# VIBRATION AND TIP DEFLECTION CONTROL OF A SINGLE-LINK FLEXIBLE MANIPULATOR

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#### **ABSTRACT**

In this paper, a hybrid control scheme for vibration and tip deflection control of a single link flexible manipulator system is presented. The purpose of this control is for input tracking, vibration control of hub angle and tip deflection control. The control scheme consists of a resonant controller and a fuzzy logic controller (FLC). The resonant controller is used as the inner loop feedback controller for vibration control using the resonant frequencies at different resonant modes of the system which were determined from experiment. The fuzzy logic controller is designed as the outer loop feedback controller for the tracking control and to achieve zero steady state error. The performance of the proposed control scheme is investigated via simulations and the results show the effectiveness of the control scheme, in addition the controller is tested to show it robustness using different values of payload.

#### Keywords

Hybrid controller, vibration control, resonant controller, fuzzy control, flexible link manipulator.

## **1. INTRODUCTION**

For the past two decades, there is a significant increase in the number of research on the control of flexible manipulator, this is due to an increase in the demand for high speed robots in our industries[1]. The need for a light-weight flexible robot for industrial applications increases significantly due to their advantages over heavy-weight robot, Which are passing to mention a few: they can be easily driven using small sized actuator that consume less energy, high-speed operation, low cost and has light weight in passing[1].

Several control techniques have been applied to solve the problems of vibration and tip deflection of flexible manipulator systems. There are two main issues that created problems in the designing of flexible manipulator controller, these problems are due to: (a) high order of the system and (b) non-minimum phase dynamics of the system that exist between the tip position and the applied input torque at the hub joint of the system explicitly in[2]. Two different approaches have been applied in literature for the control of the flexible robot arms, these are; i) linear control approach and ii) nonlinear control approach. Linear controllers such as H-infinity explicitly in [3], linear quadratic regulator (LQR) explicitly in [4], conventional PID control explicitly in [5] and integral resonant control (IRC) explicitly in [2,6], have been applied in the control of FMS. A Flexible manipulator is quite difficult to be accurately controlled by linear control approach due to their

nonlinear dynamic structure. Nonlinear control approach such as: Adaptive control technique explicitly in [7], fuzzy logic control technique explicitly in [8] and observer-based fuzzy- control explicitly in [9] have also been applied in the control of FMS.

A fuzzy logic controller is mutually exclusive to conventional dynamic model based controllers in which mathematical model is not require for the control of the plant, and is applicable to both linear and nonlinear system. Fuzzy control is a control way of applying expert knowledge to control a plant without having detail information of the plant in passing [10]. A FLC has three main component namely i) Fuzzifier which convert the input signal into fuzzy signal ii) fuzzy inference engine which process the fuzzified signal using decision rules, and iii) Defuzzifier which convert the fuzzy controller output signal to a signal used as the control input signal to the system model.

Explicitly in [10], three different fuzzy logic controllers (FLCs) are developed to control vibration and end point deflection. A Hybrid fuzzy logic control with genetic optimisation for vibration control of a single-link flexible manipulator is presented in passing [11]. An input shaping with PD-type fuzzy logic control for vibration and trajectory tracking of flexible arm robot is presented in explicitly in[12]. In passing [13], a controller is developed using fuzzy Lyapunov synthesis (FLS) to control vibration of a flexible manipulator.Inpassing [14] an experimental study using fuzzy logic and neural networks tools is presented for active vibration control of a single link flexible manipulator system. In [15],an adaptive network based interval type-2 fuzzy logic controller was developed for the control of a single flexible link carrying a pendulum. A Cascade fuzzy logic control is implemented for the vibration control of a single-link flexible-joint manipulator explicitly in [16].

In this work, a Hybrid control scheme is developed with two feedback controllers: A resonant controller and a PD-type fuzzy logic controller are designed as the inner and outer feedback control loops for a rigid body (hub joint) vibration control and tip deflection control for flexible motion of a single link flexible manipulator respectively.

The rest of this paper is organized as follows: section 2 presented the system model, section 3 presented resonant and fuzzy logic controller design, and discussion of results is presented in section 4 and finally section 5 give the conclusion.

# **2. MODEL DESCRIPTION**

The flexible link is made up of a piece of a thin aluminium alloy. The flexible manipulator model used for the controller design is as described by [17]. The dynamic model is obtained by using finite element (FE) methods and the system is described in figure 1. The parameters of the system are given in table 1. FE method is a process of decomposing a structure into number of pieces or elements. It's assumed that the elements are interconnected at a point, called node. The equation describing the behaviour of the system which is obtained by approximation technique depends on the number of elements. These elemental equations are combined together to give the system equation.

The steps involves in FE method include (1) structural discretisation into number of elements; (2) Result interpolation by an approximating function selection; (3) formulation of the element equation; (4) calculating system equation from element equations; (5) boundary conditions selection and (6) solving system equation with the boundary conditions. In this way, the manipulator system is treats as an assembly of n elements and the algorithm can be developed in

three main parts: i) FE analysis, ii) state-space representation and iii) obtaining and analysing the system transfer function [17].



Figure 1. Description of the flexible manipulator in passing [17]

Parameters	Symbols	values	units
Young modulus	Ε	$71 \times 10^{9}$	$N/m^2$
Mass density per unit volume	ρ	2710	$Kg/m^3$
Second moment of inertia	Ι	5.1924	$m^4$
Flexible link length	L	0.9	т
Beam inertia	Lb	$0.04 \times^{3}$	$g/m^2$

Table 1 system parameters

The manipulator model is presented in state space form as described in [17]

$$\dot{v} = Av + Bu \tag{1}$$

$$y = Cv + Du$$
(2)

where

$$A = \begin{bmatrix} O_m & I_m \\ -M_n^{-1}K_n & -M_n^{-1}D \end{bmatrix} \quad B = \begin{bmatrix} O_{m \times 1} \\ M_n^{-1} \end{bmatrix} \quad C = \begin{bmatrix} O_m & I_m \end{bmatrix} \quad D = \begin{bmatrix} O_{2m \times 1} \end{bmatrix}$$

Where the subscript *n* indicate number of elements,  $O_m$  is  $m \ge m$  null matrix,  $I_m$  is  $m \ge m$  identity matrix,  $O_{m \ge 1}$  is  $m \ge 1$  null vector [17],

$$u = [\tau \ 0 \ \dots \ \dots \ 0]^T, \quad v = \begin{bmatrix} \theta \ w_1 \theta_1 \ \dots \ w_n \ \theta_n \ \dot{\theta} \dot{w}_1 \ \dot{\theta}_1 \ \dots \dot{w}_n \ \dot{\theta}_n \end{bmatrix}^T$$
$$M_n = \begin{bmatrix} M_{\theta\theta} & M_{\thetaw} \\ M_{\thetaw} & M_{ww} \end{bmatrix}$$

In which  $M_{ww}$  presents matrix relates to the elastic degrees of freedom (residual motion),  $M_{\theta w}$  represents the coupling between the hub angle  $\theta$  and elastic degrees of freedom and  $M_{\theta\theta}$  is the terms relates to the system inertia about the motor axis. Similarly, the global stiffness matrix is as follows:

$$K_n = \begin{bmatrix} 0 & 0 \\ 0 & K_{ww} \end{bmatrix}$$

Where  $K_{ww}$  is relates to the elastic degrees of freedom (residual motion). As observed, the elastic degree of freedom is not related to the hub angle via the stiffness matrix. The global damping matrix D is as follows [17].

$$D = \begin{bmatrix} 0 & 0 \\ 0 & D_{ww} \end{bmatrix}$$

Where  $D_{ww}$  represents the sub-matrix associated with the material damping of the system. It is obtained as:

$$D_{ww} = \alpha M_{ww} + \beta K_{ww} \tag{3}$$

Where

$$\alpha = \frac{2f_1f_2(\epsilon_2f_2 - \epsilon_1f_1)}{f_2^2 - f_1^2} \ ; \ \beta = \frac{2(\epsilon_2f_2 - \epsilon_1f_1)}{f_2^2 - f_1^2}$$

Where

 $\epsilon_1, \epsilon_2, f_1$  and  $f_2$  representing the damping ratios and natural frequencies of modes 1 and 2 respectively. The dynamic equations of the motion of the system is represented as

$$M\ddot{Q}(t) + D\dot{Q}(t) + KQ(t) = F(t)$$
<sup>(4)</sup>

Where

 $F(t) = [\tau \ 0 \ \dots \ \dots \ 0]^T$  represent the vector associated with the applied forces and torque,  $Q(t) = [\theta \ w_0 \theta_0 \ \dots \ w_n \ \theta_n]^T$  and D is the global damping matrix, usually obtained by experimentation [17].

For *n* number of elements, the mass  $(M_n)$  and stiffness  $(K_n)$  matrices can also be present as follows:

$$M_n = \frac{\rho A l}{420} \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} \\ m_{21} & 156 & 22l & 54 & -13l \\ m_{31} & 22l & 4l^2 & 13l & -3l^2 \\ m_{41} & 54 & 13l & 156 & -22l \\ m_{51} & -13l & -3l^2 - 22l & 4l^2 \end{bmatrix}$$

$$K_n = \frac{EI}{l^3} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 6l & -12 & 6l \\ 0 & 6l & 4l^2 & -6l & 2l^2 \\ 0 & -12 & -6l & 12 & -6l \\ 0 & 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

Where

$$\begin{split} m_{11} &= 140l^2(3n^2 - 3n + 1) \\ m_{12} &= m_{21} = 21l(10n - 7) \\ m_{13} &= m_{31} = 7l^2(5n - 3) \\ m_{14} &= m_{41} = 21l(10n - 3) \\ m_{15} &= m_{51} = -7l^2(5n - 3) \end{split}$$

In this work the number of elements used were ten (i.e. n=10), which was tested with different values of payload 0 grams, 20 grams, 30 grams and 50 grams.

# **3.** Controller Design

In this section details design of the hybrid controller are explained. Figure 2 shows the Simulink block of the control scheme. The control scheme consists of two negative feedback control loops. The inner loop controller (resonant controller) is designed to add damping to the system around the hub angle so as to suppress the vibration due to rigid-body motion, and the outer loop controller (fuzzy logic controller) is designed for tracking purpose and also to achieve zero steady state error in order to have an accurate tip deflection.



Figure 2. Simulink block of the control scheme

## 3.1 Resonant Controller Design (inner loop)

The resonant controller design is based on the resonant frequency  $(w_i)$  of the flexible link at different resonant modes, the damping ratio  $(\delta_i)$  and controller gain  $alpha(\alpha_i)$ . This controller adds damping to the hub joint to suppress the rigid body vibration and also guarantee unconditional stability for the closed-loop system. It is also called collocated velocity feedback controller because it avoids closed-loop instabilities due to spill over effects. Ideally to control vibration by damping, the control should be restricted to resonant frequencies only [1].

The general model structure of a resonant controller is in form of approximation of differentiator around a resonant frequencies narrow bandwidth of the system as described in equation (5).

$$h_{i}^{\alpha} = \sum_{i=1}^{N} \frac{\alpha_{i} s^{2}}{s^{2} + 2\delta_{i} w_{i} + w_{i}^{2}}$$
(5)

Where *N* is the number of resonant modes need to be controlled,  $\delta_i$  is the damping ratio,  $w_i$  is the resonant frequency and  $\alpha_i$  is a constant parameter ranging from  $0 \le \alpha_i \le 150$  [1]. In this study the resonant controller is designed using ten (10) elements; three resonant modes are considered which are  $w_1$ ,  $w_2$  and  $w_3$ . For N=3 three resonant modes is described in (6).

$$h_i^{\alpha} = h_1 + h_2 + h_3 \tag{6}$$

For

i = 1, 2, 3

$$h_i^{\alpha} = \sum_{i=1}^3 \frac{\alpha_i s^2}{s^2 + 2\delta_i w_i + w_i^2}$$
(7)

$$h_i^{\alpha} = \frac{\alpha_1 s^2}{s^2 + 2\delta_1 w_1 s + w_1^2} + \frac{\alpha_2 s^2}{s^2 + 2\delta_2 w_2 s + w_2^2} + \frac{\alpha_3 s^2}{s^2 + 2\delta_3 w_3 s + w_2^2}$$
(8)

In this work  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$  are chosen to be 120 each, and previous study on the system in [17]have shown those three resonant modes frequencies  $w_1$ ,  $w_2$  and  $w_3$  are given by 11.99Hz, 35.22Hz and 65.2Hz and their corresponding damping ratios  $\delta_1$ ,  $\delta_2$  and  $\delta_3$  are 0.007, 0.015 and 0.314 respectively. Therefore equation (8) can be represented as

$$h_i^{\alpha} = \frac{120s^2}{s^2 + 1.056s + 5689.5} + \frac{120s^2}{s^2 + 6.6s + 48400} + \frac{120s^2}{s^2 + 41s + 167824}$$
(9)

In order to improve the system stability and increase the response speed, a phase lead can be used to shift the poles to the left half s-plane. To accomplish this, a first order lead compensator is designed using the root locus method. Alead compensator can be described as:

$$G_c(s) = K \frac{s+z}{s+p} \tag{10}$$

Where *K* is the compensator gain, *z* and *p* are the zero and pole of the compensator respectively. The values of *K*, *p* and *z* are found to be85, 75 and 5.12 respectively using root locus method. The lead compensator is therefore shown in equation (11).

$$G_c(s) = 85 \frac{s+5.12}{s+75} \tag{11}$$

Hence the new controller  $hnew_i$  now consist of the combination of resonant controller  $h_i^{\alpha}$  and the phase lead compensator  $G_c(s)$ .

$$hnew_i = \frac{120s^2}{s^2 + 1.056s + 5689.5} + \frac{120s^2}{s^2 + 6.6s + 48400} + \frac{120s^2}{s^2 + 41s + 167824} + \frac{85(s + 5.12)}{s + 75}$$
(12)

The closed-loop transfer function of the hub angle with the resonant controller is given by:

$$G_{hub}^{closed-loop} = \frac{G_{hub(s)}}{1 + hnewG_{hub}(s)}$$
(13)

#### 3.2 Fuzzy logic Controller (outer loop)

In this section a PD-type fuzzy controller is presented. This fuzzy controller has two inputs and one output, the inputs are the hub angle error (e) and its derivatives  $(\dot{e})$ , the output is the fuzzy control signal generated based on decisions designed using rule base. The controller is designed to track hub angle, the tip deflection should be regulated close to zero value with zero steady state error.

Fuzzy logic controller (FLC) design involves selection of type and number of membership function, selection of rule base, inference mechanism and defuzzification process. In this paper a triangular membership function is used. The rule base are developed using the symbols NV (negative), ZE (Zero), and PV (Positive). Table 2 shows nine (9) rule bases for the designed fuzzy logic controller.

ė/e	NV	ZE	PV
NV	NV	NV	ZE
ZE	PV	ZE	NV
PV	ZE	NV	NV

Table 2.rule based

Three membership functions are used for both the tip deflection error (*e*), its derivatives (*e*) and the output control signal. The tip deflection error membership functions are implemented with [NV,ZE,PV] with range of [-3 3], the tip deflection error derivative is also implemented with [NV,ZE,PV] with range of [-33] and the fuzzy output is implemented with [NV,ZE,PV] with range of [-33].

## 4. RESULTS AND DISCUSSION

A hybrid controller have been designed consisting of two different controllers namely resonant controller and fuzzy controller, the purpose of the controller is to eliminated the vibration effects and tip deflection of a single link flexible manipulator system. Two types of control objectives are involves in this control, these are; 1) servo (tracking) control and 2) regulation control. Figure 3 shows the result of tracking control in which the hub angle tracked or follows the desire hub angle with zero steady state error and rise time of 0.19 second and zero overshot. Similarly figure 4 shows the result of regulation control in which the tip deflection has been regulated close to zero deflection with maximum deflection (peak to peak deflection) of  $2.5 \times 10^{-3}$ m.

In addition, the controller has been tested to offered robustness to changes in an external influence such as changed in payload. Figures 5a, and 5b shows the hub angle and tip deflection with the changes in payloads values of 0g, 20g, 30g and 50g. The results shows changes both overshot, settling time and maximum tip deflection with different payload values. Table 3gives the summary of the controller results with different payload values.



Figure5a hub angle with 0, 20, 30, 50 grams payloads.



Figure5b tip deflection with 0, 20, 30, 50 grams payloads.

Payload (g)	Hub angle Overshot (degree)	Hub angle Settling time (s)	Hub angle Rise time (s)	Tip deflection Maximum(peak to peak) deflection (mm)
0	0	0.23	0.19	2.50
20	1.09	0.25	0.21	2.90
30	1.32	0.28	0.22	3.02
50	2.56	0.31	0.26	3.23

Table 3summary of the controller results with different payload value

# **5.** CONCLUSION

In this work, a hybrid control scheme for vibration control of a single link flexible manipulator system was presented. The control consist of a resonant controller, designed based on resonant frequencies of the system for vibration control, and a fuzzy logic controller for tracking control, and to attend zero steady state error. The fuzzy is designed with hub angle error and its derivative as inputs. The fuzzy logic controller was implemented in Simulink using MATLAB fuzzy tool box. The resonant controller is designed as the inner loop control and the fuzzy as the outer loop controller. The performance of the control scheme was investigated via simulation in MATLAB. The simulation results show that the controller successfully achieved both the tracking and vibration control.

To test the robustness of the control scheme, different values of payload wereapplied and the results show small changes in the overshot, rise time and settling time as compared to the result with zero payloads, which shows that the proposed control scheme is robust to some extendue to external changes and gives zero steady state error.

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