On Fuzzy Soft Multi Set and Its Application in Information Systems

Anjan Mukherjee[1] and Ajoy Kanti Das[2]

[1]Department of Mathematics,Tripura University, Agartala-799022,Tripura, India
[2] Department of Mathematics, ICV-College, Belonia -799155, Tripura, India

ABSTRACT

Research on information and communication technologies have been developed rapidly since it can be applied easily to several areas like computer science, medical science, economics, environments, engineering, among other. Applications of soft set theory, especially in information systems have been found paramount importance. Recently, Mukherjee and Das defined some new operations in fuzzy soft multi set theory and show that the De-Morgan's type of results hold in fuzzy soft multi set theory with respect to these newly defined operations. In this paper, we extend their work and study some more basic properties of their defined operations. Also, we define some basic supporting tools in information system also application of fuzzy soft multi sets in information system are presented and discussed. Here we define the notion of fuzzy multi-valued information system in fuzzy soft multi set theory and show that every fuzzy soft multi set is a fuzzy multi valued information system.

KEYWORDS

Soft set, fuzzy set, soft multi set, fuzzy soft multi set, information system.

1. INTRODUCTION

In recent years vague concepts have been used in different areas such as information and communication technologies, medical applications, pharmacology, economics and engineering since; these kinds of problems have their own uncertainties. There are many mathematical tools for dealing with uncertainties; some of them are fuzzy set theory [18] and soft set theory [12]. In soft set theory there is no limited condition to the description of objects; so researchers can choose the form of parameters they need, which greatly simplifies the decision making process and make the process more efficient in the absence of partial information. Although many mathematical tools are available for modelling uncertainties such as probability theory, fuzzy set theory, rough set theory, interval valued mathematics etc, but there are inherent difficulties associated with each of these techniques. Soft set theory is standing in a unique way in the sense that it is free from the above difficulties. Soft set theory has a rich potential for application in many directions, some of which are reported by Molodtsov [12] in his work. Later on Maji et al.[10, 11] presented some new definitions on soft sets and discussed in details the application of soft set in decision making problem. Based on the analysis of several operations on soft sets introduced in [12], Ali et al. [2] presented some new algebraic operations for soft sets and proved that certain De Morgan's law holds in soft set theory with respect to these new definitions. Combining soft sets [12] with fuzzy sets [18], Maji et al. [9] defined fuzzy soft sets, which are rich potential for solving decision making problems. Alkhazaleh and others [[1], [4], [5], [7], [16], [17]] as a generalization of Molodtsov's soft set, presented the definition of a soft multi set

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and its basic operations such as complement, union, and intersection etc. In 2012 Alkhazaleh and Salleh [3] introduced the concept of fuzzy soft multi set theory and studied the application of these sets and recently, Mukherjee and Das[14] defined some new algebraic operations for fuzzy soft multi sets and proved that certain De Morgan's law holds in fuzzy soft multi set theory with respect to these new definitions. They also presented an application of fuzzy soft multi set based decision making problems in [13].

Molodtsov [12] presented some applications of soft set theory in several directions, which includes: the study of smoothness of functions, game theory, operations research, Riemann-integration, probability, theory of measurement, etc. It has been shown that there is compact connection between soft sets and information system. From the concept and the example of fuzzy soft multisets given in the section 4.3, it can be seen that a fuzzy soft multi set is a multi-valued information system. In this paper we define the notion of fuzzy multi-valued information system in fuzzy soft multi set theory and application of fuzzy soft multi sets in information system were presented and discussed.

2. PRELIMINARY NOTES

In this section, we recall some basic notions in soft set theory and fuzzy soft multi set theory. Let U be an initial universe and E be a set of parameters. Let P(U) denotes the power set of U and A⊆E.

A pair (F, A) is called a soft set over U, where F is a mapping given by F: A→ P(U). In other words, soft set over U is a parameterized family of subsets of the universe U.

2.1. Definition [12]

Let $\{U_i : i\in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \phi$ and let $\{E_{U_i} : i\in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} FS(U_i)$ where $FS(U_i)$ denotes the set of all fuzzy subsets of U_i, $E = \prod_{i\in I} E_{U_i}$ and $A \subseteq E$. A pair (F, A) is called a fuzzy soft multi set over U, where $F: A \to U$ is a mapping given by $\forall e\in A$, $F(e) = \left(\left\{ \frac{u}{\mu_{F(e)}(u)} : u\in U_i \right\} : i\in I \right)$

For illustration, we consider the following example.

2.2. Example

Let us consider three universes $U_1 = \{h_1, h_2, h_3, h_4, h_5\}$, $U_2 = \{c_1, c_2, c_3, c_4\}$ and $U_3 = \{v_1, v_2, v_3\}$ be the sets of "houses," "cars," and "hotels", respectively. Suppose Mr. X has a budget to buy a house, a car and rent a venue to hold a wedding celebration. Let us consider a intuitionistic fuzzy soft multi set (*F, A*) which describes "houses," "cars," and "hotels" that Mr. X is considering for accommodation purchase, transportation purchase, and a venue to hold a wedding celebration, respectively. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \{e_{U_1,1} = \text{expensive},\ e_{U_1,2} = \text{cheap},\ e_{U_1,3} = \text{wooden}\}$$

$$E_{U_2} = \{e_{U_2,1} = \text{beautiful}, e_{U_2,2} = \text{cheap}, e_{U_2,3} = \text{sporty}\},$$
$$E_{U_3} = \{e_{U_3,1} = \text{expensive}, e_{U_3,2} = \text{model}, e_{U_3,3} = \text{beautiful}\}.$$

Let $U = \prod_{i=1}^{3} FS(U_i)$, $E = \prod_{i=1}^{3} E_{U_i}$ and $A \subseteq E$, such that

$$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), a_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}), a_4 = (e_{U_1,3}, e_{U_2,3}, e_{U_3,1}), a_5 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,2}), a_6 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,2})\}.$$

Suppose Mr. X wants to choose objects from the sets of given objects with respect to the sets of choice parameters. Then fuzzy soft multi set (*F, A*) can be represent as in Table 1.

Table 1. The tabular representation of the fuzzy soft multi-set (*F, A*)

U_i		a_1	a_2	a_3	a_4	a_5	a_6
	h_1	0.8	0.6	0.5	0.4	0.7	0.6
	h_2	0.4	0.5	0.7	0.5	0.5	0.4
U_1	h_3	0.9	0	1	0.1	1	0.8
	h_4	0.4	0.4	0.4	0	0.4	1
	h_5	0.1	1	0.8	0.8	1	0.1
	c_1	0.8	0.9	0.6	0.8	0	0.7
U_2	c_2	1	0.6	0.1	0	0.1	0.1
	c_3	0.8	1	0	0.8	0.7	0.9
	c_4	1	0	1	1	1	0.1
	v_1	0.8	0.6	0	1	1	0.1
U_3	v_2	0.7	0.6	0.7	0.7	0.8	0.9
	v_3	0.6	0	0.7	0	0	0.1

2.4. Definition [14]

The restricted union of two fuzzy soft multi sets (F, A) and (G, B) over U is a fuzzy soft multi set (H,C)where $C = A \cap B$ and $\forall e \in C$,

$$H(e) = \cup(F(e), G(e)) = \left(\left\{\frac{u}{\max\{\mu_{F(e)}(u), \mu_{G(e)}(u)\}} : u \in U_i\right\} : i \in I\right)$$

and is written as $(F, A)\tilde{\cup}_R (G, B) = (H, C)$.

2.5 Definition [14]

The extended union of two fuzzy soft multi sets (*F, A*) and (*G, B*) over U is a fuzzy soft multi set (*H, D*), where $D = A \cup B$ and $\forall e \in D$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ \cup(F(e), G(e)), & \text{if } e \in A \cap B, \end{cases}$$

where $\cup(F(e), G(e)) = \left(\left\{\frac{u}{\max\{\mu_{F(e)}(u), \mu_{G(e)}(u)\}} : u \in U_i\right\} : i \in I\right)$

and is written as $(F,\ A)\tilde{\cup}_E (G,\ B)=(H,D).$

2.6 Definition [14]

The restricted intersection of two intuitionistic fuzzy soft multi sets (*F, A*) and (*G, B*) over U is a fuzzy soft multi set (*H, D*) where $D=A\cap B$ and $\forall e\in D$,

$$H(e)=\bigcap\big(F(e),G(e)\big)\ =\left(\left\{\frac{u}{\min\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

and is written as $(F,\ A)\tilde{\cap}_R (G,\ B)=(H,D).$

2.7 Definition [14]

The extended intersection of two fuzzy soft multi sets (*F, A*) and (*G, B*) over U is a fuzzy soft multi set (*H, D*), where $D=A\cup B$ and $\forall e\in D$,

$$H(e)=\begin{cases}F(e), & if\ \ e\in A-B\\ G(e), & if\ \ e\in B-A\\ \bigcap\big(F(e),G(e)\big), & if\ \ e\in A\cap B\end{cases}$$

Where

$$\bigcap\big(F(e),G(e)\big)\ =\left(\left\{\frac{u}{\min\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

and is written as $(F,\ A)\tilde{\cap}_E (G,\ B)=(H,D).$

2.8. Definition [14]

If (F, A) and (G, B) be two fuzzy soft multi sets over U, then "(F,A) AND (G,B)" is a fuzzy soft multi set denoted by $(F,A)\wedge(G,B)$ and is defined by $(F,A)\wedge(G,B)=(H,A\times B)$, where H is mapping given by H: A×B→U and

$$\forall(a,b)\in A\times B,\ H(a,b)=\left(\left\{\frac{u}{\min\left\{\mu_{F(a)}(u),\mu_{G(b)}(u)\right\}}:u\in U_i\right\}:i\in I\right).$$

2.9. Definition [14]

If (*F, A*) and (*G, B*) be two fuzzy soft multi sets over U, then "(F,A) OR (G,B)" is a fuzzy soft multi set denoted by $(F,A)\vee(G,B)$ and is defined by $(F,A)\vee(G,B)=(K,A\times B)$, where K is mapping given by K: A×B→U and

$$\forall(a,b)\in A\times B,\ K(a,b)=\left(\left\{\frac{u}{\max\left\{\mu_{F(a)}(u),\mu_{G(b)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

2.10. Definition [8]

An information system is a 4-tuple $S = (U, A, V, f)$, where $U = \{u_1, u_2, u_3, \ldots, u_n\}$ is a non-empty finite set of objects, $A = \{a_1, a_2, a_3, \ldots, a_m\}$ is a non-empty finite set of attributes, $V = \cup_{a\in A} V_a$, V_a is the domain of attribute a, $f : U \times A \rightarrow V$ is an information function, such that $f(u, a) \in V_a$ for every $(u, a) \in U \times A$, called information(knowledge) function.

3. MAIN RESULTS

Mukherjee and Das [14] defined some new operations in fuzzy soft multi set theory and show that the De Morgan's types of results hold in fuzzy soft multi set theory with respect to these newly defined operations. In this section, we extend their work and study some more basic properties of their defined operations.

3.1. Proposition (Associative Laws)

Let (F, A), (G, B) and (H, C) are three fuzzy soft multi sets over U, then we have the following properties:

1. $(F,A)\tilde{\cap}_R\left((G,B)\tilde{\cap}_R(H,C)\right)=\left((F,A)\tilde{\cap}_R(G,B)\right)\tilde{\cap}_R(H,C)$
2. $(F,A)\tilde{\cup}_R\left((G,B)\tilde{\cup}_R(H,C)\right)=\left((F,A)\tilde{\cup}_R(G,B)\right)\tilde{\cup}_R(H,C)$

Proof. 1. Assume that $(G,B)\tilde{\cap}_R(H,C)=(I,D)$, where $D=B\cap C$ and $\forall e\in D$,

$$I(e)=G(e)\cap H(e)=\left(\left\{\frac{u}{\min\left\{\mu_{G(e)}(u),\mu_{H(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Since $(F,A)\tilde{\cap}_R\left((G,B)\tilde{\cap}_R(H,C)\right)=(F,A)\tilde{\cap}_R(I,D)$, we suppose that $(F,A)\tilde{\cap}_R(I,D)=(K,M)$ where $M=A\cap D=A\cap B\cap C$ and $\forall e\in M$,

$$K(e)=F(e)\cap I(e)=\left(\left\{\frac{u}{\mu_{K(e)}(u)}:u\in U_i\right\}:i\in I\right)$$

where

$$\mu_{K(e)}(u)=\min\left\{\mu_{F(e)}(u),\mu_{I(e)}(u)\right\}=\min\left\{\mu_{F(e)}(u),\min\left\{\mu_{G(e)}(u),\mu_{H(e)}(u)\right\}\right\}=\min\left\{\mu_{F(e)}(u),\mu_{G(e)}(u),\mu_{H(e)}(u)\right\}$$

Suppose that $(F,A)\tilde{\cap}_R(G,B)=(J,N)$, where $N=A\cap B$ and $\forall e\in N$,

$$J(e)=F(e)\cap G(e)=\left(\left\{\frac{u}{\min\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Since $\left((F,A)\tilde{\cap}_R(G,B)\right)\tilde{\cap}_R(H,C)=(J,N)\tilde{\cap}_R(H,C)$, we suppose that $(J,N)\tilde{\cap}_R(H,C)=(O,N\cap C)$ where

$$\forall e \in N \cap C = A \cap B \cap C,\ O(e) = J(e) \cap H(e) = \left(\left\{ \frac{u}{\mu_{O(e)}(u)} : u \in U_i \right\} : i \in I \right)$$

$$\mu_{O(e)}(u) = \min\left\{\mu_{J(e)}(u), \mu_{H(e)}(u)\right\} = \min\left\{\min\left\{\mu_{F(e)}(u), \mu_{G(e)}(u)\right\}, \mu_{H(e)}(u)\right\}$$
$$= \min\left\{\mu_{F(e)}(u), \mu_{G(e)}(u), \mu_{H(e)}(u)\right\} = \mu_{K(e)}(u)$$

Consequently, K and O are the same operators. Thus

$$(F,A)\tilde{\cap}_R\left((G,B)\tilde{\cap}_R(H,C)\right) = \left((F,A)\tilde{\cap}_R(G,B)\right)\tilde{\cap}_R(H,C).$$

2. Assume that $(G,B)\tilde{\cup}_R(H,C) = (I,D)$, where $D = B \cap C$ and $\forall e \in D$,

$$I(e) = G(e) \cup H(e) = \left(\left\{ \frac{u}{\max\left\{\mu_{G(e)}(u), \mu_{H(e)}(u)\right\}} : u \in U_i \right\} : i \in I \right)$$

Since $(F,A)\tilde{\cup}_R\left((G,B)\tilde{\cup}_R(H,C)\right) = (F,A)\tilde{\cup}_R(I,D)$, we suppose that $(F,A)\tilde{\cup}_R(I,D) = (K,M)$ where $M = A \cap D = A \cap B \cap C$ and $\forall e \in M$,

$$K(e) = F(e) \cup I(e) = \left(\left\{ \frac{u}{\mu_{K(e)}(u)} : u \in U_i \right\} : i \in I \right)$$

where

$$\mu_{K(e)}(u) = \max\left\{\mu_{F(e)}(u), \mu_{I(e)}(u)\right\} = \max\left\{\mu_{F(e)}(u), \max\left\{\mu_{G(e)}(u), \mu_{H(e)}(u)\right\}\right\} = \max\left\{\mu_{F(e)}(u), \mu_{G(e)}(u), \mu_{H(e)}(u)\right\}$$

Suppose that $(F,A)\tilde{\cup}_R(G,B) = (J,N)$, where $N = A \cap B$ and $\forall e \in N$,

$$J(e) = F(e) \cup G(e) = \left(\left\{ \frac{u}{\max\left\{\mu_{F(e)}(u), \mu_{G(e)}(u)\right\}} : u \in U_i \right\} : i \in I \right)$$

Since $\left((F,A)\tilde{\cup}_R(G,B)\right)\tilde{\cup}_R(H,C) = (J,N)\tilde{\cup}_R(H,C)$, we suppose that $(J,N)\tilde{\cup}_R(H,C) = (O, N \cap C)$

where $\forall e \in N \cap C = A \cap B \cap C,\ O(e) = J(e) \cup H(e) = \left(\left\{ \frac{u}{\mu_{O(e)}(u)} : u \in U_i \right\} : i \in I \right)$

$$\mu_{O(e)}(u) = \max\left\{\mu_{J(e)}(u), \mu_{H(e)}(u)\right\} = \max\left\{\max\left\{\mu_{F(e)}(u), \mu_{G(e)}(u)\right\}, \mu_{H(e)}(u)\right\}$$
$$= \max\left\{\mu_{F(e)}(u), \mu_{G(e)}(u), \mu_{H(e)}(u)\right\} = \mu_{K(e)}(u)$$

Consequently, K and O are the same operators. Thus

$$(F,A)\tilde{\cup}_R\left((G,B)\tilde{\cup}_R(H,C)\right) = \left((F,A)\tilde{\cup}_R(G,B)\right)\tilde{\cup}_R(H,C).$$

3.2. Proposition (Distributive Laws)

Let (F, A), (G, B) and (H, C) are three fuzzy soft multi sets over U, then we have the following properties:

(1). $(F,A)\tilde{\cup}_R\left((G,B)\tilde{\cap}_R(H,C)\right)=\left((F,A)\tilde{\cup}_R(G,B)\right)\tilde{\cap}_R\left((F,A)\tilde{\cup}_R(H,C)\right)$

(2). $(F,A)\tilde{\cap}_R\left((G,B)\tilde{\cup}_R(H,C)\right)=\left((F,A)\tilde{\cap}_R(G,B)\right)\tilde{\cup}_R\left((F,A)\tilde{\cap}_R(H,C)\right)$

Proof. (1). Assume that $(G,B)\tilde{\cap}_R(H,C)=(I,D)$, where $D=B\cap C$ and $\forall e\in D$,

$$I(e)=G(e)\cap H(e)=\left(\left\{\frac{u}{\min\left\{\mu_{G(e)}(u),\mu_{H(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Since $(F,A)\tilde{\cup}_R\left((G,B)\tilde{\cap}_R(H,C)\right)=(F,A)\tilde{\cup}_R(I,D)$, we suppose that $(F,A)\tilde{\cup}_R(I,D)=(K,M)$ where

$M=A\cap D=A\cap B\cap C$ and $\forall e\in M$,

$$K(e)=F(e)\cup I(e)=\left(\left\{\frac{u}{\mu_{K(e)}(u)}:u\in U_i\right\}:i\in I\right)$$

$$\begin{aligned}&where\ \mu_{K(e)}(u)=\max\left\{\mu_{F(e)}(u),\mu_{I(e)}(u)\right\}\\&=\max\left\{\mu_{F(e)}(u),\min\left\{\mu_{G(e)}(u),\mu_{H(e)}(u)\right\}\right\}\\&=\min\left\{\max\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\},\max\left\{\mu_{F(e)}(u),\mu_{H(e)}(u)\right\}\right\}\end{aligned}$$

Suppose that $(F,A)\tilde{\cup}_R(G,B)=(J,N)$, where $N=A\cap B$ and $\forall e\in N$,

$$J(e)=F(e)\cup G(e)=\left(\left\{\frac{u}{\max\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Again, let $(F,A)\tilde{\cup}_R(H,C)=(S,T)$, where $T=A\cap C$ and $\forall e\in T$,

$$S(e)=F(e)\cup H(e)=\left(\left\{\frac{u}{\max\left\{\mu_{F(e)}(u),\mu_{H(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Since $\left((F,A)\tilde{\cup}_R(G,B)\right)\tilde{\cap}_R\left((F,A)\tilde{\cup}_R(H,C)\right)=(J,N)\tilde{\cap}_R(S,T)$, we suppose that

$(J,N)\tilde{\cap}_R(S,T)=(O,N\cap T)$ where $\forall e\in N\cap T=A\cap B\cap C$,

$$O(e)=J(e)\cap S(e)=\left(\left\{\frac{u}{\mu_{O(e)}(u)}:u\in U_i\right\}:i\in I\right)$$

$$\mu_{O(e)}(u) = \min\left\{\mu_{J(e)}(u), \mu_{S(e)}(u)\right\}$$
$$= \min\left\{\max\left\{\mu_{F(e)}(u), \mu_{G(e)}(u)\right\}, \max\left\{\mu_{F(e)}(u), \mu_{H(e)}(u)\right\}\right\} = \mu_{K(e)}(u)$$

Consequently, K and O are the same operators. Thus

$$(F,A)\tilde{\cup}_R\left((G,B)\tilde{\cap}_R(H,C)\right) = \left((F,A)\tilde{\cup}_R(G,B)\right)\tilde{\cap}_R\left((F,A)\tilde{\cup}_R(H,C)\right)$$

(2). Assume that $(G,B)\tilde{\cup}_R(H,C) = (I,D)$, where $D = B \cap C$ and $\forall e \in D$, $I(e) = G(e) \cup H(e)$

$$= \left(\left\{\frac{u}{\max\left\{\mu_{G(e)}(u), \mu_{H(e)}(u)\right\}} : u \in U_i\right\} : i \in I\right)$$

Since $(F,A)\tilde{\cap}_R\left((G,B)\tilde{\cup}_R(H,C)\right) = (F,A)\tilde{\cap}_R(I,D)$, we suppose that $(F,A)\tilde{\cap}_R(I,D) = (K,M)$ where

$M = A \cap D = A \cap B \cap C$ and $\forall e \in M$,

$$K(e) = F(e) \cap I(e) = \left(\left\{\frac{u}{\mu_{K(e)}(u)} : u \in U_i\right\} : i \in I\right)$$

$$\textit{where } \mu_{K(e)}(u) = \min\left\{\mu_{F(e)}(u), \mu_{I(e)}(u)\right\}$$
$$= \min\left\{\mu_{F(e)}(u), \max\left\{\mu_{G(e)}(u), \mu_{H(e)}(u)\right\}\right\}$$
$$= \max\left\{\min\left\{\mu_{F(e)}(u), \mu_{G(e)}(u)\right\}, \min\left\{\mu_{F(e)}(u), \mu_{H(e)}(u)\right\}\right\}$$

Suppose that $(F,A)\tilde{\cap}_R(G,B) = (J,N)$, where $N = A \cap B$ and $\forall e \in N$,

$$J(e) = F(e) \cap G(e) = \left(\left\{\frac{u}{\min\left\{\mu_{F(e)}(u), \mu_{G(e)}(u)\right\}} : u \in U_i\right\} : i \in I\right)$$

Again, let $(F,A)\tilde{\cap}_R(H,C) = (S,T)$, where $T = A \cap C$ and $\forall e \in T$,

$$S(e) = F(e) \cap H(e) = \left(\left\{\frac{u}{\min\left\{\mu_{F(e)}(u), \mu_{H(e)}(u)\right\}} : u \in U_i\right\} : i \in I\right)$$

Since $\left((F,A)\tilde{\cap}_R(G,B)\right)\tilde{\cup}_R\left((F,A)\tilde{\cap}_R(H,C)\right) = (J,N)\tilde{\cup}_R(S,T)$, we suppose that

$(J,N)\tilde{\cup}_R(S,T) = (O, N \cap T)$ where $\forall e \in N \cap T = A \cap B \cap C$,

$$O(e) = J(e) \cup S(e) = \left(\left\{\frac{u}{\mu_{O(e)}(u)} : u \in U_i\right\} : i \in I\right)$$

$$\mu_{O(e)}(u) = \max\left\{\mu_{J(e)}(u), \mu_{S(e)}(u)\right\}$$
$$= \max\left\{\min\left\{\mu_{F(e)}(u), \mu_{G(e)}(u)\right\}, \min\left\{\mu_{F(e)}(u), \mu_{H(e)}(u)\right\}\right\} = \mu_{K(e)}(u)$$

Consequently, K and O are the same operators. Thus

$$(F,A)\tilde{\cap}_R\left((G,B)\tilde{\cup}_R(H,C)\right) = \left((F,A)\tilde{\cap}_R(G,B)\right)\tilde{\cup}_R\left((F,A)\tilde{\cap}_R(H,C)\right)$$

3.3. Proposition (Associative Laws)

Let (F, A), (G, B) and (H, C) are three fuzzy soft multi sets over U, then we have the following properties:

1. $(F,A)\tilde{\cup}_E\left((G,B)\tilde{\cup}_E(H,C)\right) = \left((F,A)\tilde{\cup}_E(G,B)\right)\tilde{\cup}_E(H,C)$
2. $(F,A)\tilde{\cap}_E\left((G,B)\tilde{\cap}_E(H,C)\right) = \left((F,A)\tilde{\cap}_E(G,B)\right)\tilde{\cap}_E(H,C)$

Proof. 1. Suppose that $(G,B)\tilde{\cup}_E(H,C) = (I,D)$, where $D = B\cup C$ and $\forall e\in D$,

$$I(e) = \begin{cases} G(e), & if\ e\in B-C \\ H(e), & if\ e\in C-B \\ \bigcup\left(G(e),H(e)\right), & if\ e\in B\cap C, \end{cases}$$

Since $(F,A)\tilde{\cup}_E\left((G,B)\tilde{\cup}_E(H,C)\right) = (F,A)\tilde{\cup}_E(I,D)$, we suppose that $(F,A)\tilde{\cup}_E(I,D) = (J,M)$ where $M = A\cup D = A\cup B\cup C$ and $\forall e\in M$,

$$J(e) = \begin{cases} G(e), & if\ e\in B-C-A \\ H(e), & if\ e\in C-B-A \\ F(e), & if\ e\in A-B-C \\ \bigcup\left(G(e),H(e)\right), & if\ e\in B\cap C-A, \\ \bigcup\left(F(e),H(e)\right), & if\ e\in A\cap C-B, \\ \bigcup\left(G(e),F(e)\right), & if\ e\in A\cap B-C, \\ \bigcup\left(F(e),G(e),H(e)\right), & if\ e\in A\cap B\cap C, \end{cases}$$

Assume that $(F,A)\tilde{\cup}_E(G,B) = (K,S)$, where $S = A\cup B$ and $\forall e\in S$,

$$K(e) = \begin{cases} F(e), & if\ e\in A-B \\ G(e), & if\ e\in B-A \\ \bigcup\left(F(e),G(e)\right), & if\ e\in A\cap B, \end{cases}$$

Since $\left((F,A)\tilde{\cup}_E(G,B)\right)\tilde{\cup}_E(H,C) = (K,S)\tilde{\cup}_E(H,C)$, we suppose that $(K,S)\tilde{\cup}_E(H,C) = (L,T)$ where $T = S\cup C = A\cup B\cup C$ and $\forall e\in T$,

$$L(e)=\begin{cases} G(e), & if\ e\in B-C-A \\ H(e), & if\ e\in C-B-A \\ F(e), & if\ e\in A-B-C \\ \bigcup\left(G(e),H(e)\right), & if\ e\in B\cap C-A, \\ \bigcup\left(F(e),H(e)\right), & if\ e\in A\cap C-B, \\ \bigcup\left(G(e),F(e)\right), & if\ e\in A\cap B-C, \\ \bigcup\left(F(e),G(e),H(e)\right), & if\ e\in A\cap B\cap C, \end{cases}$$

Therefore it is clear that $M=T$ and $\forall e\in M$, $J(e)=L(e)$, that is J and L are the same operators. Thus $(F,A)\tilde{\cup}_E\left((G,B)\tilde{\cup}_E(H,C)\right)=\left((F,A)\tilde{\cup}_E(G,B)\right)\tilde{\cup}_E(H,C)$.

2. Suppose that $(G,B)\tilde{\cap}_E(H,C)=(I,D)$, where $D=B\cup C$ and $\forall e\in D$,

$$I(e)=\begin{cases} G(e), & if\ e\in B-C \\ H(e), & if\ e\in C-B \\ \bigcap\left(G(e),H(e)\right), & if\ e\in B\cap C, \end{cases}$$

Since $(F,A)\tilde{\cap}_E\left((G,B)\tilde{\cap}_E(H,C)\right)=(F,A)\tilde{\cap}_E(I,D)$, we suppose that $(F,A)\tilde{\cap}_E(I,D)=(J,M)$ where $M=A\cup D=A\cup B\cup C$ and $\forall e\in M$,

$$J(e)=\begin{cases} G(e), & if\ e\in B-C-A \\ H(e), & if\ e\in C-B-A \\ F(e), & if\ e\in A-B-C \\ \bigcap\left(G(e),H(e)\right), & if\ e\in B\cap C-A, \\ \bigcap\left(F(e),H(e)\right), & if\ e\in A\cap C-B, \\ \bigcap\left(G(e),F(e)\right), & if\ e\in A\cap B-C, \\ \bigcap\left(F(e),G(e),H(e)\right), & if\ e\in A\cap B\cap C, \end{cases}$$

Assume that $(F,A)\tilde{\cap}_E(G,B)=(K,S)$, where $S=A\cup B$ and $\forall e\in S$,

$$K(e)=\begin{cases} F(e), & if\ e\in A-B \\ G(e), & if\ e\in B-A \\ \bigcap\left(F(e),G(e)\right), & if\ e\in A\cap B, \end{cases}$$

Since $\left((F,A)\tilde{\cap}_E(G,B)\right)\tilde{\cap}_E(H,C)=(K,S)\tilde{\cap}_E(H,C)$, we suppose that $(K,S)\tilde{\cap}_E(H,C)=(L,T)$ where $T=S\cup C=A\cup B\cup C$ and $\forall e\in T$,

$$L(e)=\begin{cases} G(e), & if\ e\in B-C-A \\ H(e), & if\ e\in C-B-A \\ F(e), & if\ e\in A-B-C \\ \bigcap\big(G(e),H(e)\big), & if\ e\in B\cap C-A, \\ \bigcap\big(F(e),H(e)\big), & if\ e\in A\cap C-B, \\ \bigcap\big(G(e),F(e)\big), & if\ e\in A\cap B-C, \\ \bigcap\big(F(e),G(e),H(e)\big), & if\ e\in A\cap B\cap C, \end{cases}$$

Therefore it is clear that $M=T$ and $\forall e\in M$, $J(e)=L(e)$, that is J and L are the same operators. Thus $(F,A)\tilde{\cap}_E\big((G,B)\tilde{\cap}_E(H,C)\big)=\big((F,A)\tilde{\cap}_E(G,B)\big)\tilde{\cap}_E(H,C)$.

3.4. Proposition (Distributive Laws)

Let (F, A), (G, B) and (H, C) are three fuzzy soft multi sets over U, then we have the following properties:

(1). $(F,A)\tilde{\cap}_E\big((G,B)\tilde{\cup}_E(H,C)\big)=\big((F,A)\tilde{\cap}_E(G,B)\big)\tilde{\cup}_E\big((F,A)\tilde{\cap}_E(H,C)\big)$

(2). $(F,A)\tilde{\cup}_E\big((G,B)\tilde{\cap}_E(H,C)\big)=\big((F,A)\tilde{\cup}_E(G,B)\big)\tilde{\cap}_E\big((F,A)\tilde{\cup}_E(H,C)\big)$

Proof. (1). Suppose that $(G,B)\tilde{\cup}_E(H,C)=(I,D)$, where $D=B\cup C$ and $\forall e\in D$,

$$I(e)=\begin{cases} G(e), & if\ e\in B-C \\ H(e), & if\ e\in C-B \\ \bigcup\big(G(e),H(e)\big), & if\ e\in B\cap C, \end{cases}$$

Since $(F,A)\tilde{\cap}_E\big((G,B)\tilde{\cup}_E(H,C)\big)=(F,A)\tilde{\cap}_E(I,D)$, we suppose that $(F,A)\tilde{\cap}_E(I,D)=(J,M)$ where $M=A\cup D=A\cup B\cup C$ and $\forall e\in M$,

$$J(e)=\begin{cases} G(e), & if\ e\in B-C-A \\ H(e), & if\ e\in C-B-A \\ F(e), & if\ e\in A-B-C \\ \bigcup\big(G(e),H(e)\big), & if\ e\in B\cap C-A, \\ \bigcap\big(F(e),H(e)\big), & if\ e\in A\cap C-B, \\ \bigcap\big(F(e),G(e)\big), & if\ e\in A\cap B-C, \\ \bigcap\big(F(e),\bigcup(G(e),H(e))\big), & if\ e\in A\cap B\cap C, \end{cases}$$

Assume that $(F,A)\tilde{\cap}_E(G,B)=(K,S)$, where $S=A\cup B$ and $\forall e\in S$,

$$K(e)=\begin{cases} F(e), & if\ e\in A-B \\ G(e), & if\ e\in B-A \\ \bigcap\big(F(e),G(e)\big), & if\ e\in A\cap B, \end{cases}$$

and $(F,A)\tilde{\cap}_E(H,C)=(N,T)$, where $T=A\cup C$ and $\forall e\in T$,

$$N(e)=\begin{cases}F(e), & if\ e\in A-C\\ H(e), & if\ e\in C-A\\ \bigcap\left(F(e),H(e)\right), & if\ e\in A\cap C,\end{cases}$$

Since $\left((F,A)\tilde{\cap}_E(G,B)\right)\tilde{\cup}_E\left((F,A)\tilde{\cap}_E(H,C)\right)=(K,S)\tilde{\cup}_E(N,T)$, we suppose that $(K,S)\tilde{\cup}_E(N,T)=(O,P)$ where $P=S\cup T=\left(A\cup B\right)\cup\left(A\cup C\right)=A\cup B\cup C$ and $\forall e\in P$,

$$O(e)=\begin{cases}G(e), & if\ e\in B-C-A\\ H(e), & if\ e\in C-B-A\\ F(e), & if\ e\in A-B-C\\ \bigcup\left(G(e),H(e)\right), & if\ e\in B\cap C-A,\\ \bigcap\left(F(e),H(e)\right), & if\ e\in A\cap C-B,\\ \bigcap\left(F(e),G(e)\right), & if\ e\in A\cap B-C,\\ \bigcap\left(F(e),\bigcup\left(G(e),H(e)\right)\right), & if\ e\in A\cap B\cap C,\end{cases}$$

Therefore it is clear that $M=P$ and $\forall e\in M$, $J(e)=O(e)$, that is “J” and “O” are the same operators. Thus $(F,A)\tilde{\cap}_E\left((G,B)\tilde{\cup}_E(H,C)\right)=\left((F,A)\tilde{\cap}_E(G,B)\right)\tilde{\cup}_E\left((F,A)\tilde{\cap}_E(H,C)\right)$.

(2). Suppose that $(G,B)\tilde{\cap}_E(H,C)=(I,D)$, where $D=B\cup C$ and $\forall e\in D$,

Since $(F,A)\tilde{\cup}_E\left((G,B)\tilde{\cap}_E(H,C)\right)=(F,A)\tilde{\cup}_E(I,D)$, we suppose that $(F,A)\tilde{\cup}_E(I,D)=(J,M)$ where $M=A\cup D=A\cup B\cup C$ and $\forall e\in M$,

$$J(e)=\begin{cases}G(e), & if\ e\in B-C-A\\ H(e), & if\ e\in C-B-A\\ F(e), & if\ e\in A-B-C\\ \bigcap\left(G(e),H(e)\right), & if\ e\in B\cap C-A,\\ \bigcup\left(F(e),H(e)\right), & if\ e\in A\cap C-B,\\ \bigcup\left(F(e),G(e)\right), & if\ e\in A\cap B-C,\\ \bigcup\left(F(e),\bigcap\left(G(e),H(e)\right)\right), & if\ e\in A\cap B\cap C,\end{cases}$$

Assume that $(F,A)\tilde{\cup}_E(G,B)=(K,S)$, where $S=A\cup B$ and $\forall e\in S$,

$$K(e)=\begin{cases}F(e), & if\ e\in A-B\\ G(e), & if\ e\in B-A\\ \bigcup\left(F(e),G(e)\right), & if\ e\in A\cap B,\end{cases}$$

and $(F,A)\tilde{\cup}_E(H,C)=(N,T)$, where $T=A\cup C$ and $\forall e\in T$,

$$N(e)=\begin{cases}F(e), & if\ e\in A-C\\ H(e), & if\ e\in C-A\\ \bigcup\left(F(e),H(e)\right), & if\ e\in A\cap C,\end{cases}$$

Since $\left((F,A)\tilde{\cup}_E(G,B)\right)\tilde{\cap}_E\left((F,A)\tilde{\cup}_E(H,C)\right)=(K,S)\tilde{\cap}_E(N,T)$, we suppose that $(K,S)\tilde{\cap}_E(N,T)=(O,P)$ where $P=S\cup T=\left(A\cup B\right)\cup\left(A\cup C\right)=A\cup B\cup C$ and $\forall e\in P$,

$$O(e)=\begin{cases}G(e), & if\ e\in B-C-A\\ H(e), & if\ e\in C-B-A\\ F(e), & if\ e\in A-B-C\\ \bigcap\left(G(e),H(e)\right), & if\ e\in B\cap C-A,\\ \bigcup\left(F(e),H(e)\right), & if\ e\in A\cap C-B,\\ \bigcup\left(F(e),G(e)\right), & if\ e\in A\cap B-C,\\ \bigcup\left(F(e),\bigcap\left(G(e),H(e)\right)\right), & if\ e\in A\cap B\cap C,\end{cases}$$

Therefore it is clear that $M=P$ and $\forall e\in M$, $J(e)=O(e)$, that is "J" and "O" are the same operators. Thus $(F,A)\tilde{\cup}_E\left((G,B)\tilde{\cap}_E(H,C)\right)=\left((F,A)\tilde{\cup}_E(G,B)\right)\tilde{\cap}_E\left((F,A)\tilde{\cup}_E(H,C)\right)$.

3.5. Proposition (Associative Laws)

Let (F, A), (G, B) and (H, C) are three fuzzy soft multi sets over U, then we have the following properties:

1. $(F,A)\wedge\left((G,B)\wedge(H,C)\right)=\left((F,A)\wedge(G,B)\right)\wedge(H,C)$
2. $(F,A)\vee\left((G,B)\vee(H,C)\right)=\left((F,A)\vee(G,B)\right)\vee(H,C)$

Proof. 1. Assume that $(G,B)\wedge(H,C)=(I,B\times C)$, where $\forall\left(b,c\right)\in B\times C$,

$$I(b,c)=G(b)\cap H(c)=\left(\left\{\frac{u}{\min\left\{\mu_{G(b)}(u),\mu_{H(c)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Since $(F,A)\wedge\left((G,B)\wedge(H,C)\right)=(F,A)\wedge(I,B\times C)$, we suppose that $(F,A)\wedge(I,B\times C)=\left(K,A\times(B\times C)\right)$

where $\forall\left(a,b,c\right)\in A\times\left(B\times C\right)=A\times B\times C$, $K(a,b,c)=F(a)\cap I(b,c)=\left(\left\{\frac{u}{\mu_{K(a,b,c)}(u)}:u\in U_i\right\}:i\in I\right)$

$$where\ \mu_{K(a,b,c)}(u)=\min\left\{\mu_{F(e)}(u),\mu_{I(b,c)}(u)\right\}=\min\left\{\mu_{F(e)}(u),\min\left\{\mu_{G(b)}(u),\mu_{H(c)}(u)\right\}\right\}$$
$$=\min\left\{\mu_{F(e)}(u),\mu_{G(b)}(u),\mu_{H(c)}(u)\right\}$$

We take $(a,b)\in A\times B$. Suppose that $(F,A)\wedge(G,B)=(J,A\times B)$ where $\forall\left(a,b\right)\in A\times B$,

$$J(a,b)=F(a)\cap G(b)=\left(\left\{\frac{u}{\min\left\{\mu_{F(a)}(u),\mu_{G(b)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Since $((F,A)\wedge(G,B))\wedge(H,C)=(J,A\times B)\wedge(H,C)$, we suppose that $(J,A\times B)\wedge(H,C)=(O,(A\times B)\times C)$ where $\forall(a,b,c)\in(A\times B)\times C=A\times B\times C$,

$$O(a,b,c)=J(a,b)\cap H(c)=\left(\left\{\frac{u}{\mu_{O(a,b,c)}(u)}:u\in U_i\right\}:i\in I\right)$$

$$\mu_{O(a,b,c)}(u)=\min\left\{\mu_{J(a,b)}(u),\mu_{H(c)}(u)\right\}=\min\left\{\min\left\{\mu_{F(e)}(u),\mu_{G(b)}(u)\right\},\mu_{H(c)}(u)\right\}$$
$$=\min\left\{\mu_{F(e)}(u),\mu_{G(b)}(u),\mu_{H(c)}(u)\right\}=\mu_{K(a,b,c)}(u)$$

Consequently, K and O are the same operators. Thus

$$(F,A)\wedge((G,B)\wedge(H,C))=((F,A)\wedge(G,B))\wedge(H,C).$$

2. Assume that $(G,B)\vee(H,C)=(I,B\times C)$, where $\forall(b,c)\in B\times C$,

$$I(b,c)=G(b)\cup H(c)=\left(\left\{\frac{u}{\max\left\{\mu_{G(b)}(u),\mu_{H(c)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Since $(F,A)\vee((G,B)\vee(H,C))=(F,A)\vee(I,B\times C)$, we suppose that $(F,A)\vee(I,B\times C)=(K,A\times(B\times C))$

where $\forall(a,b,c)\in A\times(B\times C)=A\times B\times C,\ K(a,b,c)=F(a)\cup I(b,c)=\left(\left\{\frac{u}{\mu_{K(a,b,c)}(u)}:u\in U_i\right\}:i\in I\right)$

$$\textit{where } \mu_{K(a,b,c)}(u)=\max\left\{\mu_{F(e)}(u),\mu_{I(b,c)}(u)\right\}=\max\left\{\mu_{F(e)}(u),\max\left\{\mu_{G(b)}(u),\mu_{H(c)}(u)\right\}\right\}$$
$$=\max\left\{\mu_{F(e)}(u),\mu_{G(b)}(u),\mu_{H(c)}(u)\right\}$$

We take $(a,b)\in A\times B$. Suppose that $(F,A)\vee(G,B)=(J,A\times B)$ where $\forall(a,b)\in A\times B$,

$$J(a,b)=F(a)\cup G(b)=\left(\left\{\frac{u}{\max\left\{\mu_{F(a)}(u),\mu_{G(b)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Since $((F,A)\vee(G,B))\vee(H,C)=(J,A\times B)\vee(H,C)$, we suppose that $(J,A\times B)\vee(H,C)=(O,(A\times B)\times C)$ where

$$\forall(a,b,c)\in(A\times B)\times C=A\times B\times C,\quad O(a,b,c)=J(a,b)\cup H(c)=\left(\left\{\frac{u}{\mu_{O(a,b,c)}(u)}:u\in U_i\right\}:i\in I\right)$$

$$\textit{where } \mu_{O(a,b,c)}(u)=\max\left\{\mu_{J(a,b)}(u),\mu_{H(c)}(u)\right\}=\max\left\{\max\left\{\mu_{F(e)}(u),\mu_{G(b)}(u)\right\},\mu_{H(c)}(u)\right\}$$
$$=\max\left\{\mu_{F(e)}(u),\mu_{G(b)}(u),\mu_{H(c)}(u)\right\}=\mu_{K(a,b,c)}(u)$$

Consequently, K and O are the same operators. Thus

$(F,A)\vee\left((G,B)\vee(H,C)\right)=\left((F,A)\vee(G,B)\right)\vee(H,C).$

4. An Application Of Fuzzy Soft Multi Set Theory In Information System

In this section we define some basic supporting tools in information system and also application of fuzzy soft multi sets in information system are presented and discussed.

4.1. Definition

A fuzzy multi-valued information system is a quadruple $Inf_{system}=\left(X,A,f,V\right)$ where X is a non empty finite set of objects, A is a non empty finite set of attribute, $V=\bigcup_{a\in A}V_a$, where V is the domain (a fuzzy set,) set of attribute, which has multi value and $f:X\times A\rightarrow V$ is a total function such that $f\left(x,a\right)\in V_a$ for every $\left(x,a\right)\in X\times A.$

4.1. Proposition

If $\left(F,A\right)$ is a fuzzy soft multi set over universe U, then $\left(F,A\right)$ is a fuzzy multi-valued information system.

Proof. Let $\left\{U_i:i\in I\right\}$ be a collection of universes such that $\bigcap_{i\in I}U_i=\phi$ and let $\left\{E_i:i\in I\right\}$ be a collection of sets of parameters. Let $U=\prod_{i\in I}IFS\left(U_i\right)$ where $IFS\left(U_i\right)$ denotes the set of all fuzzy subsets of U_i, $E=\prod_{i\in I}E_{U_i}$ and $A\subseteq E$. Let $\left(F,A\right)$ be an fuzzy soft multi set over U and $X=\bigcup_{i\in I}U_i$. We define a mapping f where $f:X\times A\rightarrow V$, defined as $f\left(x,a\right)=x/\mu_{F(a)}\left(x\right)$.

Hence $V=\bigcup_{a\in A}V_a$ where V_a is the set of all counts of in $F\left(a\right)$ and $\cup$ represent the classical set union. Then the fuzzy multi-valued information system $\left(X,A,f,V\right)$ represents the fuzzy soft multi set $\left(F,A\right)$.

4.1. Application in information system

Let us consider three universes $U_1=\left\{h_1,h_2,h_3,h_4,h_5\right\}$, $U_2=\left\{c_1,c_2,c_3,c_4\right\}$ and $U_3=\left\{v_1,v_2,v_3\right\}$ be the sets of "houses," "cars," and "hotels", respectively. Suppose Mr. X has a budget to buy a house, a car and rent a venue to hold a wedding celebration. Let us consider a intuitionistic fuzzy soft multi set (*F, A*) which describes "houses," "cars," and "hotels" that Mr. X is considering for accommodation purchase, transportation purchase, and a venue to hold a wedding celebration, respectively. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \{e_{U_1,1} = \text{expensive},\ e_{U_1,2} = \text{cheap},\ e_{U_1,3} = \text{wooden}\}$$
$$E_{U_2} = \{e_{U_2,1} = \text{beautiful},\ e_{U_2,2} = \text{cheap},\ e_{U_2,3} = \text{sporty}\},$$
$$E_{U_3} = \{e_{U_3,1} = \text{expensive}, e_{U_3,2} = \text{model}, e_{U_3,3} = \text{beautiful}\}.$$

Let $U = \prod_{i=1}^{3} FS(U_i)$, $E = \prod_{i=1}^{3} E_{U_i}$ and $A \subseteq E$, such that

$$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}),\ a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}),\ a_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}),$$
$$a_4 = (e_{U_1,3}, e_{U_2,3}, e_{U_3,1}),\ a_5 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,2}),\ a_6 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,2})\}.$$

Suppose Mr. X wants to choose objects from the sets of given objects with respect to the sets of choice parameters. Let

$$F(a_1) = (\{h_1/0.2, h_2/0.4, h_3/0, h_4/0.4, h_5/0\}, \{c_1/0.8, c_2/0.1, c_3/0, c_4/1\}, \{v_1/0.8, v_2/0.7, v_3/0\}),$$
$$F(a_2) = (\{h_1/0.3, h_2/0.5, h_3/0, h_4/0.4, h_5/1\}, \{c_1/0.9, c_2/0.6, c_3/1, c_4/0\}, \{v_1/0.6, v_2/0.6, v_3/0\}),$$
$$F(a_3) = (\{h_1/0.5, h_2/0.7, h_3/1, h_4/0.4, h_5/0\}, \{c_1/0.6, c_2/0.1, c_3/0, c_4/1\}, \{v_1/0, v_2/0.7, v_3/0.7\}),$$
$$F(a_4) = (\{h_1/0.4, h_2/0.5, h_3/0.1, h_4/0, h_5/1\}, \{c_1/0.8, c_2/0, c_3/0.8, c_4/1\}, \{v_1/1, v_2/0.7, v_3/0\}),$$
$$F(a_5) = (\{h_1/0.7, h_2/0.5, h_3/1, h_4/0.4, h_5/1\}, \{c_1/0, c_2/0.1, c_3/0.7, c_4/1\}, \{v_1/1, v_2/0.8, v_3/0\}),$$
$$F(a_6) = (\{h_1/0.6, h_2/0.4, h_3/0.8, h_4/1, h_5/0.1\}, \{c_1/0, c_2/0.1, c_3/1, c_4/1\}, \{v_1/1, v_2/0, v_3/1\}),$$

Then the fuzzy soft multi set (F, A) defined above describes the conditions of some “house”, “car” and “hotel” in a state. Then the quadruple (X, A, f, V) corresponding to the fuzzy soft multi set given above is a fuzzy multi-valued information system.

Where $X = \bigcup_{i=1}^{3} U_i$ and A is the set of parameters in the fuzzy soft multi set and

$$V_{a_1} = \{h_1/0.2, h_2/0.4, h_3/0, h_4/0.4, h_5/0, c_1/0.8, c_2/0.1, c_3/0, c_4/1, v_1/0.8, v_2/0.7, v_3/0\},$$
$$V_{a_2} = \{h_1/0.3, h_2/0.5, h_3/0, h_4/0.4, h_5/1, c_1/0.9, c_2/0.6, c_3/1, c_4/0, v_1/0.6, v_2/0.6, v_3/0\},$$
$$V_{a_3} = \{h_1/0.5, h_2/0.7, h_3/1, h_4/0.4, h_5/0, c_1/0.6, c_2/0.1, c_3/0, c_4/1, v_1/0, v_2/0.7, v_3/0.7\},$$
$$V_{a_4} = \{h_1/0.4, h_2/0.5, h_3/0.1, h_4/0, h_5/1, c_1/0.8, c_2/0, c_3/0.8, c_4/1, v_1/1, v_2/0.7, v_3/0\},$$
$$V_{a_5} = \{h_1/0.7, h_2/0.5, h_3/1, h_4/0.4, h_5/1, c_1/0, c_2/0.1, c_3/0.7, c_4/1, v_1/1, v_2/0.8, v_3/0\},$$
$$V_{a_6} = \{h_1/0.6, h_2/0.4, h_3/0.8, h_4/1, h_5/0.1, c_1/0, c_2/0.1, c_3/1, c_4/1, v_1/1, v_2/0, v_3/1\}$$

,

For the pair (h_1, a_1) we have $f(h_1, a_1)$ = 0.2, for (h_2, a_1), we have $f(h_2, a_1)$ = 0.4. Continuing in this way we obtain the values of other pairs. Therefore, according to the result above, it is seen that fuzzy soft multi sets are fuzzy soft multi-valued information systems. Nevertheless, it is obvious that fuzzy soft multi-valued information systems are not necessarily fuzzy soft multi sets. We can construct an information table representing fuzzy soft multi set (F, A) defined above as follows.

4.1. An Information Table

Table 2. An information table

X	a_1	a_2	a_3	a_4	a_5	a_6
h_1	0.2	0.3	0.5	0.4	0.7	0.6
h_2	0.4	0.5	0.7	0.5	0.5	0.4
h_3	0	0	1	0.1	1	0.8
h_4	0.4	0.4	0.4	0	0.4	1
h_5	0	1	0	1	1	0.1
c_1	0.8	0.9	0.6	0.8	0	0
c_2	0.1	0.6	0.1	0	0.1	0.1
c_3	0	1	0	0.8	0.7	1
c_4	1	0	1	1	1	1
v_1	0.8	0.6	0	1	1	1
v_2	0.7	0.6	0.7	0.7	0.8	0
v_3	0	0	0.7	0	0	1

5. Conclusion

The theory of soft set, fuzzy soft set and multi set are vital mathematical tools used in handling uncertainties about vague concepts. In this paper, we have introduced the notion of fuzzy multi-valued information system in fuzzy soft multi set theory. Here we shall present the application of fuzzy soft multi set in information system and show that every fuzzy soft multi set is a fuzzy multi valued information system.

References

[1] K.Alhazaymeh & N. Hassan, (2014) "Vague Soft Multiset Theory", Int. J. Pure and Applied Math., Vol. 93, pp511-523.
[2] M.I.Ali, F. Feng, X. Liu, W.K. Minc & M. Shabir, (2009) "On some new operations in soft set theory", Comp. Math. Appl., vol. 57, pp1547-1553.
[3] S.Alkhazaleh & A.R. Salleh, (2012) "Fuzzy Soft Multi sets Theory", Hindawi Publishing Corporation, Abstract and Applied Analysis, Article ID 350603, 20 pages, doi: 10.1155/2012/350603.
[4] S.Alkhazaleh, A.R. Salleh & N. Hassan, (2011) Soft Multi sets Theory, Appl. Math. Sci., Vol. 5, pp3561– 3573.
[5] K.V Babitha & S.J. John, (2013) "On Soft Multi sets", Ann. Fuzzy Math. Inform., Vol. 5 pp35-44.
[6] T. Herewan & M.M. Deris, (2011) "A soft set approach for association rules mining", Knowl.-Based Syst. Vol. 24, pp186-195.
[7] A.M. Ibrahim & H.M. Balami, (2013) "Application of soft multiset in decision making problems", J. Nig. Ass. of mathematical physics, Vol. 25, pp307- 311.
[8] H.M. Balami & A. M. Ibrahim, (2013) "Soft Multiset and its Application in Information System", International Journal of scientific research and management, Vol. 1, pp471-482.
[9] P.K. Maji, R. Biswas & A.R. Roy, (2001) "Fuzzy soft sets", J. Fuzzy Math., Vol. 9, pp589-602.

[10] P.K. Maji, R. Biswas & A.R. Roy, (2003) "Soft set theory", Comp. Math. Appl., Vol. 45 pp555-562.
[11] P.K. Maji, R. Biswas & A.R. Roy, (2002) "An application of soft sets in a decision making problem", Comput. Math. Appl., Vol. 44, pp1077-1083.
[12] D.Molodtsov, (1999) "Soft set theory-first results", Comp. Math. Appl., Vol. 37, pp19-31.
[13] A.Mukherjee & A.K. Das, (2015) "Application of fuzzy soft multi sets in decision making problems", Smart Innovation Systems and Technologies, Springer Verlag, (Accepted).
[14] A.Mukherjee & A.K. Das, (2015) "Some results on fuzzy soft multi sets. Int. J. Cybernetics & Informatics. Vol. 4, pp 51-65.
[15] A.Mukherjee & A.K. Das, (2013) "Topological structure formed by fuzzy soft multisets", Rev. Bull. Cal.Math.Soc., Vol. 21, No.2, pp193-212.
[16] A.Mukherjee & A.K. Das, (2014) "Topological structure formed by soft multi sets and soft multi compact space", Ann. Fuzzy Math. Inform. Vol. 7, pp919-933.
[17] D.Tokat & I. Osmanoglu, (2011) "Soft multi set and soft multi topology", Nevsehir Universitesi Fen Bilimleri Enstitusu Dergisi Cilt., Vol. 2, pp109-118.
[18] L.A. Zadeh, (1965) "Fuzzy sets", Inform. Control., Vol. 8, pp338-353.