ADAPTIVE CONTROL AND SYNCHRONIZATION OF A HIGHLY CHAOTIC ATTRACTOR

Sundarapandian Vaidyanathan[1]

[1]Research and Development Centre, Vel Tech Dr. RR & Dr. SR Technical University
Avadi, Chennai-600 062, Tamil Nadu, INDIA
sundarvtu@gmail.com

ABSTRACT

Recently, a novel three-dimensional highly chaotic attractor has been discovered by Srisuchinwong and Munmuangsaen (2010). This paper investigates the adaptive control and synchronization of this highly chaotic attractor with unknown parameters. First, adaptive control laws are designed to stabilize the highly chaotic system to its unstable equilibrium point at the origin based on the adaptive control theory and Lyapunov stability theory. Then adaptive control laws are derived to achieve global chaos synchronization of identical highly chaotic systems with unknown parameters. Numerical simulations are shown to demonstrate the effectiveness of the proposed adaptive control and synchronization schemes.

KEYWORDS

Adaptive Control, Stabilization, Chaos Synchronization, highly chaotic system, Srisuchinwong system.

1. INTRODUCTION

Chaotic systems are nonlinear dynamical systems, which are highly sensitive to initial conditions. The sensitive nature of chaotic systems is usually called as the *butterfly effect* [1]. In 1963, Lorenz first observed the chaos phenomenon in weather models. Since then, a large number of chaos phenomena and chaos behaviour have been discovered in physical, social, economical, biological and electrical systems.

The control of chaotic systems is to design state feedback control laws that stabilizes the chaotic systems around the unstable equilibrium points. Active control technique is used when the system parameters are known and adaptive control technique is used when the system parameters are unknown [2-4].

Chaos synchronization is a phenomenon that may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect, which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem in the chaos literature [5-16].

In 1990, Pecora and Carroll [5] introduced a method to synchronize two identical chaotic systems and showed that it was possible for some chaotic systems to be completely synchronized. From then on, chaos synchronization has been widely explored in a variety of fields including physical systems [6], chemical systems [7], ecological systems [8], secure communications [9-10], etc.

In most of the chaos synchronization approaches, the *master-slave* or *drive-response* formalism has been used. If a particular chaotic system is called the *master* or *drive* system and another

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chaotic system is called the *slave* or *response* system, then the idea of synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

Since the seminal work by Pecora and Carroll [5], a variety of impressive approaches have been proposed for the synchronization of chaotic systems such as the OGY method [11], active control method [12-16], adaptive control method [17-21], sampled-data feedback synchronization method [22], time-delay feedback method [23], backstepping method [24], sliding mode control method [25-28], etc.

In this paper, we investigate the adaptive control and synchronization of an uncertain novel three-dimensional highly chaotic attractor discovered by B. Srisuchinwong and B. Munmuangsaen ([29], 2010). First, we devise adaptive stabilization scheme using state feedback control for the highly chaotic system about its unstable equilibrium at the origin. Then, we devise adaptive synchronization scheme for identical highly chaotic systems with unknown parameters. The stability results derived in this paper are established using Lyapunov stability theory.

This paper is organized as follows. In Section 2, we give a system description of the highly chaotic system (Srisuchinwong and Munmuangsaen, 2010). In Section 3, we derive results for the adaptive stabilization of the highly chaotic system with unknown parameters. In Section 4, we derive results for the adaptive synchronization of identical highly chaotic systems with unknown parameters. In Section 5, we summarize the main results obtained in this paper.

2. SYSTEM DESCRIPTION

The highly chaotic system ([29], 2010) is a one-parameter family of three-dimensional chaotic systems, which is described by the dynamics

$$\begin{aligned} \dot{x}_1 &= 10(x_2 - x_1) \\ \dot{x}_2 &= a x_1 - 40 x_1 x_3 \\ \dot{x}_3 &= 10 x_1 x_2 - x_3 \end{aligned} \tag{1}$$

where x_i, $(i = 1, 2, 3)$ are the state variables and a is a constant positive parameter of the system. The system (1) is highly chaotic when the parameter value is taken as

$$a = 296.5 \tag{2}$$

The state orbits of the highly chaotic system (2) are described in Figure 1. In [29], it has been shown that when a is near 296.5, the maximum Lyapunov exponent $L_{(\max)} = 2.6148$ and the maximum Kaplan-Yorke dimension $D_{KY(\max)} = 2.1921$.

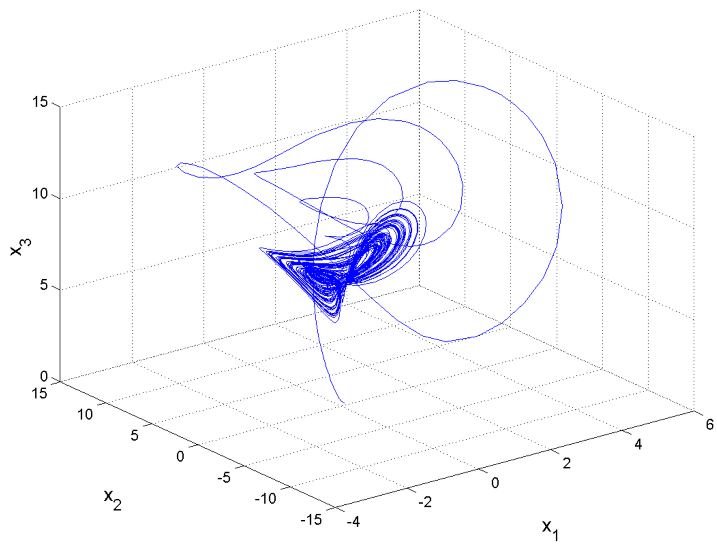

Figure 1. State Orbits of the Highly Chaotic System

When the parameter values are taken as in (2), the system (1) is highly chaotic and the system linearization matrix at the equilibrium point $E_0 = (0,0,0)$ is given by

$$A = \begin{bmatrix} -10 & 10 & 0 \\ 296.5 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

which has the eigenvalues

$$\lambda_1 = -1, \quad \lambda_2 = -59.6809 \quad \text{and} \quad \lambda_3 = 49.6809 \tag{4}$$

Since λ_3 is a positive eigenvalue, it is immediate from Lyapunov stability theory [30] that the system (1) is unstable at the equilibrium point $E_0 = (0,0,0).$

3. Adaptive Control of The Highly Chaotic System

3.1 Theoretical Results

In this section, we design adaptive control law for globally stabilizing the highly chaotic system (1) when the parameter value is unknown.

Thus, we consider the controlled highly chaotic system as follows.

$$\begin{aligned} \dot{x}_1 &= 10(x_2 - x_1) + u_1 \\ \dot{x}_2 &= ax_1 - 40x_1x_3 + u_2 \\ \dot{x}_3 &= 10x_1x_2 - x_3 + u_3 \end{aligned} \tag{5}$$

where u_1, u_2 and u_3 are feedback controllers to be designed using the states and estimates of the unknown parameter of the system.

In order to ensure that the controlled system (5) globally converges to the origin asymptotically, we consider the following adaptive control functions

$$\begin{aligned} u_1 &= -\hat{a}(x_2 - x_1) - k_1 x_1 \\ u_2 &= -\hat{a}x_1 + 40x_1x_3 - k_2x_2 \\ u_3 &= -10x_1x_2 + x_3 - k_3x_3 \end{aligned} \qquad (6)$$

where $\hat{a}$ is the estimate of the parameter a and $k_i, (i = 1, 2, 3)$ are positive constants.

Substituting the control law (6) into the highly chaotic dynamics (5), we obtain

$$\begin{aligned} \dot{x}_1 &= -k_1x_1 \\ \dot{x}_2 &= (a - \hat{a})x_1 - k_2x_2 \\ \dot{x}_3 &= -k_3x_3 \end{aligned} \qquad (7)$$

Let us now define the parameter estimation error as

$$e_a = a - \hat{a} \qquad (8)$$

Using (8), the closed-loop dynamics (7) can be written compactly as

$$\begin{aligned} \dot{x}_1 &= -k_1x_1 \\ \dot{x}_2 &= e_ax_1 - k_2x_2 \\ \dot{x}_3 &= -k_3x_3 \end{aligned} \qquad (9)$$

For the derivation of the update law for adjusting the parameter estimate $\hat{a}$, the Lyapunov approach is used.

Consider the quadratic Lyapunov function

$$V = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + x_4^2 + e_a^2\right), \qquad (10)$$

which is a positive definite function on R^5.

Note also that

$$\dot{e}_a = -\dot{\hat{a}} \qquad (11)$$

Differentiating V along the trajectories of (9) and using (11), we obtain

$$\dot{V} = -k_1x_1^2 - k_2x_2^2 - k_3x_3^2 + e_a\left[x_1x_2 - \dot{\hat{a}}\right] \qquad (12)$$

In view of Eq. (12), the estimated parameters are updated by the following law:

$$\dot{\hat{a}} = x_1x_2 + k_4e_a \qquad (13)$$

where k_4 is a positive constant.

Substituting (13) into (12), we get

$$\dot{V} = -k_1\ x_1^2 - k_2\ x_2^2 - k_3\ x_3^2 - k_4\ e_a^2 \qquad (14)$$

which is a negative definite function on R^4.

Thus, by Lyapunov stability theory [27], we obtain the following result.

Theorem 1. *The highly chaotic system (5) with unknown parameters is globally and exponentially stabilized for all initial conditions* $x(0) \in R^3$ *by the adaptive control law (6), where the update law for the parameter is given by (13) and* k_i, $(i = 1, \ldots, 4)$ *are positive constants.* ■

2.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the highly chaotic system (5) with the adaptive control law (6) and the parameter update law (13).

The parameter a of the highly chaotic system (5) is selected as

$$a = 296.5$$

For the adaptive and update laws, we take $k_i = 4, \quad (i = 1, 2, 3, 4)$.

Suppose that the initial value of the estimated parameter is taken as $\hat{a}(0) = 7$.

The initial values of the highly chaotic system (5) are taken as $x(0) = (14, 15, 22)$.

When the adaptive control law (6) and the parameter update law (13) are used, the controlled highly chaotic system (5) converges to the equilibrium $E_0 = (0, 0, 0)$ exponentially as shown in Figure 2. The parameter estimate $\hat{a}$ is shown in Figure 3, which converges to $a = 296.5$

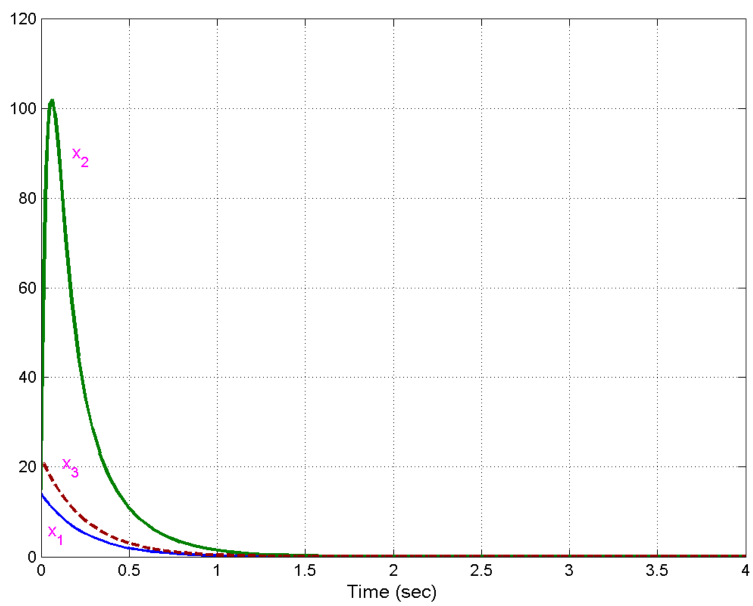

Figure 2. Time Responses of the Controlled Highly Chaotic System

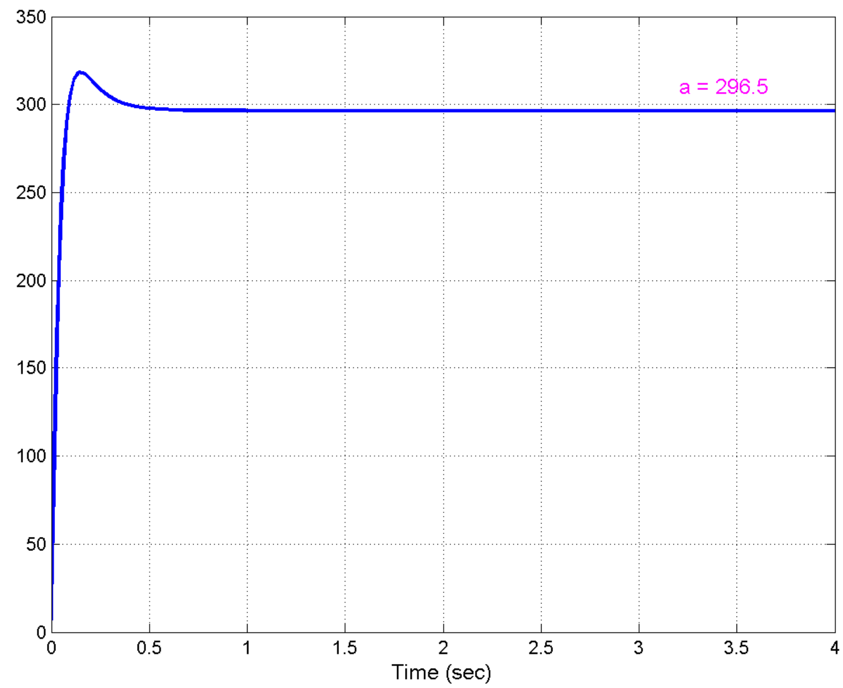

Figure 3. Parameter Estimate $\hat{a}(t)$

4. ADAPTIVE SYNCHRONIZATION OF IDENTICAL HIGHLY CHAOTIC SYSTEMS

4.1 Theoretical Results

In this section, we discuss the adaptive synchronization of identical highly chaotic systems with unknown parameter.

As the master system, we consider the highly chaotic dynamics described by

$$\begin{aligned} \dot{x}_1 &= 10(x_2 - x_1) \\ \dot{x}_2 &= a x_1 - 40 x_1 x_3 \\ \dot{x}_3 &= 10 x_1 x_2 - x_3 \end{aligned} \tag{15}$$

where $x_i, \ (i = 1, 2, 3)$ are the state variables and a is the unknown system parameter.

As the slave system, we consider the controlled highly chaotic dynamics described by

$$\begin{aligned} \dot{y}_1 &= 10(y_2 - y_1) + u_1 \\ \dot{y}_2 &= a y_1 - 40 y_1 y_3 + u_2 \\ \dot{y}_3 &= 10 y_1 y_2 - y_3 + u_3 \end{aligned} \tag{16}$$

where $y_i, \ (i = 1, 2, 3)$ are the state variables and $u_i, \ (i = 1, 2, 3)$ are the nonlinear controllers to be designed.

The synchronization error is defined by

$$e_i = y_i - x_i, \quad (i = 1,2,3) \tag{17}$$

Then the error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= 10(e_2 - e_1) + u_1 \\ \dot{e}_2 &= ae_1 - 40(y_1 y_3 - x_1 x_3) + u_2 \\ \dot{e}_3 &= 10(y_1 y_2 - x_1 x_2) - e_3 + u_3 \end{aligned} \tag{18}$$

Let us now define the adaptive control functions $u_1(t), u_2(t), u_3(t)$ as

$$\begin{aligned} u_1 &= -10(e_2 - e_1) - k_1 e_1 \\ u_2 &= -\hat{a} e_1 + 40(y_1 y_3 - x_1 x_3) - k_2 e_2 \\ u_3 &= -10(y_1 y_2 - x_1 x_2) + e_3 - k_3 e_3 \end{aligned} \tag{19}$$

where $\hat{a}$ is the estimate of the parameter a, and k_1, k_2, k_3 are positive constants.

Substituting the control law (19) into (18), we obtain the error dynamics as

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= (a - \hat{a}) e_1 - k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3 \end{aligned} \tag{20}$$

Let us now define the parameter estimation error as

$$e_a = a - \hat{a} \tag{21}$$

Substituting (21) into (20), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= e_a e_1 - k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3 \end{aligned} \tag{22}$$

For the derivation of the update law for adjusting the estimate of the parameter, the Lyapunov approach is used.

Consider the quadratic Lyapunov function

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2\right) \tag{23}$$

which is a positive definite function on R^4.

Note also that

$$\dot{e}_a = -\dot{\hat{a}} \tag{24}$$

Differentiating V along the trajectories of (22) and using (24), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a\left[e_1 e_2 - \dot{\hat{a}}\right] \tag{25}$$

In view of Eq. (25), the estimated parameter is updated by the following law:

$$\dot{\hat{a}} = e_1 e_2 + k_4 e_a \tag{26}$$

where k_4 is a positive constants.

Substituting (24) into (23), we get

$$\dot{V} = -k_1\, e_1^2 - k_2\, e_2^2 - k_3\, e_3^2 - k_4\, e_a^2, \tag{27}$$

which is a negative definite function on R^4.

Thus, by Lyapunov stability theory [30], it is immediate that the synchronization error and the parameter error decay to zero exponentially with time for all initial conditions.

Hence, we have proved the following result.

Theorem 2. *The identical highly chaotic systems (15) and (16) with unknown parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law (19), where the update law for parameter is given by (26) and* $k_i, (i = 1,2,3,4)$ *are positive constants.* ■

3.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the two systems of differential equations (15) and (16) with the adaptive control law (19) and the parameter update law (26).

Here, we take the parameter value as $a = 296.5$ and the gains as $k_i = 4$ for $i = 1,2,3,4$.

We take the initial value of the estimated parameter as $\hat{a}(0) = 10$. We take the initial state of the master system (15) as $x(0) = (2,15,10)$ and the slave system (16) as $y(0) = (18,6,4)$.

Figure 4 shows the adaptive chaos synchronization of the identical highly chaotic systems. Figure 5 shows that the estimated value $\hat{a}$ converges to the system parameter $a = 296.5$.

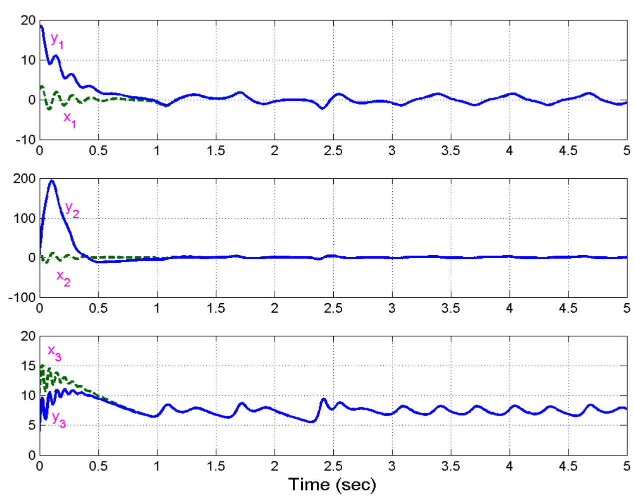

Figure 4. Adaptive Synchronization of the Highly Chaotic Systems

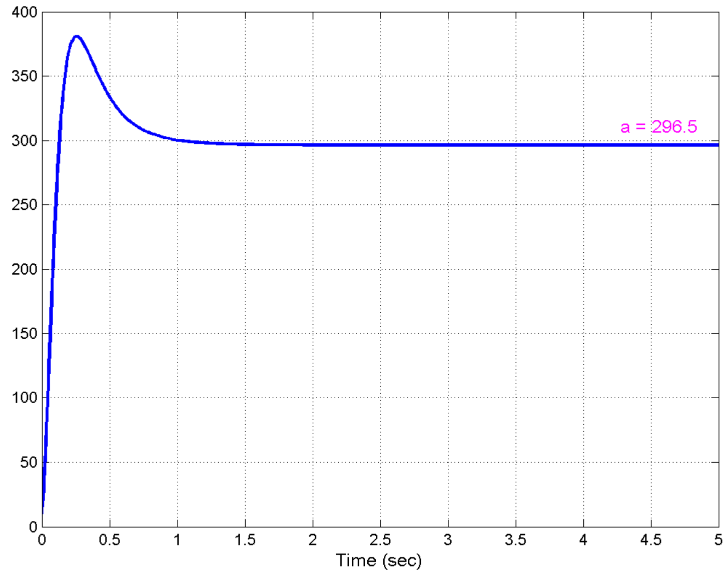

Figure 5. Parameter Estimate $\hat{a}(t)$

5. CONCLUSIONS

In this paper, we applied adaptive control theory for the stabilization and synchronization of the highly chaotic system (Srisuchinwong and Munmuangsaen, 2010) with unknown system parameters. First, we designed adaptive control laws to stabilize the highly chaotic system to its equilibrium point at the origin based on the adaptive control theory and Lyapunov stability theory. Then we derived adaptive synchronization scheme and update law for the estimation of system parameters for identical highly chaotic systems with unknown parameters. Our synchronization schemes were established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the proposed adaptive control method is very effective and convenient to achieve chaos control and synchronization of the highly chaotic system. Numerical simulations are shown to validate and illustrate the effectiveness of the adaptive stabilization and synchronization schemes derived in this paper.

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Author

Dr. V. Sundarapandian is a Professor (Systems and Control Engineering), Research and Development Centre at Vel Tech Dr. RR & Dr. SR Technical University, Chennai, India. His current research areas are: Linear and Nonlinear Control Systems, Chaos Theory, Dynamical Systems and Stability Theory, etc. He has published over 170 research articles in international journals and two text-books with Prentice-Hall of India, New Delhi, India. He has published over 50 papers in International Conferences and 90 papers in National Conferences. He is a Senior Member of AIRCC and is the Editor-in-Chief of the AIRCC journals – International Journal of Instrumentation and Control Systems, International Journal of Control Theory and Computer Modeling, etc. He is an Associate Editor of the journals – International Journal of Control Theory and Applications, International Journal of Advances in Science and Technology, International Journal of Computer Information Systems, etc. He has delivered several Key Note Lectures on Control Systems, Chaos Theory, Scientific Computing, MATLAB, SCILAB, etc.