

# Fine-tuning of $k$ in a $K$ -fold Multicast Network with Finite Queue using Markovian Model

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## **ABSTRACT**

*Multicast has brought a drastic change in the field of networking by offering bandwidth effective technology that leads to reduce tariff. It is popular for optimizing network performances. This paper evaluated a model for fine tuning the value of a  $k$  in a  $K$ -fold multicast network under different traffic loads under poisson traffic with finite queue at each node. Stationary distribution has been derived for the network states and various expressions have been developed for the network to determine throughput and the blocking probability of the network. It has been established from this research work that by increasing the fold number, the network throughput can be increased very fast. However, it has been also observed that after increasing the fold number up to a certain value, the blocking probability stops to increase and becomes constant. We have also noticed that the throughput increases with the increase of the offered traffic and the blocking probability decreases when the system parameter  $k$  is increased. Moreover, an optimum value of  $k$  ceases the blocking probability for a particular value of the offered traffic for fine-tuning the outcome of the network.*

## **KEYWORDS**

*Broadcast,  $k$ -fold network, Kendal's notation, Markov chain, Multicasting, Throughput, Traffic Theory*

## **INTRODUCTION**

Multicast, meaning transmitting information from a single source to different ends, is an essential for high-performance communication networks. Among the other collective communication operations, it is one of the most significant and preferred one in broad-band integrated services network (BISDN) and in communication-intensive applications in parallel and distributed computing systems, such as distributed database updates and cache coherence protocols. In future, the use of multicast will increase to support different interactive applications, like multimedia, teleconferencing, web servers and electronic commerce on the Internet [1]. Most of these applications demands foreseeable communications performance, for instance guaranteed multicast latency and bandwidth, which is known as quality of services (QoS) along with multicast capability.

Nevertheless, finding an optimum value of the system parameter  $k$  (the fold number) is crucial to offer a quantitative basis for the network designers. For that purpose, we have developed an analytical model to identify the appropriate value of  $k$  for different traffic loads for a  $k$ -fold multicast network under poisson traffic with finite buffers or queue at each node [2] to provide

best effort in the network. We have also developed a stationary distribution for the network states and derived expressions for the network throughput and the blocking probability of the network. In addition to that, we have illustrated the flexible parameter of  $K$  with finite users using Markovian model  $M/M/n/n+q/N$ .

## 1.LITERATURE REVIEW

### A.Background

Multicasting is a technical term which is used as a networking technique of delivering messages and information to a group simultaneously from the source. A typical multicasting service is shown in the Figure 1.

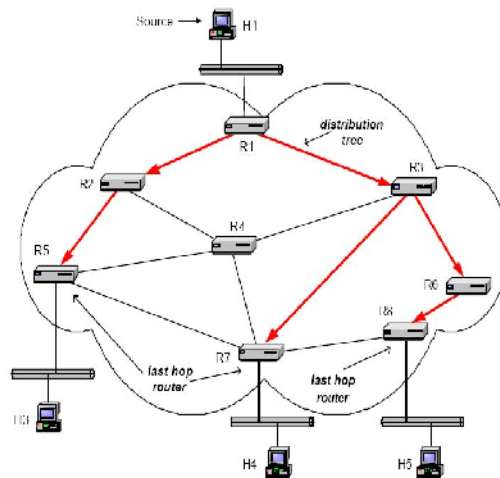


Fig1: Basic multicast service

In a  $K$ -fold multicast network, fold number indicates the number of request coming from different sources to a particular destination. On the other hand, finite queue is a data set shared by program processes which acts as a buffer for data in multicast network. In this research work, we have developed a model to help network engineers to design an effective multicast network. To do this, it is obligatory to get the optimum value of  $k$  in a  $K$ -fold multicast network. By implementing the optimum value of the system parameter  $k$  in  $K$ -fold multicast network, we have improved the network performance by fine tuning network throughput, where throughput is the number of messages successfully delivered per unit time. In this paper, the term “throughput” has been used to measure from the arrival of the first bit of data at the receiver [3].

As mentioned earlier, the primary target of this research work were to evaluate the performance of  $k$ -fold multicast network by using traffic model and get an optimum value of  $k$  to modify the output of the network. For this reason, throughout this paper, we have made the following assumptions and experimental setup on the multicast traffic:

- The probability of a destination node being involved in an incoming multicast connection request is independent of other destination nodes.
- Multicast connection requests at different source nodes are independent to each other.

- Holding time of each multicast connection is exponentially distributed with parameter and is independent to each other.
- Multicast connection requests arrive at each source node according to a Poisson process with intensity and are independent to each other [1][4].

## **B.Previous Researches**

This research work is basically the extension of previous works of Zhenghao Zhang et.al [5] who evaluated the performance of k fold network but they did not use buffers. After that Asfara R. Towfiq et.al [6] again checked the performance of K-fold network with a new look. They revealed the optimization of K-fold multicast network with buffers but for infinite users. And, in the year of 2012, Hasan et.al [1] has evaluated the performance of finite queue with buffer using M/M/n/n+q/N model. Here, we have used the same experimental setup of Hasan et.al [1] which is finite users with buffer to fine tune the performance of K-fold multicast network based on optimum value of k by using Markovian model that we have got in this research work and it is described in the results and discussion sections later on.

For this approach, a destination node may be simultaneously involved in two multicast connections. Such connections will be blocked in a network which is designed to be nonblocking or rearrangeable for only multicast assignments. Specifically, the network can realize multiple multicast assignments in a single pass with a guaranteed latency.

## **2.TRAFFIC THEORY AND K-FOLD MULTICAST NETWORK**

### **A.Basic Traffic theory and Markov Chain**

Traffic Theory describes the key models of traffic flow and associated traffic phenomena such as conflicts in traffic, congestion control and effective management of traffic.

In this paper, we have derived stationary distribution of the k- fold network from which we can obtain network throughput and the blocking probability. We assume the Markovian M/M/n/n+q/N model which is shown in the Figure 2.

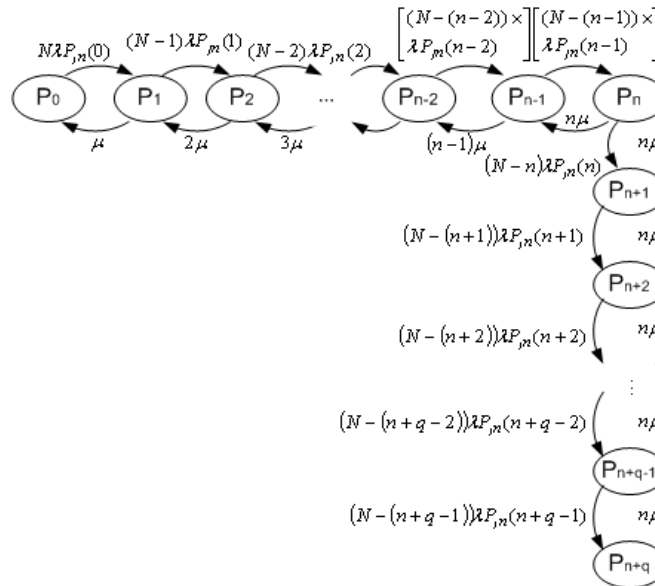


Figure 2: Markov Chain for k-fold network.

The above figure shows a glance of Markov Chain and its impact on Finite State space.

### B.K-fold Network

It is defined as a mapping from a subset of network source nodes to a subset of network destination nodes, with up to K-fold overlapping allowed among the destinations of different sources. It is an adjustable parameter. In other words, any destination node can be involved in multicast connections from up to K different sources at a time [7].

Why A k-fold network?

- A cost-effective solution to provide better quality-of-service functions in supporting real-world multicast applications.
- Predictable communications performances, such as guaranteed multicast latency and bandwidth.
- Highly demanded in communication-intensive applications in parallel and distributed computing systems, such as distributed database updates [8].

### C.Kendal's notation of queuing system

In 1953 D.G Kendall introduces special notation for queuing models. A complete notation for the paper is:

M/M/n/n+q/N Where,

M: Markov or memory less which follows exponential distribution

n: Number of servers/channels

N: Number of users

K=n+q: Sum of channels and queue

q: Length of queue

### 3. MATHEMATICAL ANALYSIS

Let us consider that there are  $j$  multicast connection requests, and let  $p_{\text{deg}}(j, m)$  be the probability that a destination node is the destination of exactly  $m$  of the multicast connection requests; or we can say that a destination node is of degree  $m$  under these  $j$  multicast connection requests. The probability that any multicast connection request chooses this destination node is  $\theta$  and is independent of other multicast connections. Thus, we have

$$p_{\text{deg}}(j, m) = \binom{j}{m} \theta^m (1-\theta)^{j-m}, \quad m \in \{0, 1, \dots, j\} \quad (1)$$

which is a binomial random variable. We assume that each destination node has the same distribution given by (1).

Furthermore, we assume that whether a destination node is chosen by a multicast connection is independent of other destination nodes. Thus, in addition to having the same distributions, the degrees of the destination nodes are also independent of each other. That is why, they are a group of independent, identically distributed (i.i.d.) random variables [9].

#### A. Mathematical Analysis of network throughput

Let  $P_{\text{mc}}(j)$  be the probability that  $j$  multicast connection requests are mutually compatible (*m.c.*) in a  $k$ -fold multicast network. We note that a set of multicast connection requests are *m.c.* when none of the destination nodes has a degree more than  $k$  when realized simultaneously in the network. From (1), it is obvious that the probability of a destination node having a degree less than or equal to  $k$  is  $\sum_{m=0}^k p_{\text{deg}}(j, m)$  for  $j > k$ , and 1 for  $j \leq k$ , because when  $j \leq k$ , no destination node can have a degree more than  $k$ . Since the degrees of destination nodes are independent of each other, we have

$$P_{\text{mc}}(j) = \begin{cases} \left( \sum_{m=0}^k p_{\text{deg}}(j, m) \right)^n, & j > k \\ 1, & \text{otherwise.} \end{cases} \quad (2)$$

Now, let us consider that a new multicast connection request arrives when  $j$  multicast connections are already in the network. If this new connection can be realized along with those ongoing connections, we say that it can *join* the ongoing connections. Let  $P_{\text{jn}}(j)$  be the probability that a new multicast connection can join  $j$  ongoing connections. It can be shown that

$$P_{\text{jn}}(j) = \frac{P_{\text{mc}}(j+1)}{P_{\text{mc}}(j)}. \quad (3)$$

By solving the Markov chain of Figure 2, the stationary states are found to have the probabilities

$$P_r = \binom{N}{r} \rho^\tau \prod_{x=0}^{r-1} P_{jn}(x) P_0, \quad 0 \leq r \leq n$$

Total number of times the network departs from state  $i$  due to the arrival of a successful connection request is,

$$TP_j(n+q-j)\lambda P_{jn}(j)$$

This is also the total numbers of successful connection requests among at the network when the network is in state  $j$  ( $j \in \{0,1,\dots,N\}$ ) during  $[0,T]$  Therefore the total number of successful connection requests carried by the network during  $[0,T]$  is obtained by summing average,

$$N_{succ} = T\lambda \sum_{j=0}^{n+q} P_j(n+q-j)P_{jn}(j)$$

Therefore the network Throughput is,

$$T_H = \frac{N_{succ}}{T} = \lambda \sum_{j=0}^{n+q} P_j P_{jn}(j)(n+q-j) \quad (4)$$

### B. Mathematical Analysis of blocking probability

The total number of connection requests arriving at the network during  $[0, T]$

$$N_{total} = (n+q)\lambda T$$

Thus the Blocking Probability,

$$P_B = \frac{N_{bl}}{N_{total}} = \left[ 1 - \frac{1}{n+q} \sum_{j=0}^{n+q} (n+q-j) P_{jn}(j) P_j \right]$$

### C. Mathematical Analysis of Probability of delay

The Probability of Delay is,

$$P_D = \sum_{s=1}^q P_{n+s}$$

$$= \sum_{s=1}^q (n+s)! \frac{P^n}{n!} \left(\frac{P}{n}\right)^s \binom{N}{n+s} \prod_{l=0}^{n+s-1} p_{jn}(l) P_0 \tag{5}$$

### 4.RESULTS AND DISCUSSIONS

For numerical appreciation of our results, we have plotted in Figures (3), (4) and (5), the throughput and the blocking probability as a function of the fold number  $k$  [10] [11].

It is seen from Figure 3 that if the fold of the network is increased, network throughput increases very fast in the lower values of the system parameter  $k$ , in our study up to  $k=5$ ; beyond this value of  $k$ , the network throughput is almost constant with respect to the system parameter  $k$  for particular offered traffic. We also observe that as the offered traffic is increased, the throughput also increases. [12]

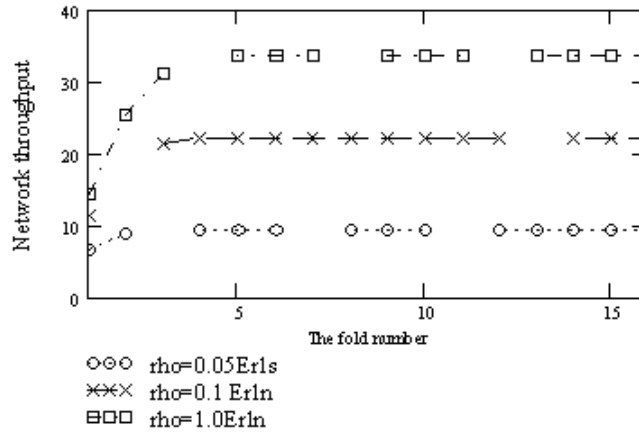


Figure 3: Network throughput as a function of the fold number under different offered traffic (N=50, n=14, q= 5,  $\theta = .31$ )

Figure 4 shows the variation of the blocking probability with respect to the fold  $k$ . It is seen from this figure that as the system parameter  $k$  increases, the blocking probability decreases. However, after an optimum value of  $k$ , in our present study it is  $\sim 5$ , the blocking probability remains constant for particular value of the offered traffic.

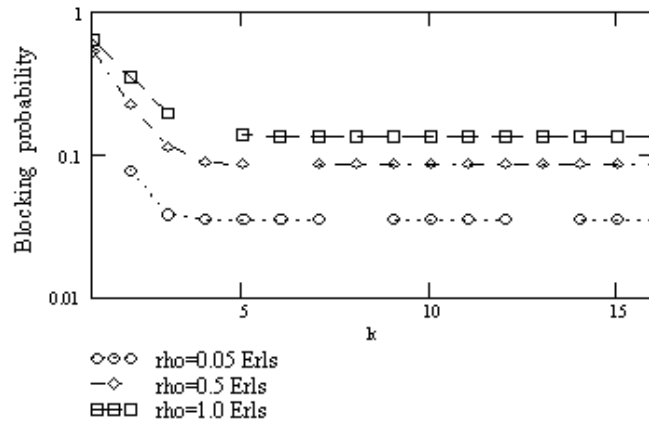


Figure 4: Blocking Probability as a function of the fold number under different offered traffic ( $N=50$ ,  $n=14$ ,  $q=5$ ,  $\theta = .31$ )

Figure 5 shows, the variation of the probability of delayed service with respect to the fold number  $k$ . It is observed that the probability of delay is almost negligible for lower values of the fold number  $k$ , whereas, it is suddenly increases as the fold number approaches the optimum value  $k \sim 5$ . However, after a certain value of  $k$ , the probability of delay becomes constant. [13]

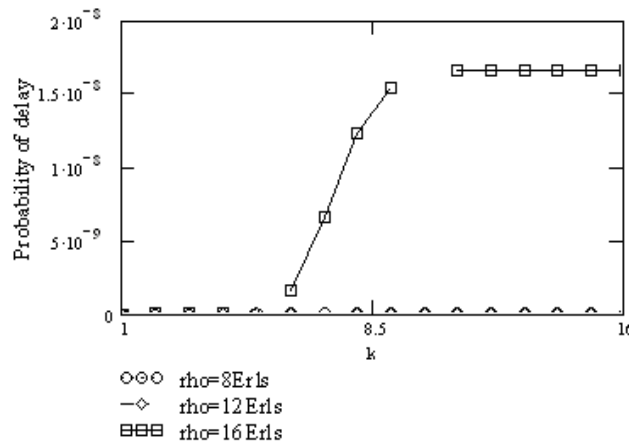


Figure 5: Probability of delay as a function of the fold number under different offered traffic ( $N=50$ ,  $n=14$ ,  $q=5$ ,  $\theta = .31$ )

## 5. CONCLUSIONS

To design an effective network, determining the system parameter  $k$  and identifying its optimum value is very crucial. Keeping that as principle, we have derived a systematical model to fix the best value for a  $k$ -fold multicast network under poisson traffic with limited queue at each node. We have also designed stationary distribution for the network states and expressions for the network throughput and the blocking probability of the network. We have observed in our study that the network throughput increases drastically if the fold number is increased. However, after a



specific value of the fold, the blocking probability stops to increase and becomes constant. Besides, the throughput increases with the rise of offered traffic.

Moreover, it is found in our research work that the blocking probability reduces in proportion to the traffic when the system parameter  $k$  in  $K$ -fold multicast network rises. Yet, after a certain value of  $k$ , which is approximately 5 in our research, the blocking probability becomes constant for the specific value of the traffic provided in multicast network.

It is to be noted here that  $k$ -fold network incurs less hardware cost whereas  $K$ -fold multicast assignment is designed by stacking  $k$  copies of one fold network together. In fact the cost of latter one is  $3-k$  times more than the former one for any  $k$ . Thus it can be said that a  $K$ -fold network will be more cost effective to ensure better QoS functions in supporting arbitrary multicast communication.

Finally, it can be concluded that this model will be helpful in future to determine an optimum value of the system parameter  $k$  in a  $K$ -fold multicast network to enhance throughput of the network and design an effective uninterrupted network.

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