ADAPTIVE CONTROLLER DESIGN FOR THE ANTI-SYNCHRONIZATION OF HYPERCHAOTIC YANG AND HYPERCHAOTIC PANG SYSTEMS

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ABSTRACT

In the anti-synchronization of chaotic systems, a pair of chaotic systems called drive and responsesystems are considered, and the design goal is to drive the sum of their respective states to zero asymptotically. This paper derives new results for the anti-synchronization of hyperchaotic Yang system (2009) and hyperchaotic Pang system (2011) with uncertain parameters via adaptive control. Hyperchaotic systems are nonlinear chaotic systems withtwo or more positive Lyapunov exponents and they have applications in areas like neural networks, encryption, secure data transmission and communication. The main results derived in this paper are illustrated with MATLAB simulations.

KEYWORDS

Hyperchaos, Adaptive Control, Anti-Synchronization, Hyperchaotic Systems.

1. INTRODUCTION

Since the discovery of a hyperchaotic system by O.E.Rössler ([1], 1979), hyperchaotic systems are known to have characteristics like high security, high capacity and high efficiency. Hyperchaotic systems are chaotic systems having two or morepositive Lyapunov exponents. They are applicable in several areas like oscillators [2], neural networks [3], secure communication [4-5], data encryption [6], chaos synchronization [7], etc.

The synchronization problem deals with a pair of chaotic systems called the drive and response chaotic systems, where the design goal is to drive the difference of their respective states to zero asymptotically [8-9].

The anti-synchronization problem deals with a pair of chaotic systems called the drive and response systems, where the design goal is to drive the sum of their respective states to zero asymptotically.

The problems of synchronization and anti-synchronization of chaotic and hyperchaotic systems have been studied via several methods like active control method [10-12], adaptive control method [13-15],backstepping method [16-19], sliding control method [20-22] etc.

This paper derives new results for the adaptive controller design for the anti-synchronization of hyperchaotic Yang systems ([23], 2009) and hyperchaotic Pang systems ([24], 2008) with unknown parameters. The main results derived in this paper were proved using adaptive control theory [25] and Lyapunov stability theory [26].

2. PROBLEM STATEMENT

The *drive system* is described by the chaotic dynamics

$$\dot{x} = Ax + f(x) \tag{1}$$

where A is the $n\times n$ matrix of the system parameters and $f: R^n \to R^n$ is the nonlinear part.
The *response system* is described by the chaotic dynamics

$$\dot{y} = By + g(y) + u \tag{2}$$

where B is the $n\times n$ matrix of the system parameters, $g: R^n \to R^n$ is the nonlinear part and $u \in R^n$ is the active controller to be designed.

For the pair of chaotic systems (1) and (2), the design goal of the anti-synchronization problemis to construct a feedback controller u, which *anti-synchronizes* their states for all $x(0), y(0) \in R^n$.

The*anti-synchronization error*is defined as

$$e = y + x, \tag{3}$$

Theerror dynamics is obtained as

$$\dot{e} = By + Ax + g(y) + f(x) + u \tag{4}$$

The design goal is to find a feedback controller $u$so that

$$\lim_{t\to\infty} \|e(t)\| = 0 \text{ for all } e(0) \in R^n \tag{5}$$

Using the matrix method, we consider a candidate Lyapunov function

$$V(e) = e^T Pe, \tag{6}$$

where P is a positive definite matrix.

It is noted that $V: R^n \to R$ is a positive definite function.
If we find a feedback controller $u$so that

$$\dot{V}(e) = -e^T Qe, \tag{7}$$

where Q is a positive definite matrix, then $\dot{V}: R^n \to R$ is a negative definite function.
Thus, by Lyapunov stability theory [26], the error dynamics (4) is globally exponentially stable.

When the system parameters in (1) and (2) are unknown, we need to construct a parameter update law for determining the estimates of the unknown parameters.

3. HYPERCHAOTIC SYSTEMS

The hyperchaotic Yang system ([23], 2009) is given by

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) \\
\dot{x}_2 &= cx_1 - x_1x_3 + x_4 \\
\dot{x}_3 &= -bx_3 + x_1x_2 \\
\dot{x}_4 &= -dx_1 - \varepsilon x_2
\end{aligned} \tag{8}$$

where a, b, c, d are constant, positive parameters of the system.
The Yang system (8) exhibits a hyperchaotic attractor for the parametric values

$$a = 35, \quad b = 3, \quad c = 35, \quad d = 2, \quad \varepsilon = 7.5 \tag{9}$$

The Lyapunov exponents of the system (8) for the parametric values in (9) are

$$\lambda_1 = 0.2747, \quad \lambda_2 = 0.1374, \quad \lambda_3 = 0, \quad \lambda_4 = -38.4117 \tag{10}$$

Since there are two positive Lyapunov exponents in (10), the Yang system (8) is hyperchaotic for the parametric values (9).

The phase portrait of the hyperchaotic Yang system is described in Figure 1.
The hyperchaotic Pang system ([24], 2011) is given by

$$\begin{aligned}
\dot{x}_1 &= \alpha(x_2 - x_1) \\
\dot{x}_2 &= \gamma x_2 - x_1x_3 + x_4 \\
\dot{x}_3 &= -\beta x_3 + x_1x_2 \\
\dot{x}_4 &= -\delta(x_1 + x_2)
\end{aligned} \tag{11}$$

where $\alpha, \beta, \gamma, \delta$ are constant, positive parameters of the system.
The Pang system (11) exhibits a hyperchaotic attractor for the parametric values

$$\alpha = 36, \quad \beta = 3, \quad \gamma = 20, \quad \delta = 2 \tag{12}$$

The Lyapunov exponents of the system (9) for the parametric values in (12) are

$$\lambda_1 = 1.4106, \quad \lambda_2 = 0.1232, \quad \lambda_3 = 0, \quad \lambda_4 = -20.5339 \tag{13}$$

Since there are two positive Lyapunov exponents in (13), the Pang system (11) is hyperchaotic for the parametric values (12).

The phase portrait of the hyperchaotic Pang system is described in Figure 2.

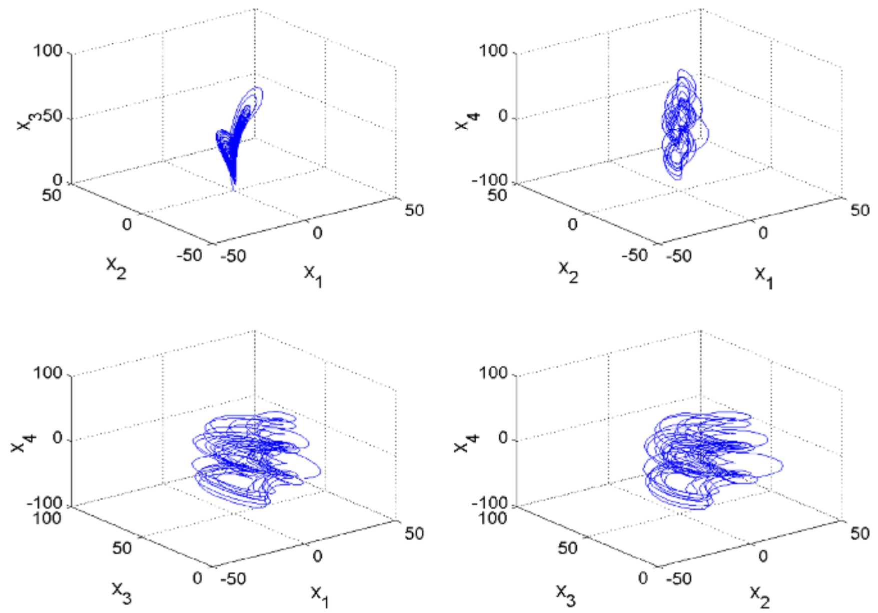

Figure 1. The Phase Portrait of the Hyperchaotic Yang System

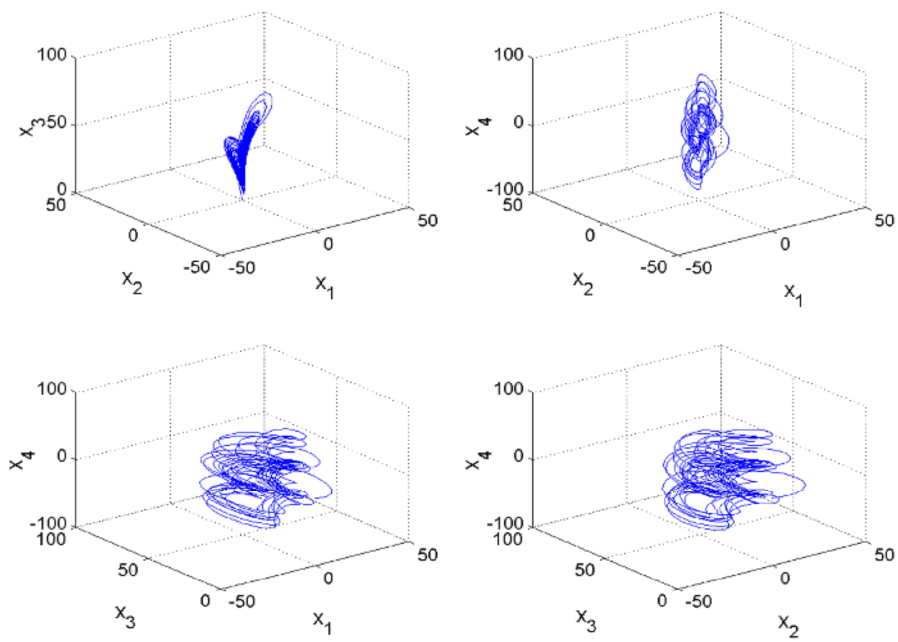

Figure 2. The Phase Portrait of the Hyperchaotic Pang System

4. ADAPTIVE CONTROL DESIGN FOR THE ANTI-SYNCHRONIZATION OF HYPERCHAOTIC YANG SYSTEMS

In this section, we design an adaptive controller for the anti-synchronization of two identical hyperchaotic Yang systems (2009) with unknown parameters.

Thedrive system is the hyperchaotic Yangdynamicsgiven by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= cx_1 - x_1x_3 + x_4 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \\ \dot{x}_4 &= -dx_1 - \varepsilon x_2 \end{aligned} \tag{14}$$

where a, b, c, d, ε are unknown parameters of the system and $x \in R^4$ is the state.
The response system is the controlled hyperchaotic Yangdynamics given by

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= cy_1 - y_1y_3 + y_4 + u_2 \\ \dot{y}_3 &= -by_3 + y_1y_2 + u_3 \\ \dot{y}_4 &= -dy_1 - \varepsilon y_2 + u_4 \end{aligned} \tag{15}$$

where $y \in R^4$ is the state and u_1, u_2, u_3, u_4 are the adaptivecontrollers to be designed.
For the anti-synchronization, the error e is defined as

$$e_1 = y_1 + x_1, \quad e_2 = y_2 + x_2, \quad e_3 = y_3 + x_3, \quad e_4 = y_4 + x_4 \tag{16}$$

Then we derive the error dynamics as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + u_1 \\ \dot{e}_2 &= ce_1 + e_4 - y_1y_3 - x_1x_3 + u_2 \\ \dot{e}_3 &= -be_3 + y_1y_2 + x_1x_2 + u_3 \\ \dot{e}_4 &= -de_1 - \varepsilon e_2 + u_4 \end{aligned} \tag{17}$$

The adaptive controller to achieve anti-synchronization is chosen as

$$\begin{aligned} u_1(t) &= -\hat{a}(t)(e_2 - e_1) - k_1e_1 \\ u_2(t) &= -\hat{c}(t)e_1 - e_4 + y_1y_3 + x_1x_3 - k_2e_2 \\ u_3(t) &= \hat{b}(t)e_3 - y_1y_2 - x_1x_2 - k_3e_3 \\ u_4(t) &= \hat{d}(t)e_1 + \hat{\varepsilon}(t)e_2 - k_4e_4 \end{aligned} \tag{18}$$

In Eq. (18), k_i, $(i = 1,2,3,4)$ are positive gains and $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{d}(t), \hat{\varepsilon}(t)$ are estimates for the unknown parameters a, b, c, d, ε respectively.

By the substitution of (18) into (17), the error dynamics is simplified as

$$\begin{aligned}
\dot{e}_1 &= (a - \hat{a}(t))(e_2 - e_1) - k_1 e_1 \\
\dot{e}_2 &= (c - \hat{c}(t))e_1 - k_2 e_2 \\
\dot{e}_3 &= -(b - \hat{b}(t))e_3 - k_3 e_3 \\
\dot{e}_4 &= -(d - \hat{d}(t))e_1 - (\varepsilon - \hat{\varepsilon}(t))e_2 - k_4 e_4
\end{aligned} \tag{19}$$

As a next step, we define the parameter estimation errors as

$$e_a(t) = a - \hat{a}(t),\ e_b(t) = b - \hat{b}(t),\ e_c(t) = c - \hat{c}(t),\ e_d(t) = d - \hat{d}(t),\ e_\varepsilon(t) = \varepsilon - \hat{\varepsilon}(t) \tag{20}$$

Upon differentiation, we get

$$\dot{e}_a(t) = -\dot{\hat{a}}(t),\ \dot{e}_b(t) = -\dot{\hat{b}}(t),\ \dot{e}_c(t) = -\dot{\hat{c}}(t),\ \dot{e}_d(t) = -\dot{\hat{d}}(t),\ \dot{e}_\varepsilon(t) = -\dot{\hat{\varepsilon}}(t) \tag{21}$$

Substituting (20) into the error dynamics (19), we obtain

$$\begin{aligned}
\dot{e}_1 &= e_a(e_2 - e_1) - k_1 e_1 \\
\dot{e}_2 &= e_c e_1 - k_2 e_2 \\
\dot{e}_3 &= -e_b e_3 - k_3 e_3 \\
\dot{e}_4 &= -e_d e_1 - e_\varepsilon e_2 - k_4 e_4
\end{aligned} \tag{22}$$

We consider the candidate Lyapunov function

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2 + e_\varepsilon^2\right) \tag{23}$$

Differentiating (23) along the dynamics (21) and (22), we obtain

$$\begin{aligned}
\dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 + e_a\left[e_1(e_2 - e_1) - \dot{\hat{a}}\right] + e_b\left(-e_3^2 - \dot{\hat{b}}\right) \\
&+ e_c\left(e_1 e_2 - \dot{\hat{c}}\right) + e_d\left(-e_1 e_4 - \dot{\hat{d}}\right) + e_\varepsilon\left(-e_2 e_4 - \dot{\hat{\varepsilon}}\right)
\end{aligned} \tag{24}$$

In view of (24), we choose the following parameter update law:

$$\begin{aligned}
&\dot{\hat{a}} = e_1(e_2 - e_1) + k_5 e_a, \quad && \dot{\hat{d}} = -e_1 e_4 + k_8 e_d \\
&\dot{\hat{b}} = -e_3^2 + k_6 e_b, && \dot{\hat{\varepsilon}} = -e_2 e_4 + k_9 e_\varepsilon \\
&\dot{\hat{c}} = e_1 e_2 + k_7 e_c
\end{aligned} \tag{25}$$

Next, we prove the following main result of this section.

Theorem 4.1 The adaptive control law defined by Eq. (18) along with the parameter update law defined by Eq. (25) achieveglobal and exponential anti-synchronization of the identical hyperchaotic Yang systems (14) and (15)with unknown parameters for all initial conditions $x(0), y(0) \in R^4$. Moreover, the parameter estimation errors $e_a(t), e_b(t), e_c(t), e_d(t), e_\varepsilon(t)$ globally and exponentially converge to zero for all initial conditions.

Proof. The proof is via Lyapunov stability theory [26] by taking *V* defined by Eq. (23) as the candidate Lyapunov function. Substituting the parameter update law (25) into (24), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 - k_5 e_a^2 - k_6 e_b^2 - k_7 e_c^2 - k_8 e_d^2 - k_9 e_\varepsilon^2 \tag{26}$$

which is a negative definite function on R^9. This completes the proof. ■

Next, we illustrate our adaptive anti-synchronization results with MATLAB simulations.The classical fourth order Runge-Kutta method with time-step $h = 10^{-8}$ has been used to solve the hyperchaotic Yang systems (14) and (15) with the nonlinear controller defined by (18).
The feedback gains in the adaptive controller (18) are taken as $k_i = 4, \ (i = 1, \ldots, 9).$

The parameters of the hyperchaotic Yang systems are taken as in the hyperchaotic case, *i.e.*

$$a = 35, \quad b = 3, \quad c = 35, \quad d = 2, \quad \varepsilon = 7.5$$

For simulations, the initial conditions of the drive system (14) are taken as

$$x_1(0) = 7, \ \ x_2(0) = 16, \ \ x_3(0) = -23, \ \ x_4(0) = -5$$

Also, the initial conditions of the response system (15) are taken as

$$y_1(0) = 34, \ \ y_2(0) = -8, \ \ y_3(0) = 28, \ \ y_4(0) = -20$$

Also, the initial conditions of the parameter estimates are taken as

$$\hat{a}(0) = 12, \ \ \hat{b}(0) = 8, \ \ \hat{c}(0) = -7, \ \ \hat{d}(0) = 5, \quad \hat{\varepsilon}(0) = 4$$

Figure 3 depicts the anti-synchronization of the identical hyperchaotic Yang systems.
Figure 4 depicts the time-history of the anti-synchronization errors e_1, e_2, e_3, e_4.
Figure 5 depicts the time-history of the parameter estimation errors $e_a, e_b, e_c, e_d, e_\varepsilon$.

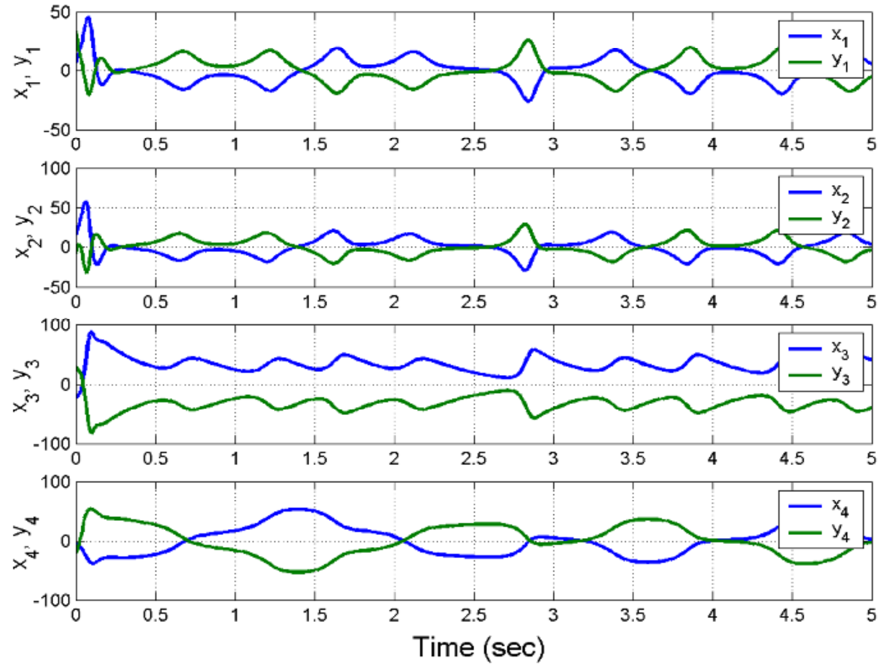

Figure 3. Anti-Synchronization of Identical Hyperchaotic Yang Systems

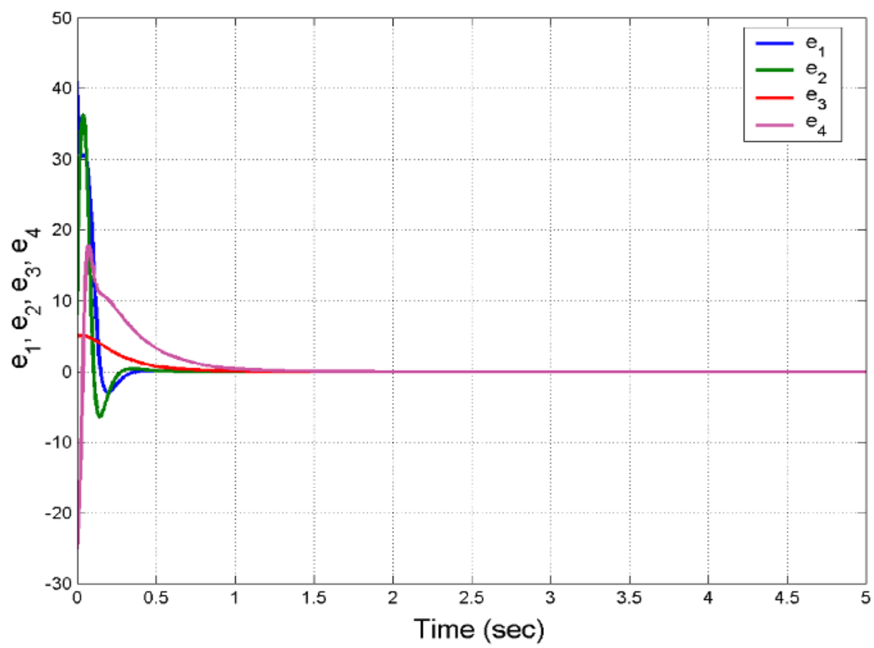

Figure 4. Time-History of the Anti-Synchronization Errors e_1, e_2, e_3, e_4

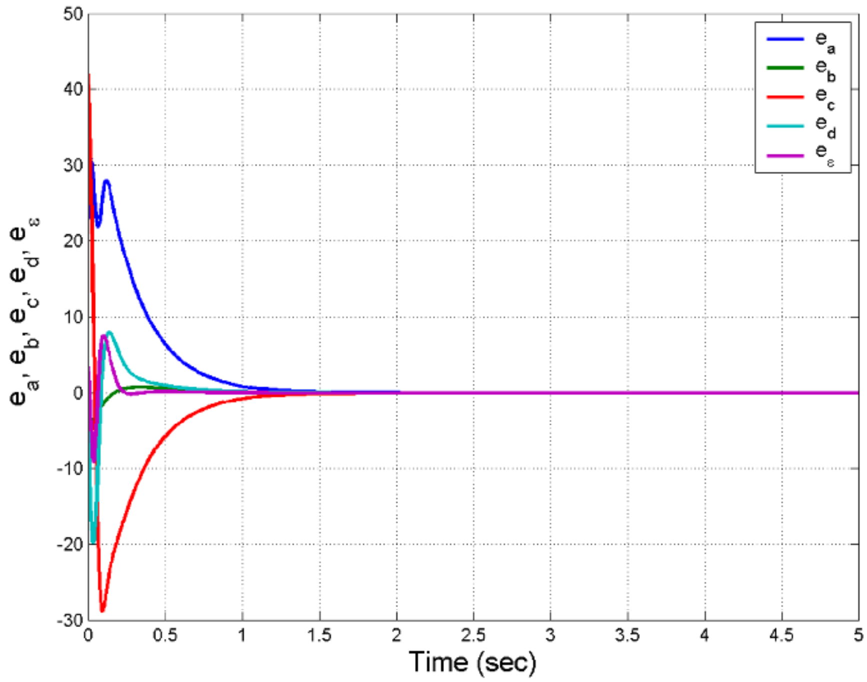

Figure 5. Time-History of the Parameter Estimation Errors $e_a, e_b, e_c, e_d, e_\varepsilon$

5. ADAPTIVE CONTROL DESIGN FOR THE ANTI-SYNCHRONIZATION OF HYPERCHAOTIC PANG SYSTEMS

In this section, we design an adaptive controller for the anti-synchronization of two identical hyperchaotic Pang systems (2011) with unknown parameters.
Thedrive system is the hyperchaotic Pangdynamics given by

$$\begin{aligned}\dot{x}_1 &= \alpha(x_2 - x_1)\\ \dot{x}_2 &= \gamma x_2 - x_1 x_3 + x_4\\ \dot{x}_3 &= -\beta x_3 + x_1 x_2\\ \dot{x}_4 &= -\delta(x_1 + x_2)\end{aligned} \tag{27}$$

where $\alpha, \beta, \gamma, \delta$ are unknown parameters of the system and $x \in R^4$ is the state.
The response system is the controlled hyperchaotic Pangdynamics given by

$$\begin{aligned}\dot{y}_1 &= \alpha(y_2 - y_1) + u_1\\ \dot{y}_2 &= \gamma y_2 - y_1 y_3 + y_4 + u_2\\ \dot{y}_3 &= -\beta y_3 + y_1 y_2 + u_3\\ \dot{y}_4 &= -\delta(y_1 + y_2) + u_4\end{aligned} \tag{28}$$

where $y \in R^4$ is the state and u_1, u_2, u_3, u_4 are the adaptivecontrollers to be designed.
For the anti-synchronization, the error e is defined as

$$e_1 = y_1 + x_1, \quad e_2 = y_2 + x_2, \quad e_3 = y_3 + x_3, \quad e_4 = y_4 + x_4 \tag{29}$$

Then we derive the error dynamics as

$$\begin{aligned}
\dot{e}_1 &= \alpha(e_2 - e_1) + u_1 \\
\dot{e}_2 &= \gamma e_2 + e_4 - y_1 y_3 - x_1 x_3 + u_2 \\
\dot{e}_3 &= -\beta e_3 + y_1 y_2 + x_1 x_2 + u_3 \\
\dot{e}_4 &= -\delta(e_1 + e_2) + u_4
\end{aligned} \tag{30}$$

The adaptive controller to achieve anti-synchronization is chosen as

$$\begin{aligned}
u_1 &= -\hat{\alpha}(t)(e_2 - e_1) - k_1 e_1 \\
u_2 &= -\hat{\gamma}(t) e_2 - e_4 + y_1 y_3 + x_1 x_3 - k_2 e_2 \\
u_3 &= \hat{\beta}(t) e_3 - y_1 y_2 - x_1 x_2 - k_3 e_3 \\
u_4 &= \hat{\delta}(t)(e_1 + e_2) - k_4 e_4
\end{aligned} \tag{31}$$

In Eq. (31), $k_i, \ (i = 1, 2, 3, 4)$ are positive gains and $\hat{\alpha}(t), \hat{\beta}(t), \hat{\gamma}(t), \hat{\delta}(t)$ are estimates for the unknown parameters $\alpha, \beta, \gamma, \delta$ respectively.

By the substitution of (31) into (30), the error dynamics is simplified as

$$\begin{aligned}
\dot{e}_1 &= (\alpha - \hat{\alpha}(t))(e_2 - e_1) - k_1 e_1 \\
\dot{e}_2 &= (\gamma - \hat{\gamma}(t)) e_2 - k_2 e_2 \\
\dot{e}_3 &= -(\beta - \hat{\beta}(t)) e_3 - k_3 e_3 \\
\dot{e}_4 &= -(\delta - \hat{\delta}(t))(e_1 + e_2) - k_4 e_4
\end{aligned} \tag{32}$$

As a next step, we define the parameter estimation errors as

$$e_\alpha(t) = \alpha - \hat{\alpha}(t), \ e_\beta(t) = \beta - \hat{\beta}(t), \ e_\gamma(t) = \gamma - \hat{\gamma}(t), \ e_\delta(t) = \delta - \hat{\delta}(t) \tag{33}$$

Upon differentiation, we get

$$\dot{e}_\alpha(t) = -\dot{\hat{\alpha}}(t), \ \dot{e}_\beta(t) = -\dot{\hat{\beta}}(t), \ \dot{e}_\gamma(t) = -\dot{\hat{\gamma}}(t), \ \dot{e}_\delta(t) = -\dot{\hat{\delta}}(t) \tag{34}$$

Substituting (33) into the error dynamics (32), we obtain

$$
\begin{aligned}
\dot{e}_1 &= e_\alpha (e_2 - e_1) - k_1 e_1 \\
\dot{e}_2 &= e_\gamma e_2 - k_2 e_2 \\
\dot{e}_3 &= -e_\beta e_3 - k_3 e_3 \\
\dot{e}_4 &= -e_\delta (e_1 + e_2) - k_4 e_4
\end{aligned}
\tag{35}
$$

We consider the candidate Lyapunov function

$$
V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_\alpha^2 + e_\beta^2 + e_\gamma^2 + e_\delta^2 \right) \tag{36}
$$

Differentiating (36) along the dynamics (34) and (35), we obtain

$$
\begin{aligned}
\dot{V} = {} & -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 + e_\alpha \left[e_1 (e_2 - e_1) - \dot{\hat{\alpha}} \right] + e_\beta \left(-e_3^2 - \dot{\hat{\beta}} \right) \\
& + e_\gamma \left(e_2^2 - \dot{\hat{\gamma}} \right) + e_\delta \left(-e_1 e_4 (e_1 + e_2) - \dot{\hat{\delta}} \right)
\end{aligned}
\tag{37}
$$

In view of (37), we choose the following parameter update law:

$$
\begin{aligned}
\dot{\hat{\alpha}} &= e_1 (e_2 - e_1) + k_5 e_\alpha \\
\dot{\hat{\beta}} &= -e_3^2 + k_6 e_\beta \\
\dot{\hat{\gamma}} &= e_2^2 + k_7 e_\gamma \\
\dot{\hat{\delta}} &= -e_1 e_4 (e_1 + e_2) + k_8 e_\delta
\end{aligned}
\tag{38}
$$

Next, we prove the following main result of this section.

Theorem 5.1 The adaptive control law defined by Eq. (31) along with the parameter update law defined by Eq. (38) achieve global and exponential anti-synchronization of the identical hyperchaotic Pang systems (27) and (28) with unknown parameters for all initial conditions $x(0), y(0) \in R^4$. Moreover, the parameter estimation errors $e_\alpha(t), e_\beta(t), e_\gamma(t), e_\delta(t)$ globally and exponentially converge to zero for all initial conditions.

Proof. The proof is via Lyapunov stability theory [26] by taking *V* defined by Eq. (36) as the candidate Lyapunov function. Substituting the parameter update law (38) into (37), we get

$$
\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 - k_5 e_a^2 - k_6 e_b^2 - k_7 e_c^2 - k_8 e_d^2 - k_9 e_\varepsilon^2 \tag{39}
$$

which is a negative definite function on R^9. This completes the proof. ■

Next, we illustrate our adaptive anti-synchronization results with MATLAB simulations. The classical fourth order Runge-Kutta method with time-step $h = 10^{-8}$ has been used to solve the hyperchaotic Pang systems (27) and (28) with the nonlinear controller defined by (31).
The feedback gains in the adaptive controller (31) are taken as $k_i = 4, \ (i = 1, \ldots, 8).$

The parameters of the hyperchaotic Pang systems are taken as in the hyperchaotic case, *i.e.*

$$\alpha = 36, \quad \beta = 3, \quad \gamma = 20, \quad \delta = 2$$

For simulations, the initial conditions of the drive system (27) are taken as

$$x_1(0) = 17, \;\; x_2(0) = -22, \;\; x_3(0) = -11, \;\; x_4(0) = 25$$

Also, the initial conditions of the response system (28) are taken as

$$y_1(0) = 24, \;\; y_2(0) = -18, \;\; y_3(0) = 24, \;\; y_4(0) = -17$$

Also, the initial conditions of the parameter estimates are taken as

$$\hat{\alpha}(0) = 3, \;\; \hat{\beta}(0) = -4, \;\; \hat{\gamma}(0) = 27, \;\; \hat{\delta}(0) = 15$$

Figure 6depicts the anti-synchronization of the identical hyperchaotic Pang systems.
Figure 7depicts the time-history of the anti-synchronization errors e_1, e_2, e_3, e_4.
Figure 8 depicts the time-history of the parameter estimation errors $e_\alpha, e_\beta, e_\gamma, e_\delta$.

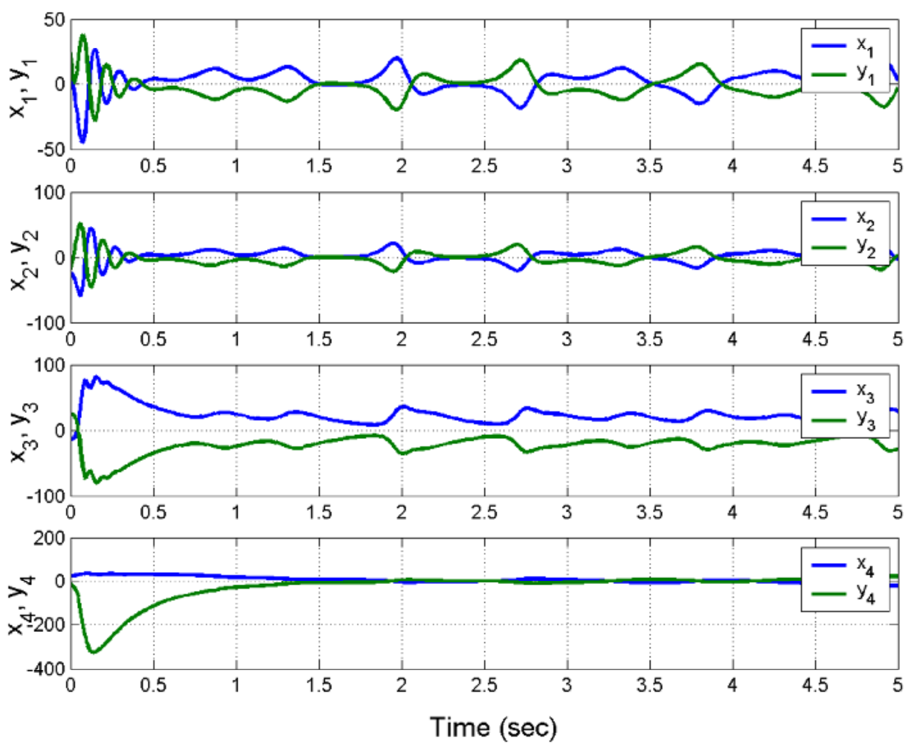

Figure 6. Anti-Synchronization of Identical Hyperchaotic Pang Systems

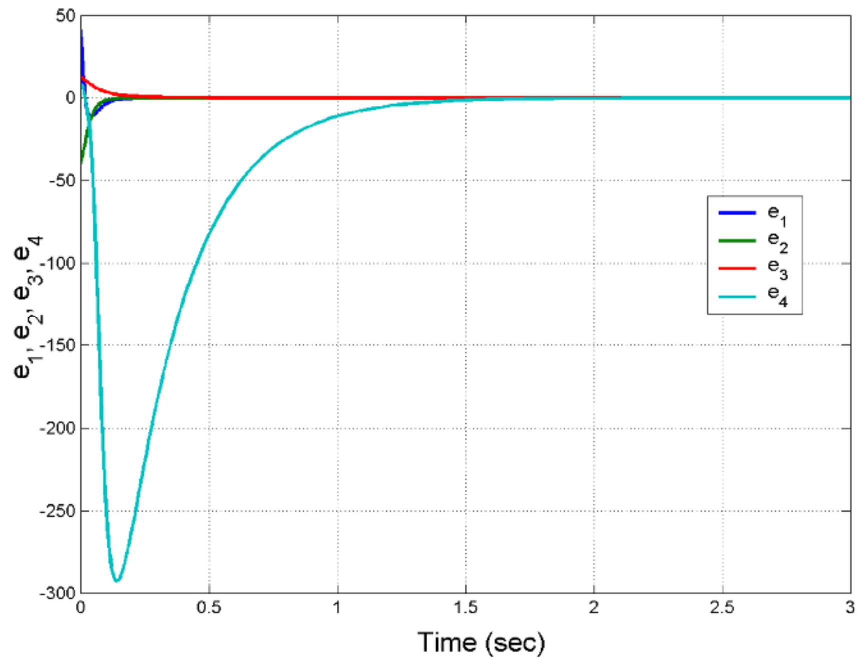

Figure 7. Time-History of the Anti-Synchronization Errors e_1, e_2, e_3, e_4

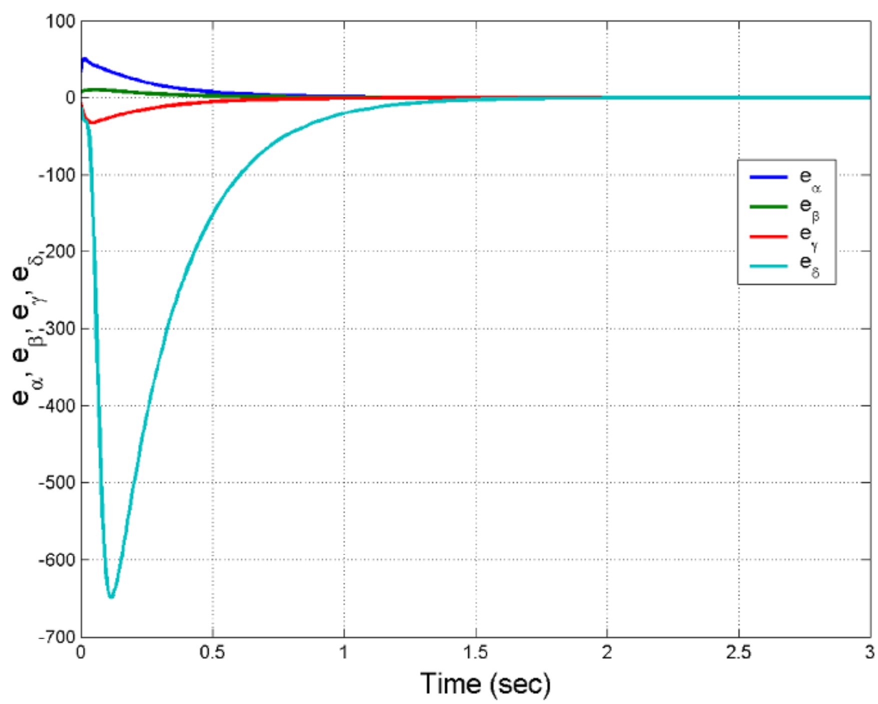

Figure 8. Time-History of the Parameter Estimation Errors $e_\alpha, e_\beta, e_\gamma, e_\delta$

6. ADAPTIVE CONTROL DESIGN FOR THE ANTI-SYNCHRONIZATION OF HYPERCHAOTIC YANG AND HYPERCHAOTIC PANG SYSTEMS

In this section, we design an adaptive controller for the anti-synchronization of non-identical hyperchaotic Yang (2009) and hyperchaotic Pang systems (2011) with unknown parameters.
Thedrive system is the hyperchaotic Yangdynamics given by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= cx_1 - x_1x_3 + x_4 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \\ \dot{x}_4 &= -dx_1 - \varepsilon x_2 \end{aligned} \tag{40}$$

where a, b, c, d, ε are unknown parameters of the system and $x \in R^4$ is the state.
The response system is the controlled hyperchaotic Pangdynamics given by

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\ \dot{y}_2 &= \gamma y_2 - y_1y_3 + y_4 + u_2 \\ \dot{y}_3 &= -\beta y_3 + y_1y_2 + u_3 \\ \dot{y}_4 &= -\delta(y_1 + y_2) + u_4 \end{aligned} \tag{41}$$

where $\alpha, \beta, \gamma, \delta$ are unknown parameters, $y \in R^4$ is the state and u_1, u_2, u_3, u_4 are the adaptivecontrollers to be designed.
For the anti-synchronization, the error e is defined as

$$e_1 = y_1 + x_1, \quad e_2 = y_2 + x_2, \quad e_3 = y_3 + x_3, \quad e_4 = y_4 + x_4 \tag{42}$$

Then we derive the error dynamics as

$$\begin{aligned} \dot{e}_1 &= \alpha(y_2 - y_1) + a(x_2 - x_1) + u_1 \\ \dot{e}_2 &= \gamma y_2 + e_4 + cx_1 - y_1y_3 - x_1x_3 + u_2 \\ \dot{e}_3 &= -\beta y_3 - bx_3 + y_1y_2 + x_1x_2 + u_3 \\ \dot{e}_4 &= -\delta(y_1 + y_2) - dx_1 - \varepsilon x_2 + u_4 \end{aligned} \tag{43}$$

The adaptive controller to achieve anti-synchronization is chosen as

$$\begin{aligned} u_1 &= -\hat{\alpha}(t)(y_2 - y_1) - \hat{a}(t)(x_2 - x_1) - k_1e_1 \\ u_2 &= -\hat{\gamma}(t)y_2 - e_4 - \hat{c}(t)x_1 + y_1y_3 + x_1x_3 - k_2e_2 \\ u_3 &= \hat{\beta}(t)y_3 + \hat{b}(t)x_3 - y_1y_2 - x_1x_2 - k_3e_3 \\ u_4 &= \hat{\delta}(t)(y_1 + y_2) + \hat{d}(t)x_1 + \hat{\varepsilon}(t)x_2 - k_4e_4 \end{aligned} \tag{44}$$

In Eq. (44), k_i, $(i = 1,2,3,4)$ are positive gains, $\hat{a}(t),\hat{b}(t),\hat{c}(t),\hat{d}(t),\hat{\varepsilon}(t)$ are estimates for the unknown parameters a,b,c,d,ε respectively, and $\hat{\alpha}(t),\hat{\beta}(t),\hat{\gamma}(t),\hat{\delta}(t)$ are estimates for the unknown parameters $\alpha,\beta,\gamma,\delta$ respectively.

By the substitution of (44) into (43), the error dynamics is simplified as

$$\begin{aligned}
\dot{e}_1 &= (\alpha - \hat{\alpha}(t))(y_2 - y_1) + (a - \hat{a}(t))(x_2 - x_1) - k_1 e_1 \\
\dot{e}_2 &= (\gamma - \hat{\gamma}(t))y_2 + (c - \hat{c}(t))x_1 - k_2 e_2 \\
\dot{e}_3 &= -(\beta - \hat{\beta}(t))y_3 - (b - \hat{b}(t))x_3 - k_3 e_3 \\
\dot{e}_4 &= -(\delta - \hat{\delta}(t))(y_1 + y_2) - (d - \hat{d}(t))x_1 - (\varepsilon - \hat{\varepsilon}(t))x_2 - k_4 e_4
\end{aligned} \tag{45}$$

As a next step, we define the parameter estimation errors as

$$\begin{aligned}
&e_a(t) = a - \hat{a}(t),\ e_b(t) = b - \hat{b}(t),\ e_c(t) = c - \hat{c}(t),\ e_d(t) = d - \hat{d}(t),\ e_\varepsilon(t) = \varepsilon - \hat{\varepsilon}(t) \\
&e_\alpha(t) = \alpha - \hat{\alpha}(t),\ e_\beta(t) = \beta - \hat{\beta}(t),\ e_\gamma(t) = \gamma - \hat{\gamma}(t),\ e_\delta(t) = \delta - \hat{\delta}(t)
\end{aligned} \tag{46}$$

Upon differentiation, we get

$$\begin{aligned}
&\dot{e}_a(t) = -\dot{\hat{a}}(t),\ \dot{e}_b(t) = -\dot{\hat{b}}(t),\ \dot{e}_c(t) = -\dot{\hat{c}}(t),\ \dot{e}_d(t) = -\dot{\hat{d}}(t),\ \dot{e}_\varepsilon(t) = -\dot{\hat{\varepsilon}}(t) \\
&\dot{e}_\alpha(t) = -\dot{\hat{\alpha}}(t),\ \dot{e}_\beta(t) = -\dot{\hat{\beta}}(t),\ \dot{e}_\gamma(t) = -\dot{\hat{\gamma}}(t),\ \dot{e}_\delta(t) = -\dot{\hat{\delta}}(t)
\end{aligned} \tag{47}$$

Substituting (46) into the error dynamics (45), we obtain

$$\begin{aligned}
\dot{e}_1 &= e_\alpha(y_2 - y_1) + e_a(x_2 - x_1) - k_1 e_1 \\
\dot{e}_2 &= e_\gamma y_2 + e_c x_1 - k_2 e_2 \\
\dot{e}_3 &= -e_\beta y_3 - e_b x_3 - k_3 e_3 \\
\dot{e}_4 &= -e_\delta(y_1 + y_2) - e_d x_1 - e_\varepsilon x_2 - k_4 e_4
\end{aligned} \tag{48}$$

We consider the candidate Lyapunov function

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2 + e_\varepsilon^2 + e_\alpha^2 + e_\beta^2 + e_\gamma^2 + e_\delta^2\right) \tag{49}$$

Differentiating (49) along the dynamics (47) and (48), we obtain

$$\begin{aligned}
\dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 + e_a\left[e_1(x_2 - x_1) - \dot{\hat{a}}\right] + e_b\left[-e_3 x_3 - \dot{\hat{b}}\right] + e_c\left[e_2 x_1 - \dot{\hat{c}}\right] \\
&+ e_d\left[-e_4 x_1 - \dot{\hat{d}}\right] + e_\varepsilon\left[-e_4 x_2 - \dot{\hat{\varepsilon}}\right] + e_\alpha\left[e_1(y_2 - y_1) - \dot{\hat{\alpha}}\right] + e_\beta\left[-e_3 y_3 - \dot{\hat{\beta}}\right] \\
&+ e_\gamma\left[e_2 y_2 - \dot{\hat{\gamma}}\right] + e_\delta\left[-e_4(y_1 + y_2) - \dot{\hat{\delta}}\right]
\end{aligned} \tag{50}$$

In view of (50), we choose the following parameter update law:

$$\begin{aligned} &\dot{\hat{a}} = e_1(x_2 - x_1) + k_5 e_a, && \dot{\hat{\alpha}} = e_1(y_2 - y_1) + k_{10} e_\alpha \\ &\dot{\hat{b}} = -e_3 x_3 + k_6 e_b, && \dot{\hat{\beta}} = -e_3 y_3 + k_{11} e_\beta \\ &\dot{\hat{c}} = e_2 x_1 + k_7 e_c, && \dot{\hat{\gamma}} = e_2 y_2 + k_{12} e_\gamma \\ &\dot{\hat{d}} = -e_4 x_1 + k_8 e_d, && \dot{\hat{\delta}} = -e_4(y_1 + y_2) + k_{13} e_\delta \\ &\dot{\hat{\varepsilon}} = -e_4 x_2 + k_9 e_\varepsilon \end{aligned} \tag{51}$$

Theorem 6.1 The adaptive control law defined by Eq. (44) along with the parameter update law defined by Eq. (51) achieve global and exponential anti-synchronization of the non-identical hyperchaotic Yang system (40) and hyperchaotic Pang system (41) with unknown parameters for all initial conditions $x(0), y(0) \in R^4$. Moreover, all the parameter estimation errors globally and exponentially converge to zero for all initial conditions.

Proof. The proof is via Lyapunov stability theory [26] by taking V defined by Eq. (49) as the candidate Lyapunov function. Substituting the parameter update law (51) into (50), we get

$$\begin{aligned} \dot{V}(e) = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 - k_5 e_a^2 - k_6 e_b^2 - k_7 e_c^2 - k_8 e_d^2 - k_9 e_\varepsilon^2 \\ &- k_{10} e_\alpha^2 - k_{11} e_\beta^2 - k_{12} e_\gamma^2 - k_{13} e_\delta^2 \end{aligned} \tag{52}$$

which is a negative definite function on R^{13}. This completes the proof. ■

Next, we illustrate our adaptive anti-synchronization results with MATLAB simulations. The classical fourth order Runge-Kutta method with time-step $h = 10^{-8}$ has been used to solve the hyperchaotic systems (40) and (41) with the nonlinear controller defined by (44). The feedback gains in the adaptive controller (31) are taken as $k_i = 4, \ (i = 1, \ldots, 8)$.

The parameters of the two hyperchaoticsystems are taken as in the hyperchaotic case, *i.e.*

$$a = 35, \quad b = 3, \quad c = 35, \quad d = 2, \quad \varepsilon = 7.5, \quad \alpha = 36, \quad \beta = 3, \quad \gamma = 20, \quad \delta = 2$$

For simulations, the initial conditions of the drive system (40) are taken as

$$x_1(0) = 29, \quad x_2(0) = 14, \quad x_3(0) = -23, \quad x_4(0) = -9$$

Also, the initial conditions of the response system (41) are taken as

$$y_1(0) = 14, \quad y_2(0) = -18, \quad y_3(0) = 29, \quad y_4(0) = -14$$

Also, the initial conditions of the parameter estimates are taken as

$$\hat{a}(0) = 9,\ \hat{b}(0) = 4,\ \ \hat{c}(0) = -3,\ \hat{d}(0) = 8,\ \hat{\varepsilon}(0) = -4,$$
$$\hat{\alpha}(0) = 6,\ \hat{\beta}(0) = 2,\ \hat{\gamma}(0) = 11,\ \hat{\delta}(0) = -9$$

Figure 9depicts the anti-synchronization of the non-identical hyperchaotic Yang and hyperchaotic Pang systems. Figure 10depicts the time-history of the anti-synchronization errors e_1, e_2, e_3, e_4. Figure 11 depicts the time-history of the parameter estimation errors $e_a, e_b, e_c, e_d, \mathrm{e}_\varepsilon$. Figure 12 depicts the time-history of the parameter estimation errors $e_\alpha, e_\beta, e_\gamma, e_\delta$.

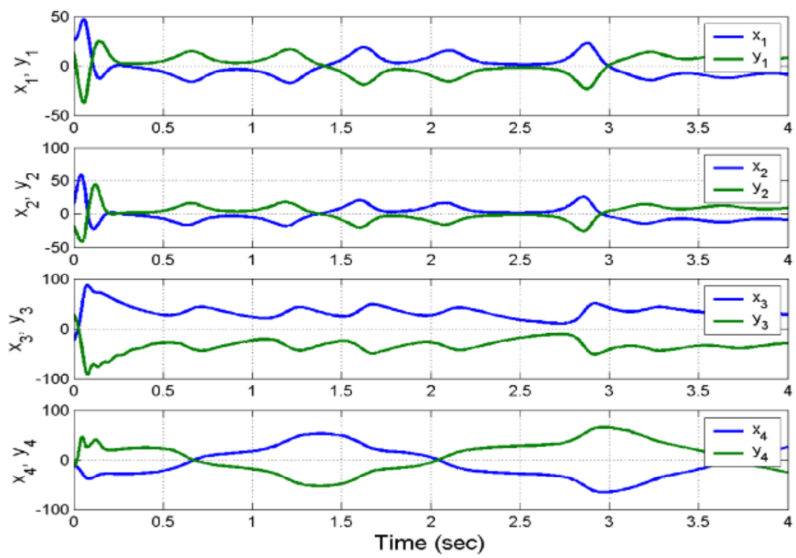

Figure 9. Anti-Synchronization of Hyperchaotic Yang and Hyperchaotic Pang Systems

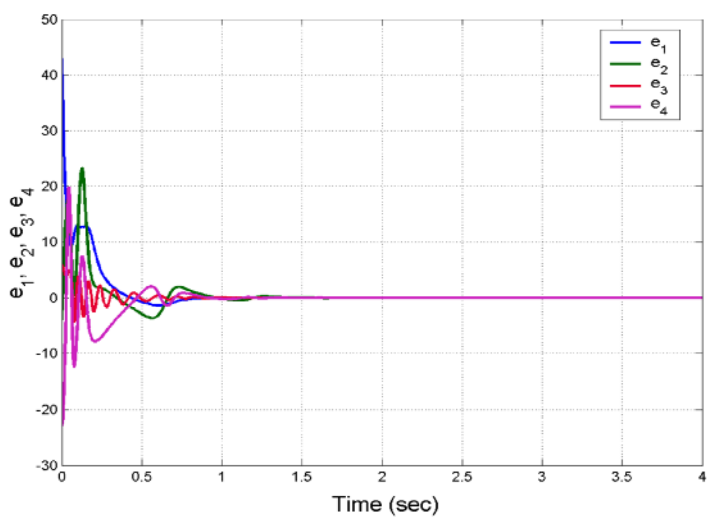

Figure 10. Time-History of the Anti-Synchronization Errors e_1, e_2, e_3, e_4

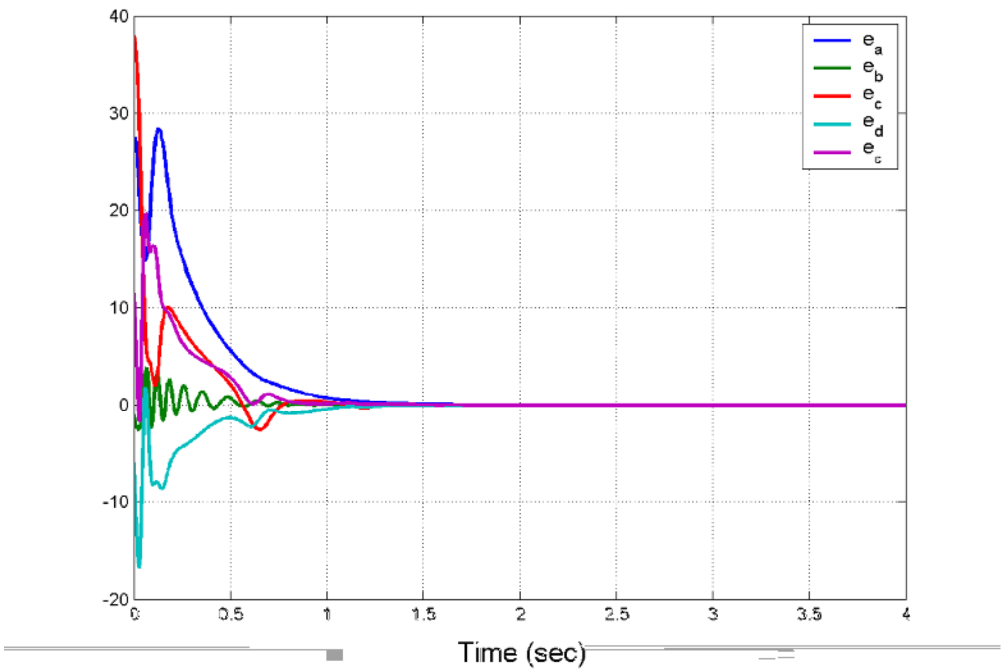

Figure 11. Time-History of the Parameter Estimation Errors $e_a, e_b, e_c, e_d, e_\varepsilon$

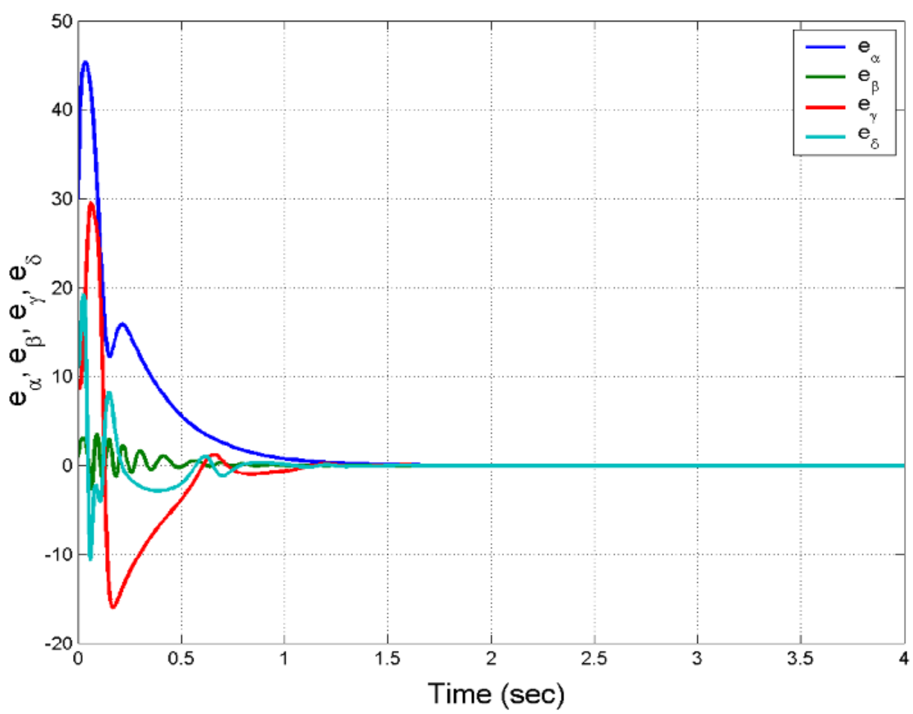

Figure 12. Time-History of the Parameter Estimation Errors $e_\alpha, e_\beta, e_\gamma, e_\delta$

7. CONCLUSIONS

In this paper, we have used adaptive control to derive new results for the anti-synchronization of hyperchaotic Yang system (2009) and hyperchaotic Pang system (2011) with unknown parameters. Main results of anti-synchronization design results for hyperchaotic systems addressed in this paper were proved using adaptive control theory and Lyapunov stability theory. Hyperchaotic systems have important applications in areas like secure communication, data encryption, neural networks, etc.MATLAB simulations have been shown to validate and demonstrate the adaptive anti-synchronization results for hyperchaotic Yang and hyperchaotic Pang systems.

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