).

By Theorem 2.1, we obtain the following result.

Theorem 3.1. *The identical hyperchaotic Xu systems (21) and (22) are globally and asymptotically hybrid synchronized for all initial conditions with the sliding mode controller* u *defined by (31).*

3.2 Numerical Results

In this section For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the hyperchaotic Xu systems (21) and (22) with the sliding mode controller u given by (31) using MATLAB.

In the hyperchaotic case, the parameter values are given by

$$a = 10, \quad b = 40, \quad c = 2.5, \quad d = 2, \quad \varepsilon = 1 \quad \text{and} \quad f = 16$$

The sliding mode gains are chosen as $k = 6$ and $q = 0.2.$

The initial values of the master system (21) are taken as

$$x_1(0) = 14, \; x_2(0) = 9, \; x_3(0) = 27, \quad x_4(0) = -10$$

The initial values of the slave system (22) are taken as

$$y_1(0) = 4, \; y_2(0) = 12, \; y_3(0) = -6, \; y_4(0) = 22$$

Figure 2 illustrates the hybrid synchronization of the hyperchaotic Xu systems (21) and (22).

Figure 3 illustrates the time-history of the synchronization errors e_1, e_2, e_3, e_4.

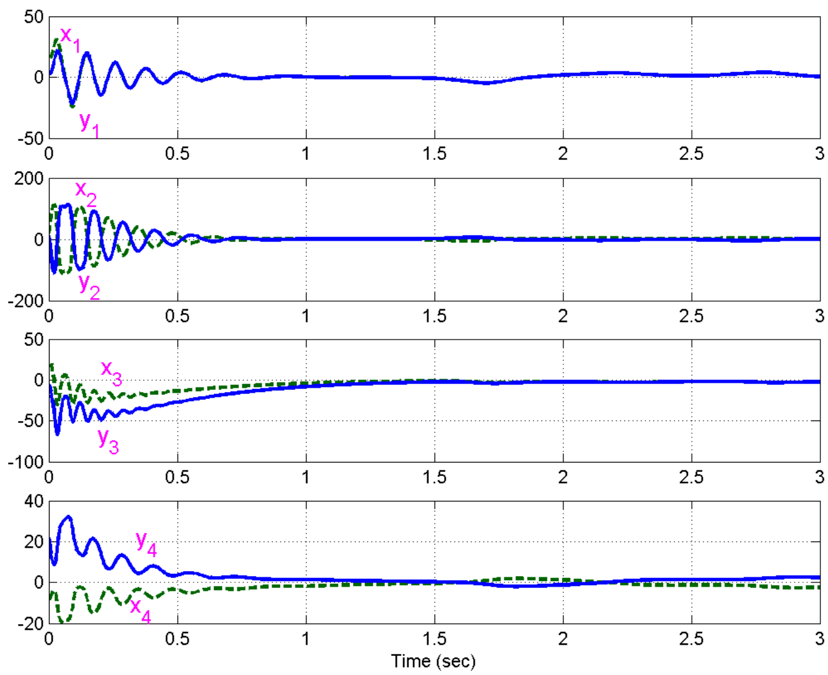

Figure 2. Hybrid Synchronization of Identical Hyperchaotic Xu Systems

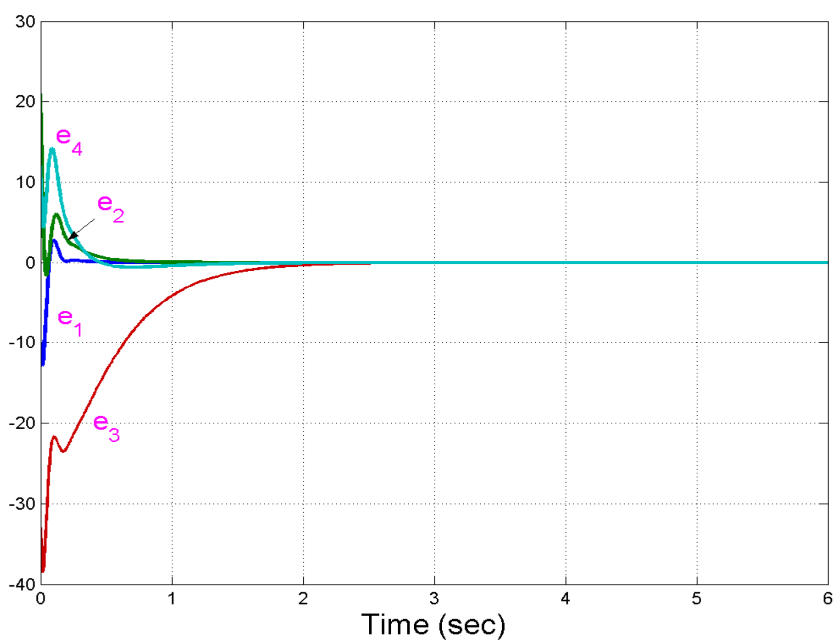

Figure 3. Time-History of the Hybrid Synchronization Error

4. CONCLUSIONS

In this paper, we have derived sliding mode controllers to achieve hybrid synchronization for the identical hyperchaotic Xu systems (2009). Our hybrid synchronization results for the identical hyperchaotic Xu systems have been proved using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the sliding mode control method is very effective and convenient to achieve hybrid synchronization for the identical hyperchaotic Xu systems. Numerical simulations are also shown to illustrate the effectiveness of the hybrid synchronization results derived in this paper using the sliding mode control.

REFERENCES

[1] Alligood, K.T., Sauer, T. & Yorke, J.A. (1997) Chaos: An Introduction to Dynamical Systems, Springer, New York.

[2] Pecora, L.M. & Carroll, T.L. (1990) "Synchronization in chaotic systems", Phys. Rev. Lett., Vol. 64, pp 821-824.

[3] Lakshmanan, M. & Murali, K. (1996) Nonlinear Oscillators: Controlling and Synchronization, World Scientific, Singapore.

[4] Han, S.K., Kerrer, C. & Kuramoto, Y. (1995) "Dephasing and burstling in coupled neural oscillators", Phys. Rev. Lett., Vol. 75, pp 3190-3193.

[5] Blasius, B., Huppert, A. & Stone, L. (1999) "Complex dynamics and phase synchronization in spatially extended ecological system", Nature, Vol. 399, pp 354-359.

[6] Cuomo, K.M. & Oppenheim, A.V. (1993) "Circuit implementation of synchronized chaos with applications to communications," Physical Review Letters, Vol. 71, pp 65-68.

[7] Kocarev, L. & Parlitz, U. (1995) "General approach for chaotic synchronization with applications to communication," Physical Review Letters, Vol. 74, pp 5028-5030.

[8] Tao, Y. (1999) "Chaotic secure communication systems – history and new results," Telecommun. Review, Vol. 9, pp 597-634.

[9] Ott, E., Grebogi, C. & Yorke, J.A. (1990) "Controlling chaos", Phys. Rev. Lett., Vol. 64, pp 1196-1199.

[10] Ho, M.C. & Hung, Y.C. (2002) "Synchronization of two different chaotic systems using generalized active network," Physics Letters A, Vol. 301, pp 424-428.

[11] Huang, L., Feng, R. & Wang, M. (2005) "Synchronization of chaotic systems via nonlinear control," Physical Letters A, Vol. 320, pp 271-275.

[12] Chen, H.K. (2005) "Global chaos synchronization of new chaotic systems via nonlinear control," Chaos, Solitons & Fractals, Vol. 23, pp 1245-1251.

[13] Sundarapandian, V. (2011) "Global chaos synchronization of four-scroll and four-wing chaotic attractors by active nonlinear control," International Journal on Computer Science and Engineering, Vol. 3, No. 5, pp. 2145-2155.

[14] Sundarapandian, V. (2011) "Global chaos synchronization of Li and Liu-Chen-Liu chaotic systems by active nonlinear control," International Journal of Advances in Science and Technology, Vol. 3, No. 1, pp. 1-12.

[15] Lu, J., Wu, X., Han, X. & Lü, J. (2004) "Adaptive feedback synchronization of a unified chaotic system," Physics Letters A, Vol. 329, pp 327-333.

[16] Chen, S.H. & Lü, J. (2002) "Synchronization of an uncertain unified system via adaptive control," Chaos, Solitons & Fractals, Vol. 14, pp 643-647.

[17] Sundarapandian, V. (2011) "Adaptive control and synchronization of hyperchaotic Cai system", International Journal of Control Theory and Computer Modelling, Vol. 1, No. 1, pp 1-13.

[18] Sundarapandian, V. (2011) "Adaptive synchronization of hyperchaotic Lorenz and hyperchaotic Liu systems", International Journal of Instrumentation and Control Systems, Vol. 1, No. 1, pp 1-18.

[19] Sundarapandian, V. (2011) "Adaptive control and synchronization of Liu's four-wing chaotic system with cubic nonlinearity," International Journal of Computer Science, Engineering and Applications, Vol. 1, No. 4, pp 127-138.

[20] Sundarapandian, V. & Karthikeyan, R. (2011) "Global chaos synchronization of Pan and Lü chaotic systems via adaptive control," International Journal of Information Technology, Convergence and Services, Vol. 1, No. 5, pp. 49-66.
[21] Park, J.H. & Kwon, O.M. (2003) "A novel criterion for delayed feedback control of time-delay chaotic systems," Chaos, Solitons & Fractals, Vol. 17, pp 709-716.
[22] Wu, X. & Lü, J. (2003) "Parameter identification and backstepping control of uncertain Lü system," Chaos, Solitons & Fractals, Vol. 18, pp 721-729.
[23] Zhao, J. & J. Lu (2006) "Using sampled-data feedback control and linear feedback synchronization in a new hyperchaotic system," Chaos, Solitons & Fractals, Vol. 35, pp 376-382.
[24] Slotine, J.E. & Sastry, S.S. (1983) "Tracking control of nonlinear systems using sliding surface with application to robotic manipulators," International Journal of Control, Vol. 38, pp 465-492.
[25] Utkin, V.I. (1993) "Sliding mode control design principles and applications to electric drives," IEEE Trans. Industrial Electronics, Vol. 40, pp 23-36, 1993.
[26] Sundarapandian, V. (2011) "Global chaos synchronization of four-wing chaotic systems by sliding mode control", International Journal of Control Theory and Computer Modelling, Vol. 1, No. 1, pp 15-31.
[27] Xu, J., Cai, G. & Zheng, Song. (2009) "A novel hyperchaotic system and its control", Journal of Uncertain Systems, Vol. 3, No. 2, pp. 137-144.
[28] Hahn, W. (1967) The Stability of Motion, Springer, New York.

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