AN OPTIMAL ALGORITHM FOR CONFLICT-FREE COLORING FOR TREE OF RINGS

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ABSTRACT

An optimal algorithm is presented about Conflict-Free Coloring for connected subgraphs of tree of rings. Suppose the number of the rings in the tree is |T| and the maximum length of rings is |R|. A presented algorithm in [1] for a Tree of rings used O(log|T|.log|R|) colors but this algorithm uses O(log|T|+log|R|) colors. The coloring earned by this algorithm has the unique-min property, that is, the unique color is also minimum.

KEYWORDS

Conflict-Free Coloring, Tree , Tree of Rings

1. INTRODUCTION

A vertex coloring of graph G=(V,E) is an assignment of colors to the vertices such that two adjacent vertices are assigned different colors. A hypergraph H = (V,E) is a generalization of a graph for which hyperedges can be arbitrary-sized non-empty subsets of V. A vertex coloring C of hypergraph H is called conflict-free if in every hyperedge there is a vertex whose color is unique among all other colors in the hyperedge. Suppose the hypergraph H=(V,D) of a graph G=(V,E) be defined as follows: The set of vertices V of H is the same as that of G and the set of hyperedges D consists of all possible subsets of V that induce connected subgraphs of G. Another possible generalization [5] is the following one:

Definition 1. A vertex coloring of a hypergraph H=(V,D) is called conflict-free if in every hyperedge e there exists at least one vertex which has a unique color among all other colors used for vertices in that hyperedge.

A vertex coloring of a hypergraph such that the minimum (maximum) color of any vertex of a hyperedge is unique (assigned to only one vertex in this hyperedge) is conflict-free and is called unique-min (resp. unique-max) (confict-free) coloring. The problems of computing a unique-min coloring is equivalent to computing a unique-max coloring since we can replace every color i by $c_{\max} - i + 1$, where $c_{\max}$ is the maximum color among all vertices [1].

In this paper, first i study unique-min (confict-free) coloring in chain, ring and tree, second, present a new algorithm for a tree of rings.

Conflict-free coloring have various applications. For Example in [2] consider the following scenario: vertices represent base stations of a cellular network interconnected through a backbone.

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Mobile client connect to the network by radio links and the reception range of each agent is a connected subgraph of the base stations graph. Then it may be desirable that in each agent's range there is a base station transmitting in a unique frequency, in order to avoid interference. The problem of minimizing the number of necessary frequencies is equivalent to Connected Subgraphs Conflict-Free Coloring.

Related work. The study of conflict-free coloring was initiated in [2] as a geometric problem with applications to cellular networks. Some of the problems proposed in that paper can be defined as hypergraph conflict-free coloring problems. The algorithm that uses $O(\log^2 n)$ colors (where n is the number of vertices) is given in [1] about CF-coloring for trees and trees of rings. Some of the problems presented in [2] can be defined as hypergraph conflict-free coloring problems. In [3,4] the conflict-free coloring was studied for grids. In [6] the conflict-free coloring of n points with respect to (closed) disks were studied and were proved a lower bound of $\Omega(\log n)$ colors. In [7] the conflict-free coloring of n points with respect to axis-parallel rectangles were studied. Various other conflict-free coloring problems have been considered in very recent papers [8,12,13,14,15,16,17,18].

The problem becomes more interesting when the vertices are given online by an adversary. For example, at every given time step i, a new vertex v_i is given and the algorithm must assign v_i a color such that the coloring is a conflict-free coloring of the hypergraph that is induced by the vertices $V = \{v_1, v_2, ..., v_i\}$. Once v_i is assigned a color, that color cannot be changed in the future. This is an online setting, so the algorithm has no knowledge of how vertices will be given in the future. In [5] there is the online version of conflict-free coloring of a hypergraph. The online version of Connected Subgraphs Conflict-Free Coloring in chains was presented in [8]. Also, in the case of intervals, there are several algorithms [11]. Their randomized algorithm uses $O(\log n \log\log n)$ colors with high probability. Their deterministic algorithm uses $O(\log^2 n)$ colors in the worst case. Recently, randomized algorithms that use $O(\log n)$ colors have been found in [9,10].

2. PRELIMINARIES

The topologies i study during this paper are chain, ring, tree and tree of rings. A graph is a ring when all its vertices V are connected in such a way that they form a cycle of length |V|. A tree of rings can be defined recursively in the following manner [18]: it is either a single ring or a ring R attached to a tree of rings T by identifying exactly one vertex of R to one vertex of T. An Example of a tree of rings is displayed in Figure 1.

Algorithm for unique-minimum conflict-free coloring in a chain: in [2] there exists an algorithm that uses $\lfloor \log n \rfloor + 1$ colors for chains. The algorithm for a chain {1,2,…,n} as follows:

step 1: Color vertex $\left\lceil \frac{n}{2^1} \right\rceil$ with color 1

step 2: Color vertices $\left\lceil \frac{n}{2^2} \right\rceil$, $\left\lceil \frac{n}{2^1} + \frac{n}{2^2} \right\rceil$ with color 2

step 3: Color vertices $\left\lceil \frac{n}{2^3} \right\rceil$, $\left\lceil \frac{n}{2^2} + \frac{n}{2^3} \right\rceil$, $\left\lceil \frac{n}{2^1} + \frac{n}{2^3} \right\rceil$, $\left\lceil \frac{n}{2^1} + \frac{n}{2^2} + \frac{n}{2^3} \right\rceil$ with color 3

…….

step i: Color verices $\left\lceil \frac{n}{2^i} \right\rceil, \ . \ \ . \ \ . \ \ . \ , \left\lceil \frac{n}{2^1}+\frac{n}{2^2}+\frac{n}{2^3}+....+\frac{n}{2^i} \right\rceil$ with color i

Color i is used only if $\left\lceil \frac{n}{2^i} \right\rceil = 1$, so in fact $\lfloor \log n \rfloor + 1$ colors are used by the algorithm.

For example, if n=8, the coloring is 32313234. It is clearly to see that the coloring is unique-minimum conflict-free coloring.

The above algorithm with a small change can be used to solve the unique-minimum conflict-free coloring in a ring. Pick an arbitrary vertex v and color it with a color 1 (not to be reused anywhere else in the coloring). The remaining vertices form a chain that color with the algorithm described above. This algorithm colors a ring of n vertices with $\lfloor \log(n-1) \rfloor + 2$ colors. For example, if n=8, the coloring is 14342434, where `1' is the first unique color used for v. It is not difficult to see that the coloring is conflict-free: All paths that include v are conflict-free colored, and the remaining graph G–v is a chain of n–1 vertices, so paths of G–v are also conflict-free colored.

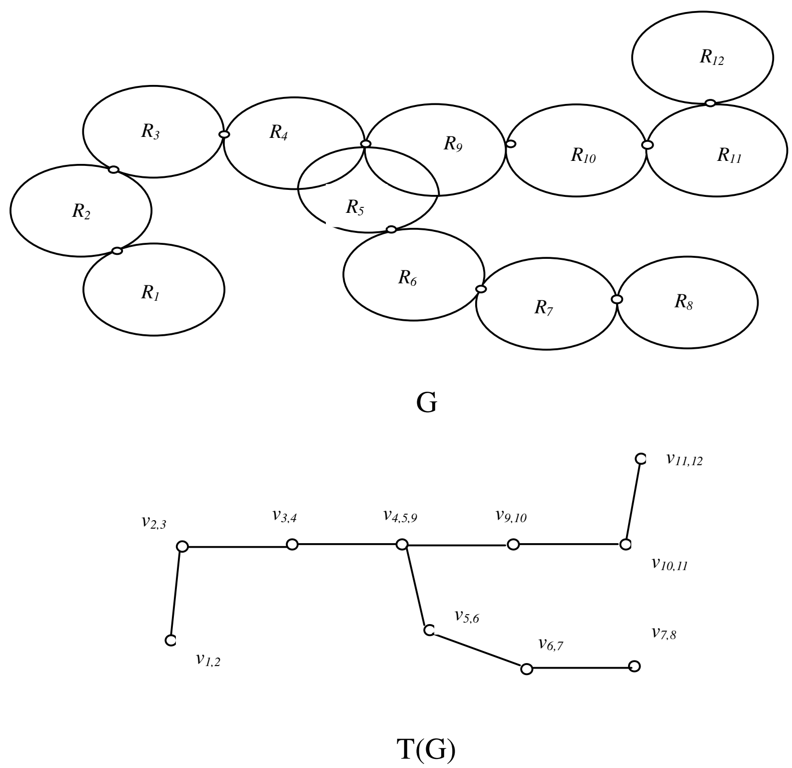

Figure 1. A tree of rings G and the corresponding tree representation T(G).

An important notion for my algorithm is α-separator.

Definition 2. An α-separator (α<1) of a graph G=(V,E) is a vertex u the removal of which partitions G to connected components of size at most α|V|.

It is obvious from the above definition that on a general graph an α-separator does not always exist. It is a folklore result that in trees a (1/2)-separator always exists; moreover it can be found in polynomial time [19]. In my algorithm i will often make use of (1/2)-separators.

Algorithm for unique-minimum conflict-free coloring in a tree: in [1] there exists an algorithm that uses $\lfloor \log n \rfloor$ colors for trees. The algorithm for a tree is displayed in Figure 2.

Algorithm 1: Unique-Min Coloring for a Tree

Input: a tree T
Output: a coloring of vertices of *T*.
1: Set T_1:=T , i:=1.
2: while $T_i \neq ¢$ **do**
3: Find (1/2)-separators on all connected components of forest T_i.
4: Add these separators to set V_i.
5: Color vertices in V_i with color i.
6: Construct forest T_{i+1} by removing vertices V_i from T_i.
7: Set i:=i+1.
8: **end while**

Figure 2. Algorithm 1

3. AN ALGORITHM FOR TREES OF RINGS

In order to present my algorithm for a tree of rings, i will use the notion of tree representation of a tree of rings. Assume a tree of rings G is $R_1, R_2, ..., R_{|T|}$. Let me first describe how to construct such a representation T(G) of a tree of rings G: Connect all vertices together that lied in intersection of rings. An Example of a tree of rings and its tree representation is displayed in Figure 1. The algorithm for a tree of rings is displayed in Figure 3.

Algorithm 2: Unique-Min Coloring for a Tree of Rings

Input: a tree of rings G by names $R_1, R_2, ..., R_{|T|}$
Output: a coloring of vertices of G
1: Construct the tree representation T(G) of the tree of rings G.
2: Color the tree T(G) with algorithm 1.
3: for i:=1 to |T|-1 do
Color the vertex in intersection (if exists and before didn't colour) of the rings R_i, R_{i+1} by color vertex $v_{i,(i+1)}$ in tree T(G).
end for
4: for i:=1 to |T| do
5: set cm:=a max color of the colored vertices of ring R_i .
6: Delete the colored vertices of ring R_i and connect the neighbors of them.
7: Let R'_i denote the resulting cycle.
8: Color cycle R'_i with said algorithm in section 2 by using colors from $\{cm+1, ..., cm + \lfloor \log |R'_i| \rfloor + 2\}$.
9: end for

Figure 3. Algorithm 2

3.1. Analysis of the algorithm

Lemma 1. The coloring obtained by Algorithm 2 is a connected-subgraphs unique-min conflict-free coloring.

Proof. Assume that C is a path in G. There are two cases for C. **Case 1**: C is part of a ring or a ring itself. if C does not contain the common vertices of the rings, C will be colored in a unique-min way because C colored in line 8 from algorithm 2. if C contains the common vertices of the rings, C will be colored in a unique-min way because the coloring of it start from the max of the colors of the common vertices of the rings (see lines 5,8 from algorithm 2). **Case 2**: C lies on a connected subset of rings, say $R_i,...,R_j$; the corresponding vertices of these rings in T(G), say $v_{i,(i+1)}...v_{(j-1),j}$. Since these vertices of T(G) in line 2 from algorithm 2 are colored in a unique-min way, and each ring R_k in C lies between vertices $v_{(k-1)k}, v_{k,(k+1)}$ that colored in line 8 from algorithm 2, therefore C has been colored in a unique-min way.

Lemma 2. The Algorithm 2 uses O(log|T|+log|R|) colors.

Proof. The number of colors for coloring T(G) equal log|T|. For coloring the rings, in line 5 from algorithm 2, the maximum of cm's is log|T|, therefore the maximum color is used in line 8 are log|T|+2+log|R|. Thus the Algorithm 2 uses O(log|T|+log|R|) colors.

4. CONCLUSIONS

I have presented an optimal algorithm for coloring a tree of rings such that each connected subgraph has a vertex with a unique minimum color. Also i have proved this algorithm uses O(log|T|+log|R|) colors.

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