DISTANCE TWO LABELING FOR MULTI-STOREY GRAPHS

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ABSTRACT

An L (2, 1)-labeling of a graph G (also called *distance two labeling*) *is a function f from the vertex set V (G) to the non negative integers {0,1,…, k }such that |f(x)-f(y)| ≥2 if d(x, y) =1 and | f(x)- f(y)| ≥1 if d(x, y) =2. The L (2, 1)-labeling number λ (G) or span of G is the smallest k such that there is a f with max {f (v) : v∈ V(G)}= k. In this paper we introduce a new type of graph called multi-storey graph. The distance two labeling of multi-storey of path, cycle, Star graph, Grid, Planar graph with maximal edges and its span value is determined. Further maximum upper bound span value for Multi-storey of simple graph are discussed.*

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KEYWORDS

Labeling, Cycle, Complete, Grid, Planar

1. INTRODUCTION

More and more Communication networks have been developed, but the available radio frequencies allocated to communications are not sufficient. It is important to find an efficient way to assign these frequencies. The main difficulty is given by inferences caused by unconstrained simultaneous transmissions which will damage communications. The interference can be avoided by means of suitable Channel assignment. In 1980, Hale introduced the Graph theory Model of the channel assignment problem where it was represented as a vertex coloring problem. Vertices on the graph correspond to the radio stations and the edges show the proximity of the stations. In 1991, Roberts proposed a variation of the channel assignment problem in which the FM radio stations were considered either "Close" or "Very Close". "Close" stations were vertices of distance two apart on the graph and were assigned different channels; stations that were considered "Very Close" were adjacent vertices on the graph and were assigned channels that differed by distance two. More precisely, In 1992 Grigs and Yeh defined the L(2, 1)-labeling (Distance two labeling) as follows.

An *L* (2, 1)*-labeling* of a graph *G* is a function *f* from the vertex set *V (G)* to the non negative integers {0,1,…, *k* }such that |*f*(*x*)-*f*(*y*)| ≥2 if *d*(*x*, *y*) =1 and | *f*(*x*)- *f*(*y*)| ≥1 if *d*(*x*, *y*) =2. The *L* (2, 1)*-labeling number* λ (*G*) or *span of G* is the smallest *k* such that *G* has an *L* (2, 1)-labeling *f* with *max* {*f* (*v*) : *v*∈*V* (*G*)}= *k*.

Our work is motivated the conjecture of Griggs and Yeh [9] that asserts that $\lambda_{2,1}(G) \leq \Delta^2$ for every graph G with maximum degree $\Delta \geq 2$. The conjecture has been verified only for several classes of graphs such as graphs of maximum degree two, chordal graphs [20] (see also [6, 16]) and Hamiltonian cubic graphs [12, 13]. For general graphs, the original bound $\lambda_{2,1}(G) \leq \Delta^2 + 2\Delta$ of [9] was improved to $\lambda_{2,1}(G) \leq \Delta^2 + \Delta$ in [6]. A more general result of [15] yields $\lambda_{2,1}(G) \leq \Delta^2 + \Delta - 1$ and the present record of $\lambda_{2,1}(G) \leq \Delta^2 + \Delta - 2$ was proven by

Goncalves [10]. The algorithmic aspects of L(2, 1)-labelings have also been investigated [4, 7, 8, 14, 17] because of their potential applications in practice. In this Paper we construct a new Graph called Multi-storey Graph and distance two labeling of Multi-storey of some class of graphs also determined. Upper bound for the span value of Multi-storey Simple Graph is also discussed.

2. CONSTRUCTION OF MULTI-STOREY GRAPH

A graph G consists of a pair $(V(G), E(G))$ where $V(G)$ is a non-empty finite set whose elements are called vertices and $E(G)$ is a set of unordered pairs of distinct elements of $V(G)$. The elements of $E(G)$ are called edges of the graph G. A graph G with out loops and parallel edges is called Simple Graph. If the given number of stations (vertices) is too large and we have to assign a channel (label) for these stations with out interference then we can arrange the stations in m layers and each stations of any layer is connected to corresponding stations by a link(edges). This idea creates a construction of Multi-storey graph. Consider any simple graph 'G' with n vertices. Arrange m copies of G like a Multi-storey building every layer of it using mn edges like pillar of a Multi-storey building. Formally we define a Multi-storey graph as follows

$\bigcup_{i=1}^{m} G^j = \left(V(G^j), E(G^j)\right)$ where $V(G^j) = \{v_1^j, v_2^j, ..., v_n^j\}$ for $j = 1,2,...,m$ and

$E(G^j) = \{v_i^j v_k^j : v_i v_k \in E(G)\}$. Now we construct a new graph $G^M = (V, E)$ called Multi-storey

graph where $V(G^M) = V(G^1) \cup V(G^2) \cup \ldots \cup V(G^m) = \bigcup_{j=1}^{m} V(G^j)$ and $E(G^M) = \bigcup_{j=1}^{m} E(G^j) \bigcup E'$

where $E' = \{v_i^k v_i^{k+1} : 1 \le i \le n; 1 \le k \le m-1\}$. The construction is given in Fig. 1.1.

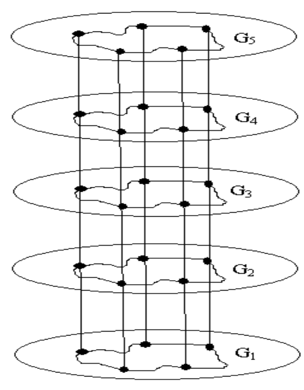

Figure 1. Multi-storey Graph

Number of vertices in this Multi-storey Graph G^M is mn, where m is the number of copies of G and n is the number of vertices of a simple Graph G. Number of edges in this Multi-storey Graph G^M is $(m-1)n + m|E|$. In general the Maximum degree of a graph G is denoted as Δ. In our Multi-storey graph Maximum degree is denoted as $\Delta_M = \Delta + 2 \le n+1$.

For example, Multi-storey of Path will give the structure of Grid. In [21] $L(h, k)$-labeling problem on the product of paths is studied by Tiziana Calamoneri. For a squared grid S, following results are proved in [20]:

Result 1:If $k \le h \le 2k$, then $2h + 2k \le \lambda_{h,k}(S) \le \min(4h, 2h + 3k - 1, 6k)$.

Result 2:If $2k \le h \le 3k$ then $2h + 2k \le \lambda_{h,k}(S) \le \min(3h, 2h + 3k - 1, 8k)$.

3. DISTANCE TWO LABELING - MULTI-STOREY GRAPH OF CYCLES AND STARS

In this section distance two labeling for Multi-storey of Cycle and Star are discussed.

Definition 3.1. The cycle graph or a Circuit graph C_n is a graph which has n vertices and n edges.

In the following algorithm, we give the construction and distance two labeling for Multi-storey cycle C_n^M. M copies of the cycles C_n are placed in M stores and the n vertices of cycle linked to the corresponding n vertices of the cycles in succeeding layers. Next distance two labeling of C_n^M is discussed in two cases. In the first case, distance two labeling for Multi-storey cycle C_n^M with $M < 3$ is given. In the second case, distance two labeling for Multi-storey cycle C_n^M with $M \geq 3$ is given in three sub cases $M \equiv 1(\bmod 3)$, $M \equiv 2(\bmod 3)$ and $M \equiv 0(\bmod 3)$.

Algorithm 3.2.

Input: Number of vertices mn of C_n^M

Output: Distance two labeling of vertices of the graph C_n^M
begin

$$V(C_n^M) = \left\{ \bigcup_{j=1}^{m} V(C_n^j) : V(C_n^j) = \{v_1^j, v_2^j, \ldots, v_n^j\} \right\}$$

$$E(C_n^M) = \bigcup_{j=1}^{m} E(C_n^j) \bigcup E' \text{ where } E(C_n^j) = \{v_i^j v_{i+1}^j : 1 \leq i \leq n-1; 1 \leq j \leq m\} \bigcup \{v_n^j v_1^j\}$$

$$E' = \{v_i^k v_i^{k+1} : 1 \leq i \leq n; 1 \leq k \leq m-1\}$$

if $(m < 3)$
for $j = 1$ to 2
  for $i = 1$ to $n-1$
    {
    $f(v_i^j) = 2(i-1) + 3(j-1);$
    $f(v_n^1) = 2(n-1);$
    $f(v_n^2) = 1;$
    }
elseif $(m \geq 3)$ and $(m \equiv 1 \pmod 3)$
for $j = 1$ to $\frac{m-1}{3}$
  for $i = 1$ to $n-1$
    {
    $f(v_i^{3j-2}) = f(v_i^m) = 2(i-1);$
    $f(v_i^{3j-1}) = 2(i-1) + 3;$
    $f(v_i^{3j}) = 2(i-1) + 6;$
    $f(v_n^{3j-2}) = f(v_n^m) = 2(n-1);$
    $f(v_n^{3j-1}) = 1;$
    $f(v_n^{3j}) = 4;$
    }

else if($m \geq 3$) and ($m \equiv 2 \pmod 3$)

for $j = 1$ to $\frac{m-2}{3}$

for $i = 1$ to $n-1$

{

$f(v_i^{3j-2}) = f(v_i^{m-1}) = 2(i-1);$

$f(v_i^{3j-1}) = f(v_i^{m}) = 2(i-1)+3;$

$f(v_i^{3j}) = 2(i-1)+6;$

$f(v_n^{3j-2}) = f(v_n^{m-1}) = 2(n-1);$

$f(v_n^{3j-1}) = f(v_n^{m}) = 1;$

$f(v_n^{3j}) = 4;$

}

elseif ($m \geq 3$) and ($m \equiv 0 \pmod 3$)

for $j = 1$ to $\frac{m}{3}$

for $i = 1$ to $n-1$

{

$f(v_i^{3j-2}) = 2(i-1);$

$f(v_i^{3j-1}) = 2(i-1)+3;$

$f(v_i^{3j}) = 2(i-1)+6;$

$f(v_n^{3j-2}) = 2(n-1);$

$f(v_n^{3j-1}) = 1;$

$f(v_n^{3j}) = 4;$

}

end

Theorem 3.3: The Multi-storey of Cycle graph C_n^M has distance two labeling and its span of G $\lambda(C_n^M) = 2n+2$.

Proof: Consider the Multi-storey of Cycle graph C_n^M with the vertex and edge set defined as in algorithm 3.2. Let the function f: $V(C_n^M) \rightarrow \{0,1,2,\ldots,2n+2\}$ be defined as in algorithm 3.2.

Now we have to prove this theorem using two cases.

Case (i): If $d(v_i, v_j)=1$ then to prove $|f(v_i)-f(v_j)| \geq 2$.

$d(v_i, v_j)=1$occur for the edges $\{v_i^j v_{i+1}^j : 1 \leq i \leq n-1; 1 \leq j \leq m\} \cup \{v_n^j v_1^j\}$;

$\{v_{i-1}^j v_i^j : 2 \leq i \leq n; 1 \leq j \leq m\}$; $\{v_i^j v_i^{j+1} : 1 \leq i \leq n; 1 \leq j \leq m-1\}$; $\{v_i^j v_i^{j-1} : 1 \leq i \leq n; 2 \leq j \leq m\}$

By algorithm 3.2, In all cases,

We get $|f(v_n^j)-f(v_1^j)| \geq 2$.

$|f(v_i^j)-f(v_i^{j+1})| = 3$ for $1 \leq i \leq n-1$ and $1 \leq j \leq m-1$.

$|f(v_i^j)-f(v_i^{j-1})| = 3$ for $1 \leq i \leq n-1$ and $2 \leq j \leq m$.

$|f(v_n^j)-f(v_n^{j-1})| = |f(v_n^j)-f(v_n^{j+1})| \geq 3$ for$1 \leq j \leq m-1$and $2 \leq j \leq m$.

$|f(v_i^j)-f(v_{i+1}^j)| \geq 2$for $1 \leq i \leq n-1$ and $1 \leq j \leq m$; $|f(v_{i-1}^j)-f(v_i^j)| \geq 2$for $2 \leq i \leq n$ and $1 \leq j \leq m$.

Thus for all i,j, we have $|f(v_i)-f(v_j)| \geq 2$.

Case(ii): If $d(v_i, v_j)=2$ then to prove $|f(v_i)-f(v_j)| \geq 1$.

$d(v_i, v_j)=2$ occur for the edges $\{v_i^j v_i^{j+2} : 1 \leq i \leq n-1; 1 \leq j \leq m-2\}$;

$\{v_i^j v_i^{j-2} : 1 \leq i \leq n-1; 3 \leq j \leq m\}$; $\{v_i^j v_{i+2}^j, v_n^j v_2^j : 1 \leq i \leq n-2; 1 \leq j \leq m\}$

$\{v_i^j v_{i-2}^j, : 3 \leq i \leq n; 1 \leq j \leq m\}$; $\{v_i^j v_{i+1}^{j+1} : 1 \leq i \leq n-1; 1 \leq j \leq m-1\}$ $\{v_i^j v_{i+1}^{j-1} : 1 \leq i \leq n-1; 2 \leq j \leq m\}$

$\{v_i^j v_{i-1}^{j+1} : 2 \leq i \leq n; 1 \leq j \leq m-1\}$; $\{v_i^j v_{i-1}^{j-1} : 2 \leq i \leq n; 2 \leq j \leq m\}$.

all the vertices at distance two have distinct labeling according to the algorithm 3.2.1.
Thus in this case we have $|f(v_i)-f(v_j)| \geq 1$ for all i, j.
By algorithm 3.2, the vertices $v_{n-1}{}^{m}$ has the maximum label i.e., $f(v_{n-1}{}^{m}) = 2n+2$.
Hence the Multi-storey of cycle graph $C_n{}^M$ has distance two labeling and $\lambda(C_n{}^M) = 2n+2$.

Definition 3.4. A Star graph $K_{1,k}$ is a tree in which one of the vertex have degree n and all the other vertices are pendent vertices.

In the following algorithm, we give the construction and distance two labeling for Multi-storey Star $K_{1,n}{}^M$. In this graph, the star $K_{1,n}$ is placed in M stores and the n+1 vertices of star linked to the corresponding n+1 vertices of the stars in succeeding layers. Next distance two labeling of $K_{1,n}{}^M$ is discussed in two cases. In the first case, distance two labeling for Multi-storey star $K_{1,n}{}^M$ with $M < 4$ is given. In the second case, distance two labeling for Multi-storey star $K_{1,n}{}^M$ with $M \geq 4$ is given in four sub cases $M \equiv 1 (\mathrm{mod}\, 4)$, $M \equiv 2 (\mathrm{mod}\, 4)$, $M \equiv 3 (\mathrm{mod}\, 4)$ and $M \equiv 0 (\mathrm{mod}\, 4)$.

Algorithm 3.5.

Input: Number of vertices of $K_{1,n}{}^M$
Output: Distance two labeling of vertices of the graph $K_{1,n}{}^M$
begin

$$V(K_{1,n}^M) = \left\{ \bigcup_{j=1}^{m} V(K_{1,n}^j) : V(K_{1,n}^j) = \{v_1^j, v_2^j, \ldots, v_n^j, v_{n+1}^j\} \right\}$$

$$E(K_{1,n}^M) = \bigcup_{j=1}^{m} E(K_{1,n}^j) \bigcup E' \text{ where } E(K_{1,n}^j) = \{v_1^j v_{i+1}^j : 1 \leq i \leq n; 1 \leq j \leq m\} \cup$$

$$E' = \{v_i^j v_i^{j+1} : 1 \leq i \leq n+1; 1 \leq j \leq m-1\}$$

if $(m < 4)$
{
for $j = 1$ to m
  for $i = 1$ to n
    $f(v_i^j) = 2(i-1) + 3(j-1);$
    $f(v_{n+1}^1) = 2(n-1); f(v_{n+1}^2) = 1; f(v_{n+1}^3) = 4;$
}
if $(m \geq 4) and (m \equiv 1 (\mathrm{mod} 4))$
for $j = 1$ to $\dfrac{m-1}{4}$
  for $i = 1$ to n
    {
    $f(v_i^{4j-3}) = f(v_i^m) = 2(i-1);$
    $f(v_i^{4j-2}) = 2(i-1) + 3;$
    $f(v_i^{4j-1}) = 2(i-1) + 6;$
    $f(v_i^{4j}) = 2(i-1) + 9;$
    $f(v_{n+1}^{4j-3}) = f(v_{n+1}^m) = 2(n-1);$
    $f(v_{n+1}^{4j-2}) = 1;$
    $f(v_{n+1}^{4j-1}) = 4;$
    $f(v_{n+1}^{4j}) = 7;$
    }

else if$(m \geq 4)$ and $(m \equiv 2(\text{mod}4))$

for $j=1$ to $\frac{m-2}{4}$

for $i=1$ to n

{

$f(v_i^{4j-3}) = f(v_i^{m-1}) = 2(i-1);$

$f(v_i^{4j-2}) = f(v_i^{m}) = 2(i-1)+3;$

$f(v_i^{4j-1}) = 2(i-1)+6;$

$f(v_i^{4j}) = 2(i-1)+9;$

$f(v_{n+1}^{4j-3}) = f(v_{n+1}^{m-1}) = 2(n-1);$

$f(v_{n+1}^{4j-2}) = f(v_{n+1}^{m}) = 1;$

$f(v_{n+1}^{4j-1}) = 4;$

$f(v_{n+1}^{4j}) = 7;$

}

else if $(m \geq 4)$and $(m \equiv 3(\text{mod}4))$

for $j=1$ to $\frac{m-3}{4}$

for $i=1$ to n

{

$f(v_i^{4j-3}) = f(v_i^{m-2}) = 2(i-1);$

$f(v_i^{4j-2}) = f(v_i^{m-1}) = 2(i-1)+3;$

$f(v_i^{4j-1}) = f(v_i^{m}) = 2(i-1)+6;$

$f(v_i^{4j}) = 2(i-1)+9;$

$f(v_{n+1}^{4j-3}) = f(v_{n+1}^{m-2}) = 2(n-1);$

$f(v_{n+1}^{4j-2}) = f(v_{n+1}^{m-1}) = 1;$

$f(v_{n+1}^{4j-1}) = f(v_{n+1}^{m}) = 4;$

$f(v_{n+1}^{4j}) = 7;$

}

elseif$(m \geq 4)$and $(m \equiv 0(\text{mod}4))$

for $j=1$ to $\frac{m}{4}$

for $i=1$ to n

{

$f(v_i^{4j-3}) = 2(i-1);$

$f(v_i^{4j-2}) = 2(i-1)+3;$

$f(v_i^{4j-1}) = 2(i-1)+6;$

$f(v_i^{4j}) = 2(i-1)+9;$

$f(v_{n+1}^{4j-3}) = 2n;$

$f(v_{n+1}^{4j-2}) = 1;$

$f(v_{n+1}^{4j-1}) = 4;$

$f(v_{n+1}^{4j}) = 7;$

}

end.

Theorem 3.6: The Multi-storey of Star graph $K_{1,n}^{M}$ has distance two labeling and its span $\lambda(K_{1,n}^{M}) = 2n+7$.

Proof: Consider the Multi-storey of Star graph $K_{1,n}^{M}$ with the vertex and edge set defined as in algorithm 3.5. Let the function f: $V(K_{1,n}^{M}) \rightarrow \{0,1,2,\ldots, 2n+7\}$ be defined as in algorithm 3.5. Now we have to prove this theorem using two cases.

Case (i): If $d(v_i, v_j)=1$ then to prove $|f(v_i)-f(v_j)| \geq 2$.

$d(v_i,v_j)$=1occur for the edges $\{v_i^j v_{n+1}^j : 1 \leq i \leq n; 1 \leq j \leq m\}$ and

$\{v_i^j v_i^{j+1} : 1 \leq i \leq n+1; 1 \leq j \leq m-1\}$; $\{v_i^j v_i^{j-1} : 1 \leq i \leq n+1; 2 \leq j \leq m\}$.

By algorithm 3.5, in all cases,

We get $|f(v_{n+1}^{j})-f(v_i^{j})| \geq 2$ for $1 \leq i \leq n$; and $1 \leq j \leq m$.

$|f(v_i^j)-f(v_i^{j+1})| = 3$ for $1 \leq i \leq n+1$ and $1 \leq j \leq m-1$.

$|f(v_i^j)-f(v_i^{j-1})| = 3$ for $1 \leq i \leq n+1$ and $2 \leq j \leq m$.

Thus for all i,j, we have $|f(v_i)-f(v_j)| \geq 2$.

Case(ii): If $d(v_i, v_j)=2$ then to prove $|f(v_i)-f(v_j)| \geq 1$.

$d(v_i, v_j)$=2 occur for the edges $\{v_i^j v_{n+1}^{j+1} : 1 \leq i \leq n; 1 \leq j \leq m-1\}$,

$\{v_i^j v_{n+1}^{j-1} : 1 \leq i \leq n; 2 \leq j \leq m\}$; $\{v_i^j v_i^{j+2} : 1 \leq i \leq n+1; 1 \leq j \leq m-2\}$; $\{v_i^j v_i^{j-2} : 1 \leq i \leq n+1; 3 \leq j \leq m\}$

and $\{v_i^j v_k^j : 1 \leq i,k \leq n \text{ and } i \neq k; 1 \leq j \leq m\}$.

all the vertices at distance two have distinct labeling. Thus in this case we have $|f(v_i)-f(v_j)| \geq 1$ for all i, j. By algorithm 3.5, the vertices v_{n+1}^{4j} has the maximum label i.e., $f(v_{n+1}^{4j}) = 2n+7$.

Hence the Multi-storey of Star graph $K_{1,n}^{M}$ has distance two labeling and $\lambda(K_{1,n}^{M}) = 2n+7$.

4. DISTANCE TWO LABELING - MULTI-STOREY GRAPH OF SQUARED GRID

Definition 4.1. A two-dimensional grid graph is a graph which is Cartesian product of the path P_m and P_n.

We investigate distance two labeling for Multi-story of squared grids. Construction of Multi-storey of grids is shown in below figure 2.

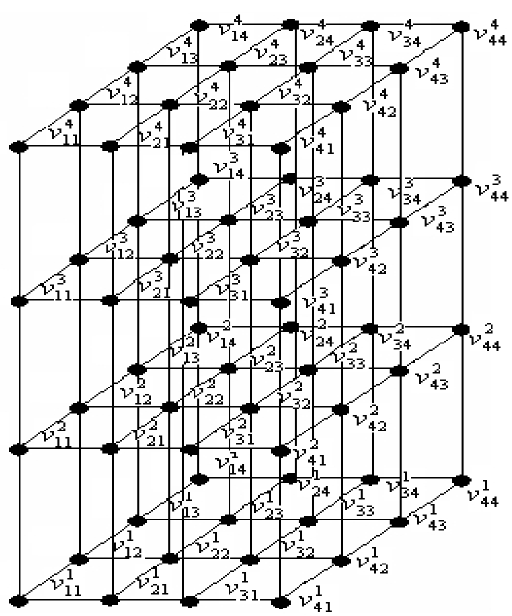

Figure 2. Multi-storey of $P_4 \times P_4$

Example 4.2.

Distance two labelling of $(P_5 \times P_5)^5$ is shown in below Matrix form

Stores M = 1 2 3 4 5

$$\begin{pmatrix} 0 & 2 & 4 & 0 & 2 \\ 6 & 8 & 10 & 6 & 8 \\ 12 & 14 & 16 & 12 & 14 \\ 0 & 2 & 4 & 0 & 2 \\ 6 & 8 & 10 & 6 & 8 \end{pmatrix} \begin{pmatrix} 3 & 5 & 7 & 3 & 5 \\ 9 & 11 & 13 & 9 & 11 \\ 15 & 17 & 19 & 15 & 17 \\ 3 & 5 & 7 & 3 & 5 \\ 9 & 11 & 13 & 9 & 11 \end{pmatrix} \begin{pmatrix} 6 & 8 & 10 & 6 & 8 \\ 12 & 14 & 16 & 12 & 14 \\ 18 & 20 & 22 & 18 & 20 \\ 6 & 8 & 10 & 6 & 8 \\ 12 & 14 & 16 & 12 & 14 \end{pmatrix} \begin{pmatrix} 9 & 11 & 13 & 9 & 11 \\ 15 & 17 & 19 & 15 & 17 \\ 21 & 23 & 25 & 21 & 23 \\ 9 & 11 & 13 & 9 & 11 \\ 15 & 17 & 19 & 15 & 17 \end{pmatrix} \begin{pmatrix} 0 & 2 & 4 & 0 & 2 \\ 6 & 8 & 10 & 6 & 8 \\ 12 & 14 & 16 & 12 & 14 \\ 0 & 2 & 4 & 0 & 2 \\ 6 & 8 & 10 & 6 & 8 \end{pmatrix}$$

The entries of the matrices are the labels given to the vertices of the graph. There are 5 matrices which represent the labeling of the five different layers of the Multi-story squared grid. The ijth entry of the kth matrices represent the distance two labelng of ijth vertex lying in the k^{th} layer of the Multi-story Graph. After the 4th store the same values of the first four levels will be repeated. In general for all values of n ≥3 and $M \geq 4$ the maximum value used to label (span) the Multi-storey of grid $P_n \times P_n$ is λ =25.

Distance two labelling of first store in the Multi-storey of grid is given in below algorithm 4.3. In this algorithm, the distance two labeling for grid $P_n \times P_n$ is given in three cases $n \equiv 1(\bmod 3)$, $n \equiv 2(\bmod 3)$ and $n \equiv 0(\bmod 3)$.

Algorithm 4.3.

Input: Number of n^2 vertices of $P_n \times P_n$
Output: Distance two labelling $f(v_{ij})$ of n^2 vertices of the graph $P_n \times P_n$
begin

$V(P_{n\times n}) = \bigcup_{i,j=1}^{n} v_{ij}$;

$E(P_{n\times n}) = \{v_{ij}v_{ij+1} : 1 \leq i \leq n; 1 \leq j \leq n-1\} \cup \{v_{ij}v_{i+1j} : 1 \leq i \leq n-1; 1 \leq j \leq n\}$

if ($n \geq 3$) and ($n \equiv 1$(mod3))

for $i = 1$ to $\frac{n-1}{3}$

for $j = 1$ to $\frac{n-1}{3}$

{

$f(v_{3i-2,3j-2}) = f(v_{n,3j-2}) = f(v_{3i-2,n}) = f(v_{n,n})$ =0;

$f(v_{3i-2,3j-1}) = f(v_{n,3j-1}) = 2$;

$f(v_{3i-2,3j}) = f(v_{n,3j}) = 4$;

$f(v_{3i-1,3j-2}) = f(v_{3i-1,n}) = 6$;

$f(v_{3i-1,3j-1}) = 8$;

$f(v_{3i-1,3j}) = 10$;

$f(v_{3i,3j-2}) = f(v_{3i,n}) = 12$;

$f(v_{3i,3j-1}) = 14$;

$f(v_{3i,3j}) = 16$;

}

elseif ($n \geq 3$) and ($n \equiv 2$(mod3))

for $i = 1$ to $\frac{n-2}{3}$

for $j=1$ to $\frac{n-2}{3}$
{
$f(v_{3i-2,3j-2}) = f(v_{n-1,3j-2}) = f(v_{3i-2,n-1}) = f(v_{n-1,n-1}) = 0;$
$f(v_{3i-2,3j-1}) = f(v_{3i-2,n}) = f(v_{n-1,n}) = 2;$
$f(v_{3i-2,3j}) = f(v_{n-1,3j}) = 4;$
$f(v_{3i-1,3j-2}) = f(v_{3i-1,n-1}) = f(v_{n,3j-2}) = f(v_{n,n-1}) = 6;$
$f(v_{3i-1,3j-1}) = f(v_{3i-1,n}) = f(v_{n,3j-1}) = f(v_{n,n}) = 8;$
$f(v_{3i-1,3j}) = f(v_{n,3j}) = 10;$
$f(v_{3i,3j-2}) = f(v_{3i,n-1}) = 12;$
$f(v_{3i,3j-1}) = f(v_{3i,n}) = 14;$
$f(v_{3i,3j}) = 16;$
}
elseif $(n \geq 3)$ and $(n \equiv 0(\text{mod}3))$
for $i=1$ to $\frac{n}{3}$
for $j=1$ to $\frac{n}{3}$
{
$f(v_{3i-2,3j-2}) = 0;$
$f(v_{3i-2,3j-1}) = 2;$
$f(v_{3i-2,3j}) = 4;$
$f(v_{3i-1,3j-2}) = 6;$
$f(v_{3i-1,3j-1}) = 8;$
$f(v_{3i-1,3j}) = 10;$
$f(v_{3i,3j-2}) = 12;$
$f(v_{3i,3j-1}) = 14;$
$f(v_{3i,3j}) = 16;$
}
end.

In algorithm 4.4, we give the construction of Multi-storey grid $(P_n \times P_n)^M$ and assign the distance two labeling. The grids $P_n \times P_n$ are placed in M stores and the n vertices of grid linked to the corresponding n vertices of the grids in succeeding layers. In the first step we define Vertex set and edge sets. In the successive steps 2 to 5 we assign distance two labeling of $(P_n \times P_n)^M$, $M \geq 4$ for four cases $M \equiv 1(\text{mod}\,4)$, $M \equiv 2(\text{mod}\,4)$, $M \equiv 3(\text{mod}\,4)$ and $M \equiv 0(\text{mod}\,4)$ respectively.

Algorithm 4.4.

Input: Number of mn^2 vertices of $(P_n \times P_n)^M$
Output: Distance two labelling $f(v_{ij}^m)$ of mn^2 vertices of the graph $(P_n \times P_n)^M$
begin

Step1. Take vertex and edge set of grid as $V(P_{n\times n}^M) = \left\{ \bigcup_{k=1}^{m} V(P_{n\times n}^k) : V(P_{n\times n}^k) = \bigcup_{i,j=1}^{n} v_{ij}^k \right\}$

$$E(P^M_{n\times n})=\bigcup_{k=1}^{m}E(P^k_{n\times n})\bigcup E^{'} \text{ where } E(P^k_{n\times n})=\left\{v^k_{ij}v^k_{ij+1}:1\le i\le n;1\le j\le n-1\right\}\cup\left\{v^k_{ij}v^k_{i+1j}:1\le i\le n-1;1\le j\le n\right\}$$

$$E^{'}=\{v^k_{ij}v^{k+1}_{ij}:1\le i\le n;1\le j\le n;1\le k\le m-1\}$$

Step 2. Consider the distance two labeling $f(v_{ij})$ from the above algorithm3.4.2., When m ≥ 4 and m ≡ 1(mod 4), the labeling of Multi-storey of grid will be as follows

for $k=1$ to $\frac{m-1}{4}$

{
$f(v^{4k-3}_{ij})=f(v^m_{ij})=f(v_{ij});$
$f(v^{4k-2}_{ij})=f(v_{ij})+3;$
$f(v^{4k-1}_{ij})=f(v_{ij})+6;$
$f(v^{4k}_{ij})=f(v_{ij})+9;$
}

Step 3. When m ≥ 4 and m ≡ 2(mod 4), the labeling of Multi-storey of grid will be as follows

for $k=1$ to $\frac{m-2}{4}$

{
$f(v^{4k-3}_{ij})=f(v^{m-1}_{ij})=f(v_{ij});$
$f(v^{4k-2}_{ij})=f(v^m_{ij})=f(v_{ij})+3;$
$f(v^{4k-1}_{ij})=f(v_{ij})+6;$
$f(v^{4k}_{ij})=f(v_{ij})+9;$
}

Step 4. When m ≥ 4 and m ≡ 3(mod 4), the labeling of Multi-storey of grid will be as follows

for $k=1$ to $\frac{m-3}{4}$

{

$f(v^{4k-3}_{ij})=f(v^{m-2}_{ij})=f(v_{ij});$
$f(v^{4k-2}_{ij})=f(v^{m-1}_{ij})=f(v_{ij})+3;$
$f(v^{4k-1}_{ij})=f(v^m_{ij})=f(v_{ij})+6;$
$f(v^{4k}_{ij})=f(v_{ij})+9;$
}

Step 5. When m ≥ 4 and m ≡ 0(mod 4), the labeling of Multi-storey of grid will be as follows

for $k=1$ to $\frac{m}{4}$

{

$f(v_{ij}^{4k-3}) = f(v_{ij});$
$f(v_{ij}^{4k-2}) = f(v_{ij}) + 3;$
$f(v_{ij}^{4k-1}) = f(v_{ij}) + 6;$
$f(v_{ij}^{4k}) = f(v_{ij}) + 9;$
}
end.

Theorem 4.5: The Multi-storey of Grid $(P_n \times P_n)^M$ has distance two labeling and its span $\lambda((P_n \times P_n)^M) = 25$.

Proof: Consider the Multi-storey of grid $(P_n \times P_n)^M$ with the vertex and edge set defined as in algorithm 4.4. Let the function f: $V((P_n \times P_n)^M) \rightarrow \{0,1,2,\ldots, 25\}$ be defined as in algorithm 4.4. Now we have to prove this theorem using two cases.

Case (i): If $d(v_i, v_j)=1$ then to prove $|f(v_i)-f(v_j)| \geq 2$.

$d(v_i, v_j)=1$occur for the edges $\left\{v_{ij}^k v_{ij+1}^k : 1 \leq i \leq n; 1 \leq j \leq n-1; 1 \leq k \leq m\right\}$;

$\left\{v_{ij}^k v_{ij-1}^k : 1 \leq i \leq n; 2 \leq j \leq n; 1 \leq k \leq m\right\}$; $\left\{v_{ij}^k v_{i+1j}^k : 1 \leq i \leq n-1; 1 \leq j \leq n; 1 \leq k \leq m\right\}$;

$\left\{v_{ij}^k v_{i-1j}^k : 2 \leq i \leq n; 1 \leq j \leq n; 1 \leq k \leq m\right\}$; $\left\{v_{ij}^k v_{ij}^{k+1} : 1 \leq i \leq n; 1 \leq j \leq n; 1 \leq k \leq m-1\right\}$ and

$\left\{v_{ij}^k v_{ij}^{k-1} : 1 \leq i \leq n; 1 \leq j \leq n; 2 \leq k \leq m\right\}$.

By algorithm 4.4, in all cases,

We get $|f(v_{ij}^k)-f(v_{ij+1}^k)| \geq 2$ for $1 \leq i \leq n$; $1 \leq j \leq n-1$ and $1 \leq k \leq m$.
$|f(v_{ij}^k)-f(v_{ij-1}^k)| \geq 2$ for $1 \leq i \leq n$; $2 \leq j \leq n$ and $1 \leq k \leq m$.
$|f(v_{ij}^k)-f(v_{i+1\,j}^k)| > 2$ for $1 \leq i \leq n-1; 1 \leq j \leq n$ and $1 \leq k \leq m$.
$|f(v_{ij}^k)-f(v_{i-1\,j}^k)| > 2$ for $1 \leq i \leq n; 2 \leq j \leq n$ and $1 \leq k \leq m$.
$|f(v_{ij}^k)-f(v_{ij}^{k+1}| =3$for $1 \leq i \leq n; 1 \leq j \leq n$ and $1 \leq k \leq m-1$.
$|f(v_{ij}^k)-f(v_{ij}^{k-1}| =3$for $1 \leq i \leq n; 1 \leq j \leq n$ and $2 \leq k \leq m$.

Thus for all i,j, we have $|f(v_i)-f(v_j)| \geq 2$.

Case(ii): If $d(v_i, v_j)=2$ then to prove $|f(v_i)-f(v_j)| \geq 1$.

$d(v_i, v_j)=2$ occur for the edges

$\left\{v_{ij}^k v_{ij+2}^k : 1 \leq i \leq n; 1 \leq j \leq n-2; 1 \leq k \leq m\right\}$; $\left\{v_{ij}^k v_{ij-2}^k : 1 \leq i \leq n; 3 \leq j \leq n; 1 \leq k \leq m\right\}$;

$\left\{v_{ij}^k v_{i+2j}^k : 1 \leq i \leq n-2; 1 \leq j \leq n; 1 \leq k \leq m\right\}$; $\left\{v_{ij}^k v_{i-2j}^k : 3 \leq i \leq n; 1 \leq j \leq n; 1 \leq k \leq m\right\}$;

$\left\{v_{ij}^k v_{ij+1}^{k+1} : 1 \leq i \leq n; 1 \leq j \leq n-1; 1 \leq k \leq m-1\right\}$; $\left\{v_{ij}^k v_{ij-1}^{k+1} : 1 \leq i \leq n; 2 \leq j \leq n; 1 \leq k \leq m-1\right\}$;

$\left\{v_{ij}^k v_{i+1j}^{k+1} : 1 \leq i \leq n-1; 1 \leq j \leq n; 1 \leq k \leq m-1\right\}$; $\left\{v_{ij}^k v_{i-1j}^{k+1} : 2 \leq i \leq n; 1 \leq j \leq n; 1 \leq k \leq m-1\right\}$;

$\left\{v_{ij}^k v_{ij+1}^{k-1} : 1 \leq i \leq n; 1 \leq j \leq n-1; 2 \leq k \leq m\right\}$; $\left\{v_{ij}^k v_{ij-1}^{k-1} : 1 \leq i \leq n; 2 \leq j \leq n; 2 \leq k \leq m\right\}$;

$\left\{v_{ij}^k v_{i+1j}^{k-1} : 1 \leq i \leq n-1; 1 \leq j \leq n; 2 \leq k \leq m\right\}$; $\left\{v_{ij}^k v_{i-1j}^{k-1} : 2 \leq i \leq n; 1 \leq j \leq n; 2 \leq k \leq m\right\}$; and

$\left\{v_{ij}^k v_{ij}^{k+2} : 1 \leq i \leq n; 1 \leq j \leq n; 1 \leq k \leq m-2\right\}$; $\left\{v_{ij}^k v_{ij}^{k-2} : 1 \leq i \leq n; 1 \leq j \leq n; 3 \leq k \leq m\right\}$.

all the vertices at difference two have distinct labeling. Thus in this case we have $|f(v_i)-f(v_j)| \geq 1$ for all i, j. By algorithm 4.4, the maximum labelling number is 25.

Hence the Multi-storey of Grid $(P_n \times P_n)^M$ has distance two labeling and its span $\lambda((P_n \times P_n)^M) =25$.

5. PLANAR GRAPH WITH MAXIMAL EDGES

5.1. Construction for Planar Graph with Maximal Edges Pl_n

The following definition and properties of planar are introduced by J. Baskar Babujee [1]

Definition 5.1.1: [1]**:**Let K_n = (V,E) be the complete graph with n vertices V(K_n) ={1,2,…n}and $n(n-1)/2$ edges $E(K_n) = \{(i.j): 1 \leq i ,j \leq n \text{ and } i<j\}$.The class of planar graphs with maximal edges over n vertices derived from complete graph K_n is defined as Pl_n = (V, E), where V=V(K_n) and $E = E(K_n) \setminus \{(k,l): k = 3 \text{ to } n-2, l = k+2 \text{ to } n\}$.

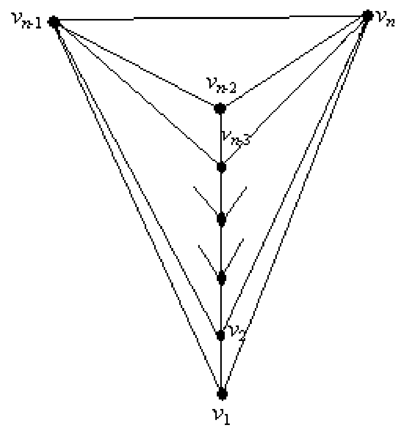

Fig 3.

The number of edges in Pl_n: $n \geq 5$ is $3(n-2)$. A planar graph divides the plain into areas which we call faces. By Eulers formula m – n + 2 = f (where f is the number of faces). In Pl_n the number of faces f = $3(n-2)-n+2 = 2(n-2)$. Hence In Pl_n: 2m – 3f = 0 i.e., ∑deg (v) = 3f where deg(v) denotes degree of a vertex v. There are exactly two vertices with degree 3, two vertices with degree $(n – 1)$ and all other vertices with degree 4.

In the following algorithm, we give the construction and distance two labeling for Multi-storey Planar graph Pl_n^M. In this graph, the planar graph Pl_n is placed in M stores and the n vertices of Pl_n linked to the corresponding n vertices of the planar Pl_n's in succeeding layers. The distance two labeling of Pl_n^M is discussed in two cases. In the first case, distance two labeling for Multi-storey planar graph Pl_n^M with M < 4 is given. In the second case, distance two labeling for Multi-storey planar graph Pl_n^M with $M \geq 4$ is given in four sub cases $M \equiv 1 (\bmod 4)$, $M \equiv 2 (\bmod 4)$, $M \equiv 3 (\bmod 4)$ and $M \equiv 0 (\bmod 4)$.

Algorithm 5.2.

Input: Number of vertices mn of Pl_n^M

Output: Distance two labeling of vertices of the graph Pl_n^M

begin

$$V(Pl_n^M) = \left\{ \bigcup_{j=1}^{m} V(Pl_n^j) : V(Pl_n^j) = \{v_1^j, v_2^j, ..., v_n^j\} \right\}$$

$$E(Pl_n^M) = \bigcup_{j=1}^{m} E(Pl_n^j) \bigcup E^{'} \text{ where } E(Pl_n^j) = \left\{ v_i^j v_{n-1}^j, v_i^j v_n^j : \ 1 \leq i \leq n-2 \ \right\} \cup \left\{ v_k^j v_{k+1}^j : 1 \leq k \leq n-3 \right\} \cup \left\{ v_{n-1}^j v_n^j \right\}$$

$$E^{'} = \{v_i^k v_i^{k+1} : 1 \leq i \leq n; 1 \leq k \leq m-1\}$$

if $(m < 4)$
for $j = 1$ to m
for $i = 1$ to $n-1$
{
$f(v_i^j) = 2(i-1) + 3(j-1);$
$f(v_n^1) = 1;$
$f(v_n^2) = 4;$
$f(v_n^3) = 7;$
}
if $(m \geq 4)$ and $(m \equiv 1 (\text{mod} 4))$
for $j = 1$ to $\frac{m-1}{4}$
for $i = 1$ to $n-1$
{
$f(v_i^{4j-3}) = f(v_i^m) = 2(i-1);$
$f(v_i^{4j-2}) = 2(i-1) + 3;$
$f(v_i^{4j-1}) = 2(i-1) + 6;$
$f(v_i^{4j}) = 2(i-1) + 9;$
$f(v_n^{4j-3}) = f(v_n^m) = 2(n-1);$
$f(v_n^{4j-2}) = 1;$
$f(v_n^{4j-1}) = 4;$
$f(v_n^{4j}) = 7;$
}
else if$(m \geq 4)$ and $(m \equiv 2 (\text{mod} 4))$
for $j = 1$ to $\frac{m-2}{4}$
for $i = 1$ to $n-1$
{
$f(v_i^{4j-3}) = f(v_i^{m-1}) = 2(i-1);$
$f(v_i^{4j-2}) = f(v_i^m) = 2(i-1) + 3;$
$f(v_i^{4j-1}) = 2(i-1) + 6;$
$f(v_i^{4j}) = 2(i-1) + 9;$
$f(v_n^{4j-3}) = f(v_n^{m-1}) = 2(n-1);$
$f(v_n^{4j-2}) = f(v_n^m) = 1;$
$f(v_n^{4j-1}) = 4;$
$f(v_n^{4j}) = 7;$
}

else if $(m \geq 4)$ and $(m \equiv 3 (\text{mod} 4))$

for $j = 1$ to $\frac{m-3}{4}$

for $i = 1$ to $n-1$

{

$f(v_i^{4j-3}) = f(v_i^{m-2}) = 2(i-1);$

$f(v_i^{4j-2}) = f(v_i^{m-1}) = 2(i-1)+3;$

$f(v_i^{4j-1}) = f(v_i^{m}) = 2(i-1)+6;$

$f(v_i^{4j}) = 2(i-1)+9;$

$f(v_n^{4j-3}) = f(v_{n+1}^{m-2}) = 2(n-1);$

$f(v_n^{4j-2}) = f(v_{n+1}^{m-1}) = 1;$

$f(v_n^{4j-1}) = f(v_{n+1}^{m}) = 4;$

$f(v_n^{4j}) = 7;$

}

elseif $(m \geq 4)$ and $(m \equiv 0 (\text{mod} 4))$

for $j = 1$ to $\frac{m}{4}$

for $i = 1$ to $n-1$

{

$f(v_i^{4j-3}) = 2(i-1);$

$f(v_i^{4j-2}) = 2(i-1)+3;$

$f(v_i^{4j-1}) = 2(i-1)+6;$

$f(v_i^{4j}) = 2(i-1)+9;$

$f(v_{n+1}^{4j-3}) = 2(n-1);$

$f(v_n^{4j-2}) = 1;$

$f(v_n^{4j-1}) = 4;$

$f(v_n^{4j}) = 7;$

}

end.

Theorem 5.3: The Multistage of Planar graph Pl_n^M with maximal edges has distance two labeling and its span $\lambda(Pl_n^M) = 2n+7$.

Proof: Consider the Multistage of Planar graph Pl_n^M with the vertex and edge set defined as in algorithm 5.2. Let the function f: $V(Pl_n^M) \to \{0,1,2,\ldots, 2n+7\}$ be defined as in algorithm 5.2. Now we have to prove this theorem using two cases.

Case (i): If $d(v_i, v_j)=1$ then to prove $|f(v_i)-f(v_j)| \geq 2$.

$d(v_i,v_j)$=1occur or the edges $\{v_i^j v_{n-1}^j, v_i^j v_n^j : 1 \leq i \leq n-2; 1 \leq j \leq m\}$;

$\{v_i^j v_{i+1}^j : 1 \leq i \leq n-3; 1 \leq j \leq m\}$; $\{v_i^j v_{i-1}^j : 2 \leq i \leq n-3; 1 \leq j \leq m\}$; $\{v_{n-1}^j v_n^j : 1 \leq k \leq m\}$

$\{v_i^j v_i^{j+1} : 1 \leq i \leq n; 1 \leq j \leq m-1\}$ and $\{v_i^j v_i^{j-1} : 1 \leq i \leq n; 2 \leq j \leq m\}$

By algorithm 5.2.,

We get $|f(v_i^j)-f(v_{n-1}^j)| \geq 2$ and $|f(v_i^j)-f(v_n^j)| \geq 2$ for $1 \leq i, \leq n-2$ and $1 \leq j \leq m$.

$|f(v_i^j)-f(v_{i+1}^j)| = 2$ for $1 \leq i \leq n-3$ and $1 \leq j \leq m$; $|f(v_i^j)-f(v_{i-1}^j)| = 2$ for $2 \leq i \leq n-2$ and $1 \leq j \leq m$.

$|f(v_{n-1}^j)-f(v_n^j)| = 2$ for $1 \leq j \leq m$.

$|f(v_i^j)-f(v_i^{j+1})| \geq 2$ for $1 \leq i, \leq n$ and $1 \leq j \leq m-1$.

$|f(v_i^j)-f(v_i^{j-1})| \geq 2$ for $1\leq i,\leq n$ and $2\leq j \leq m$.
Thus for all i,j, we have $|f(v_i)-f(v_j)| \geq 2$.
Case(ii): If $d(v_i, v_j)=2$ then to prove $|f(v_i)-f(v_j)| \geq 1$.
$d(v_i, v_j)=2$ occur for the edges $\{v_i^j v_{i+2}^j : 1 \leq i \leq n-4; 1 \leq j \leq m\}; \{v_i^j v_{i-2}^j : 3 \leq i \leq n-4; 1 \leq j \leq m\}$;
$\{v_i^j v_{i+1}^{j+1} : 1 \leq i \leq n-3; 1 \leq j \leq m-1\}; \{v_i^j v_{i-1}^{j+1} : 2 \leq i \leq n-3; 1 \leq j \leq m-1\}$
$\{v_i^j v_{i+1}^{j-1} : 1 \leq i \leq n-3; 2 \leq j \leq m\}; \{v_i^j v_{i-1}^{j-1} : 2 \leq i \leq n-3; 2 \leq j \leq m\}$
$\{v_{n-1}^j v_n^{j+1}; v_n^j v_{n-1}^{j+1} : 1 \leq j \leq m-1\}; \{v_{n-1}^j v_n^{j-1}; v_n^j v_{n-1}^{j-1} : 2 \leq j \leq m\}$ and
$\{v_i^j v_i^{j+2} : 1 \leq i \leq n; 1 \leq j \leq m-2\}; \{v_i^j v_i^{j-2} : 1 \leq i \leq n; 3 \leq j \leq m\}$.
all the vertices at difference two have distinct labeling. Thus in this case we have $|f(v_i)-f(v_j)| \geq 1$ for all i, j. By algorithm 5.2., the vertices v_{n-1}^{4j} has the maximum label i.e., $f(v_{n-1}^{4j}) = 2n+7$.
Hence the Multistage of Planar graph Pl_n^M has distance two labeling and $\lambda(Pl_n^M) = 2n+7$.

6. UPPER BOUND FOR SPAN OF MULTI-STOREY SIMPLE GRAPHS

A Simple graph said to be complete if every pair of vertices is joined by an edge. The Maximum degree for Multi-storey of Complete Graph is $\Delta_M = n+1$. By the section 2, Maximum degree for Multi-storey of simple graphs is less than $n+1$, So the upper bound for span of any Multi-storey simple graph is equal to the span value of Multi-storey of Complete graph.

In the following algorithm, we give the construction and distance two labeling for Multi-storey Complete graph K_n^M. In this graph, the Complete graph K_n is placed in M stores and the n vertices of K_n linked to the corresponding n vertices of the planar K_n's in succeeding layers. The distance two labeling of Pl_n^M is discussed in two cases. In the first case, distance two labeling for Multi-storey Complete graph K_n^M with $M < 4$ is given. In the second case, distance two labeling for Multi-storey Complete graph K_n^M with $M \geq 4$ is given in four sub cases $M \equiv 1 (\mathrm{mod}\, 4)$, $M \equiv 2 (\mathrm{mod}\, 4)$, $M \equiv 3 (\mathrm{mod}\, 4)$ and $M \equiv 0 (\mathrm{mod}\, 4)$.

Algorithm 6.1.

Input: Number of vertices mn of K_n^M
Output: Distance two labeling of vertices of the graph K_n^M
begin

$$V(K_n^M) = \left\{ \bigcup_{j=1}^{m} V(K_n^j) : V(K_n^j) = \{v_1^j, v_2^j, \ldots, v_n^j\} \right\}$$

$$E(K_n^M) = \bigcup_{j=1}^{m} E(K_n^j) \bigcup E' \text{ where } E(K_n^j) = \{v_i^j v_k^j : 1 \leq i, k \leq n \text{ and } i \neq k\}$$

$$E' = \{v_i^k v_i^{k+1} : 1 \leq i \leq n; 1 \leq k \leq m-1\}$$

if $(m < 4)$
for $j = 1$ to m
  for $i = 1$ to n
    {
   $f(v_i^j) = (i-1) + 3(j-1);$
    }

if $(m \geq 4)$ and $(m \equiv 1(\text{mod} 4))$
for $j = 1$ to $\frac{m-1}{4}$
  for $i = 1$ to n
    {
    $f(v_i^{4j-3}) = f(v_i^m) = 2(i-1);$
    $f(v_i^{4j-2}) = 2(i-1)+3;$
    $f(v_i^{4j-1}) = 2(i-1)+6;$
    $f(v_i^{4j}) = 2(i-1)+9;$
    }
else if$(m \geq 4)$ and $(m \equiv 2(\text{mod} 4))$
for $j = 1$ to $\frac{m-2}{4}$
  for $i = 1$ to n
    {
    $f(v_i^{4j-3}) = f(v_i^{m-1}) = 2(i-1);$
    $f(v_i^{4j-2}) = f(v_i^m) = 2(i-1)+3;$
    $f(v_i^{4j-1}) = 2(i-1)+6;$
    $f(v_i^{4j}) = 2(i-1)+9;$
    }
else if$(m \geq 4)$ and $(m \equiv 3(\text{mod} 4))$
for $j = 1$ to $\frac{m-3}{4}$
  for $i = 1$ to n
    {
    $f(v_i^{4j-3}) = f(v_i^{m-2}) = 2(i-1);$
    $f(v_i^{4j-2}) = f(v_i^{m-1}) = 2(i-1)+3;$
    $f(v_i^{4j-1}) = f(v_i^m) = 2(i-1)+6;$
    $f(v_i^{4j}) = 2(i-1)+9;$
    }
elseif $(m \geq 4)$ and $(m \equiv 0(\text{mod} 4))$
for $j = 1$ to $\frac{m}{4}$
  for $i = 1$ to n
    {
    $f(v_i^{4j-3}) = 2(i-1);$
    $f(v_i^{4j-2}) = 2(i-1)+3;$
    $f(v_i^{4j-1}) = 2(i-1)+6;$
    $f(v_i^{4j}) = 2(i-1)+9;$
    }
end.

Theorem 6.2: The Multi-storey of Complete graph K_n^M has distance two labeling and its span $\lambda(K_n^M) = 2\Delta_M + 5$.

Proof: Consider the Multi-storey of Complete graph K_n^M with the vertex and edge set defined as in algorithm 6.1. Let the function f: $V(K_n^M) \rightarrow \{0,1,2,\ldots, 2\Delta_M + 5\}$ be defined as in algorithm 6.1. Now we have to prove this theorem using two cases.

Case (i): If $d(v_i, v_j)=1$ then to prove $|f(v_i)-f(v_j)| \geq 2$.

$d(v_i, v_j)=1$occur for the edges $\{v_i^j v_k^j : 1 \leq i, k \leq n \text{ and } i \neq k; 1 \leq j \leq m\}$ and for the edges $\{v_i^j v_i^{j+1} : 1 \leq i \leq n; 1 \leq j \leq m-1\}$.

By algorithm 6.1., in all cases,

We get $|f(v_i^j)-f(v_k^j)| \geq 2$ for $1 \leq i, k \leq n$; $i \neq k$ and $1 \leq j \leq m$.

$|f(v_i^j)-f(vi^{j+1})| = 3$ for $1 \leq i \leq n$ and $1 \leq j \leq m-1$.

Thus for all i,j, we have $|f(v_i)-f(v_j)| \geq 2$.

Case(ii): If $d(v_i, v_j)=2$ then to prove $|f(v_i)-f(v_j)| \geq 1$.

$d(v_i, v_j)=2$ occur for the edges $\{v_i^j v_k^{j+1} : 1 \leq i, k \leq n \text{ and } i \neq k; 1 \leq j \leq m-1\}$; $\{v_i^j v_i^{j+2} : 1 \leq i \leq n; 1 \leq j \leq m-2\}$ and $\{v_i^j v_i^{j-2} : 1 \leq i \leq n; 2 \leq j \leq m\}$.

all the vertices at difference two have distinct labelling. Thus in this case we have $|f(v_i)-f(v_j)| \geq 1$ for all i, j. By algorithm 6.1., the vertices v_n^m has the maximum label i.e., $f(v_n^m) = 2\Delta_M + 5$.

Hence the Multi-storey of Complete graph K_n^M has distance two labeling and $\lambda(K_n^M) = 2\Delta_M + 5$.

Corollary 6.3: Upper bound span value $\lambda(G^M)$ for Multi-storey of any simple graph is $2\Delta_M + 5$.

Proof: To prove the upper bound, ingeneral we are consider the Complete graph K_n with n vertices. Let K_n^M be the Multi-storey of K_n.

From the theorem 6.2.,

When m $\equiv 1$ (mod4), The labeling numbers will be {0, 2, 4, …, 2(n-1)}.

When m $\equiv 2$ (mod4), The labeling numbers will be {3, 5, 7, …, 2(n-1) +3}.

When m $\equiv 3$ (mod4), The labeling numbers will be {6, 8, 10, …, 2(n-1) +6}.

When m $\equiv 0$ (mod4), The labeling numbers will be {9, 11, 13, …, 2(n-1) +9}. Then the Maximum labeling number used or span is 2(n-1) +9 = $2\Delta_M + 5$, where $\Delta_M = n+1$. Since the upper bound span value of any simple graph cannot be more than the Complete graph. Hence the upper bound span value $\lambda(G^M)$ for any simple graph is $2\Delta_M + 5$.

7. CONCLUSIONS

A Multi-storey graph is clearly a three dimensional graph. As more and more transmitters are introduced for communication networks, to avoid interference we can arrange the graphs in different layers to interpret a Multi-storey graph. This distance two labelling notion will be useful to reduce the interference by choosing the frequency in such type of networks in different layers.

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