# Constructing Minimum Connected Dominating Set: Algorithmic approach

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***Abstract:*** *Connected Dominating Set is popularly used for constructing virtual backbones for broadcasting operation in WSNs. UD Graph is the most suitable model for a wireless sensor network. In this paper we provide an algorithm to find MCDS in UD Graph. It is based on the computation of convex hulls of sensor nodes or vertices. Constructing a virtual backbone in WSNs is an important issue because it reduces unnecessary message transmission or flooding in the network. It helps in reducing interference and energy consumption because a limited number of sensors are engaged in message transmission and thus it helps in improving the Quality of Service (QoS) in the network.*

## 1. Introduction

Wireless sensor network has wide variety of applications such as battle field surveillance, target tracking, security, environmental control, habitat monitoring, source localization, fire detection, oil and gas pumping etc.

A wireless sensor network is composed of a set of battery powered sensors. These sensors can communicate with one another through wireless links if they are within their transmission range, otherwise they can communicate via other sensors between them.

Unit Disk Graph [1] is an important class of graphs. A wireless sensor network can be modeled as a UD Graph [2] since the transmission range of sensors is based on Euclidean distance. In this modeling sensors are denoted as vertices. The sensing coverage area of a sensor is represented by a unit disk centered at corresponding vertex. The connectivity between two sensor nodes is determined if the first sensor is within the sensing coverage range of the second sensor. Thus there is an edge between two vertices u and v iff $d(u,v)\leq 1$, where d(u,v) is the Euclidean distance between u and v. In this way UD Graph is most suitable model for a wireless sensor network.

Dominating set in a graph G = (V, E) is a subset D of V such that every node $u\in V$ is in D or adjacent to some node $v\in D$. A dominating set D is called Connected Dominating Set (CDS) if it is a induced connected subgraph of G. A Minimum Connected Dominating Set (MCDS) is a connected dominating set with smallest possible cardinality

DOI : 10.5121/jgraphoc.2010.2305

among all the CDSs of G. Connected Dominating Sets [4] are popularly used for constructing virtual backbones for broadcasting operation in WSNs. Establishing a virtual backbone in WSNs is an important issue because it reduces unnecessary message transmission or flooding in the network. It helps in reducing interference and energy consumption because a limited number of sensors are engaged in message transmission and thus it helps in improving the Quality of Service (QoS) in the network.

Virtual backbone is basically a subset of sensor nodes which can transmit and receive messages throughout the network. CDS is one of the earliest approaches to construct a virtual backbone in WSNs [4]. To find the CDS in UD Graph is a well studied problem. The MCDS problem was shown. The MCDS remains NP hard [1] for UD graph. A CDS is also useful for location based routing. In location based routing messages are forwarded based on the geographical co-ordinates of the hosts and topological connectivity.

In this paper we provide an algorithm to find MCDS in UD Graph. It is based on the computation of convex hulls of sensor nodes or vertices. This paper is organized as follows. In section 2, related work is given. Auxiliary definition and notation are given in section 3. Section 4 contains the main algorithm of the paper and its verification through an example. In the Section 5, discussion and conclusion is given. Last Section 6 contains references.

## 2. Related Work

Guha and Khuller [6] studied the MCDS problem and showed that this problem is NP hard in an arbitrary undirected graph. Later it was shown in [1] that computing an MCDS for a Unit Disk (UD) Graph is also a NP hard problem. Alzoutri et al. [3] proposed the first distributed algorithm guaranteeing a constant approximation factor for CDS construction based on MIS in UD Graph. A distributed algorithm to construct small sized connected dominating set for UD Graph was provided in [4]. It is based on the computation of convex hull of sensor nodes which are considered as vertices. An analysis of the size of MIS and the size of an MCDS also has been provided. It is also shown that this algorithm produces an optimal CDS if the graph is a tree. In the case of grid the approximation factor is 2. A technique is presented that produces a CDS with the size at most $38*|MCDS|$, where $|MCDS|$ is the size of a minimum CDS.

In [7] a distributed algorithm was proposed that also utilized MIS for CDS construction in UD Graphs. This technique requires large amount of message exchanges and transmission for the construction of spanning tree for constructing CDS. An algorithm is proposed to find MCDS using dominating set in UD Graphs in [2]. This algorithm is implemented in three phases. In first phase, dominating sets are found. In second phase, connectors are identified, connected through Steiner tree. In third phase, the CDS obtained a MCDS. Network needs to adapt to the continuous topological changes due to deactivation of a node due to exhaustion of battery. These changes are taken care by a local repair algorithm that reconstructs the MCDS. i.e. power aware MCDS using only neighborhood information.

## 3. Auxiliary Definition

In order to develop the algorithm, we state some definition and introduce some terminology relevant to the paper.

**3*(i)* Dominating Set** – Dominating Set for a graph G = (V,E) is a subset D of the Vertex Set V such that each vertex u∈ V is either in D or adjacent to some vertex v in D. The elements of dominating set are called dominators. Examples of dominating set in a graph G are given below:

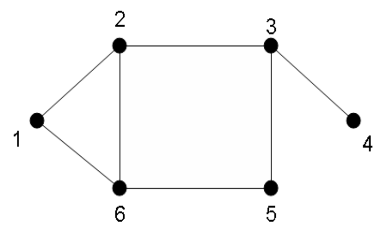



**Figure 1: {1, 3}, {2, 3, 5} and {1, 2, 3, 4} are Dominating Sets**

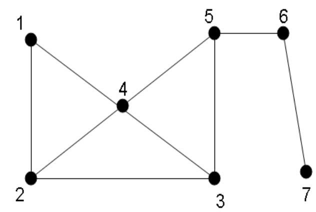



**Figure 2: {4, 6}, {1, 5, 7} and {4, 5, 6} are Dominating Sets**

**3*(ii)* Connected Dominating Set** – A Connected Dominating Set (CDS) of a graph G = (V,E) is a set of vertices with two properties:

1. D is a dominating set in G.
2. D induces a connected subgraph of G.

In Fig.1, {2, 3, 5} and {1, 2, 3, 4} are Connected Dominating Sets. Similarly in Fig. 2, {4, 5, 6} is a Connected Dominating Set.

**3*(iii)* Minimum Connected Dominating Set** – A minimum Connected Dominating Set (MCDS) is a connected dominating set with smallest possible cardinality among all the CDSs of G. As in Figs. 1 and 2, {2, 3, 5} and {4, 5, 6} are Minimum Connected Dominating Sets respectively.

**3*(iv)* Independent Set** – Independent Set of a graph G is a subset of the set of vertices such that no two vertices are adjacent in the subset. For example in Fig.1 {1, 3}, {1, 4, 5}, {2, 4, 5} are independent sets.

**3*(v)* Maximal Independent Set** – Maximal independent set (MIS) is an independent set, which is not a subset of any other independent set. i.e. it is a set S such that every edge of the graph has atleast one end point not in S and every vertex not in S has atleast one neighbor in S. An MIS is also a dominating set. Six different Maximal Independent Set of following cubic graph are {1,6}, {2,5}, {3, 8},{4, 7}, {1, 5, 7} and {4, 6, 8, 2}.

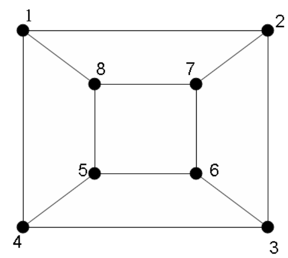



**Figure 3: MIS in Cubic Graph**

**3*(vi)* Convex hull** – the convex hull for a set of points X in real vector space is the minimum convex set containing X. it is also called convex envelop and denoted by CH(X). It is represented by a sequence of the vertices of the line segment forming the boundary of the convex polygon. As in the following example convex hull of the set {1, 2, 3, 4, 5, 6, 7} of points is shown.

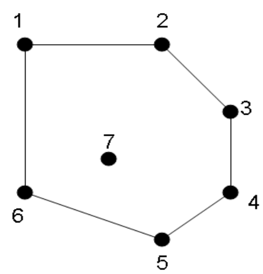



**Figure 4: CH ({1, 2, 3, 4, 5, 6, 7})**

**3*(vii)* Unit Disk Graph** – A graph G is a Unit Disk graph if there is an assignment of unit disks centered at its vertices such two vertices are adjacent if and only if one vertex is within the unit disk centered at the other vertex.

**3*(viii)* Neighborhood of a vertex** – Neighborhood of a vertex u in a graph G = (V,E) is a set of vertices which are adjacent to u in G. It is denoted by N(u) or N[u]. If neighborhood does not include u itself, then it is called *open neighborhood* of u and denoted by N(u). As in figure 1, N(1) is {2, 6}, N(2) is {1, 3, 6}, N(3) is {2, 4, 5} and so on. If neighborhood includes u itself, then it is called *closed neighborhood* of u and denoted by N[u]. For example in Fig. 2, N[1] is {1, 2, 4} and N[2] is {1, 2, 3, 4}and so on.

## 4. Algorithm

Now we describe an algorithm to find minimum connected dominating set from a connected dominating set. This CDS is found by algorithm described in [4]. We have to do following steps:

Step 1: Select a minimum degree vertex u from the CDS.
Step 2: Calculate CH(N[u]).
Step 3: Calculate CH(N[i]) $\forall$ i $\in$ N(u).
Step 4: Check if CH (N[u]) is contained in $\bigcup_i CH(N[i])$ where i$\in$ N(u).
Step 5: If step 2 returns true then remove vertex u and go to step 1.
Step 6: Otherwise do not remove vertex u and go to step 1.
Step 7: Algorithm terminates when all the nodes in C are considered and the node remains in C construct the MCDS.

The above algorithm can be understood with the help of following examples:

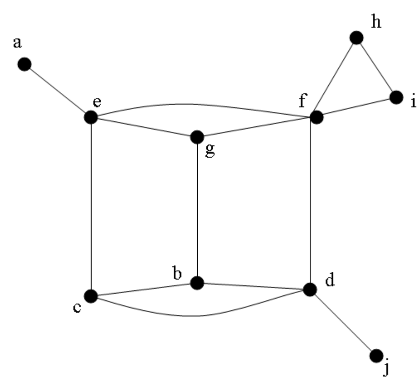



**Figure 5: G = (V, E) and CDS = {c, e, d, f, g}**

Connected Dominating Set (CDS) in the above graph found by the algorithm described in [4] is {c, d, e, f, g} say it C. Now we apply our given algorithm to find Minimum Connected Dominating set (MCDS).

Step1: Select the minimum degree vertex in C i.e. c.

Step2: Calculate CH(N[c]) i.e. convex hull ecd.

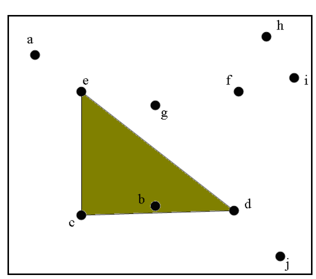
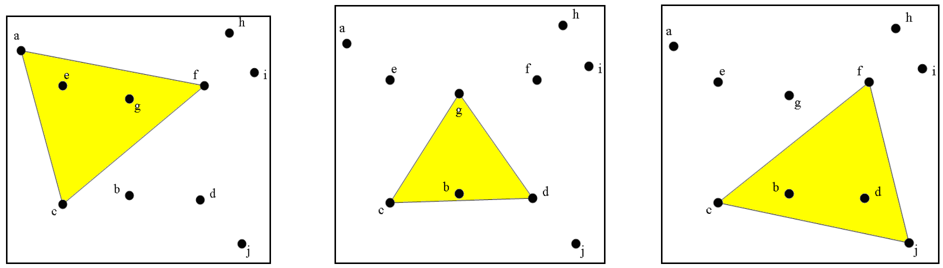



**Figure 6: CH(N[c]) = ecd**

Step3: Calculate CH(N[i]) $\forall$ i$\in$ N(c) = {e, b, d}.




**CH(N[e]) = acf** **CH(N[b]) = gcd** **CH(N[d]) = cfj**

**Figure 7: CH(N[i])**

Step 4: CH (N[c]) is contained in $\bigcup_i CH(N[i])$ where i∈ N(c) = {e, b, d}.

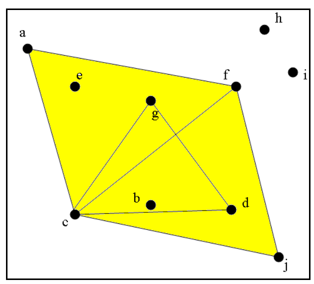



**Figure 8:** $\bigcup_i CH(N[i])$

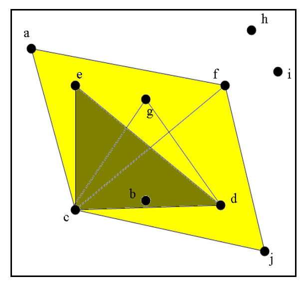



**Figure 9:** $CH(N[c]) \subseteq \bigcup_i CH(N[i])$

Step 5: Step 4 returns true. Therefore we remove the vertex c from CDS and go to Step1.

Step 6: Select the next minimum degree vertex i.e. g and proceed as previously and we find that g will also remove by above process and go to step 1.

Step 7: Select the next minimum degree vertex i.e. d.

Step 8: Calculate CH(N[d]) i.e. convex hull cfj.

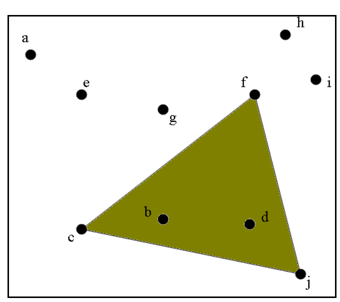
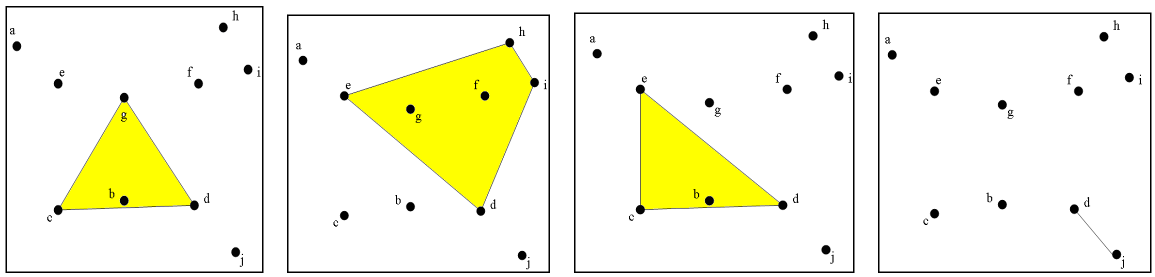



**Figure 14: CH(N[d]) = cfj**

Step 9: Calculate CH(N[i]) $\forall i \in$ N(d) = {b, f, c, j}.




**CH(N[b]) = gcd** **CH(N[f]) = ehid** **CH(N[c]) = ecd** **CH(N[j]) =dj**

**Figure 15: CH(N[i])**

Step 10: CH (N[d]) is not contained in $\bigcup_i CH(N[i])$ where $i \in$ N(d) = {b, f, c, j}.

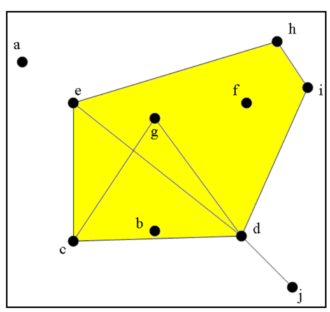



**Figure 16:** $\bigcup_i CH(N[i])$

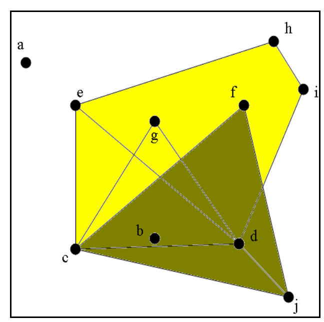



**Figure 17:** $CH(N[d]) \not\subset \bigcup_i CH(N[i])$

Step 11: Step 10 returns false. Therefore we do not remove the vertex d from CDS and go to Step1.

Step 12: Select the next minimum degree vertices e and f, and perform the steps 6 to 11. Likewise d these two vertices are not deleted from CDS.

Finally CDS left with the vertices e, d and f.

Thus we get the MCDS = {e, d, f}.

## 5. Conclusion

UD graph is the most suitable model for WSNs. Virtual backbone in a WSNs is a subset of sensor nodes which can transmit and receive the messages through out the network.

Construction of a CDS in UD graph is a common way to generate virtual backbone in WSNs. We introduce an algorithm to find Minimum Connected Dominating Set (MCDS) for UD Graph based on computation of convex hull of sensor nodes. This is an important issue due to many causes such as it precludes unnecessary message transmission of flooding in the network. It reduces interference between nodes and energy consumption because only the nodes of CDS are engage in message transmission. These all above reasons help to improve Quality of Service in the network. We also calculated the complexity of our algorithm. It turns out to be $O(n^2 logn)$. Though this complexity is large, however it is better than existing algorithms which are NP hard ([1] and [6]). So our algorithm appears to be superior as compare to available algorithms.

## 6. References

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