ANTI-SYNCHRONIZATON OF TWO DIFFERENT HYPERCHAOTIC SYSTEMS VIA ACTIVE GENERALIZED BACKSTEPPING METHOD

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ABSTRACT

This paper presents hyperchaos anti-synchronization of different hyperchaotic systems using Active Generalized Backstepping Method (AGBM). The proposed technique is applied to achieve hyperchaos anti-synchronization for the Lorenz and Lu dynamical systems. Generalized Backstepping Method (GBM) is similarity to Backstepping and more applications in systems than it. Bckstepping method is used only to strictly feedback systems but GBM expand this class. The hybrid active control method and generalized backstepping method forces the system error to decay to zero rapidly that it causes the system to have a short setteling time, overshoot.

KEYWORDS

Active Generalized Backstepping Method, Anti-Synchronization, Hyperchaotic Lorenz System, Hyperchaotic Lu System, Lyapunov Direct Method.

1. INTRODUCTION

An interesting phenomenon of nonlinear systems is chaos. In recent years, studies of chaos and hyperchaos generation, control and synchronization have attracted considerable attentions, laser, nonlinear circuit and neural network, etc [1-13]. Therefore, various effective methods have been proposed one the past decades to achieve the control and stabilization of chaotic system, such as Robust Control [1], the sliding method control [2], linear and nonlinear feedback control [3], adaptive control [4], active control [5], backstepping control [6] and generalized backsteppig method control [7-9], ect. Hyperchaotic system has more complex dynamical behaviors than chaotic system. Historically, the more well know hyperchaotic systems are the 4D hyperchaotic Rossler system [14], the 4D hyperchaotic Chua's circuit [15], the generalized Lorenz system [16], Chen system [17] and Lu system[18]. Synchronization of chaotic systems has become more and more interesting topics to engineering and science communities [19-27]. The concept of synchronization has been extended to the scope, such as phase synchronization [28], lag synchronization [29] and even anti-synchronization (anti-phase synchronization) [30-32].

This paper is organized as follows: in Section 2, studies the Generalized Backstepping Method. In Section 3, involves the basic properties for the Hyperchaotic systems. In Section 4, studies the ani-synchronization of the two diferent hyperchaotic system with an Active Generalized Backstepping Method. In Section 5, numerical simulation of output presented. Conclusion are given in final section.

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2. THE GENERALIZED BACKSTEPPING METHOD

Generalized Backstepping Method [7-9] will be applied to a certain class of autonomous nonlinear systems which are expressed as follow

$$\begin{cases} \dot{X} = F(X) + G(X)\eta \\ \dot{\eta} = f_0(X,\eta) + g_0(X,\eta)u \end{cases} \tag{1}$$

In which $\eta\epsilon□$ and $x = [x_1, x_2, \cdots, x_n]\epsilon□$. In order to obtain an approach to control these systems, we may need to prove a new theorem as follow.

Theorem : suppose equation (1) is available, then suppose the scalar function $\varphi_i(x)$ for the i_{th} state could be determined i a manner which by inserting the i_{th} term for η, the function $v(x)$ would be a positive definite equation (3) with negative definite derivative.

$$V(X) = \frac{1}{2}\sum_{i=1}^{n} x_i^2 \tag{2}$$

Therefore, the control signal and also the general control lyapunov function of this system can be obtained by equation (3),(4).

$$u = \frac{1}{g_0(X,\eta)}\left\{ \begin{array}{c} \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial\varphi_i}{\partial x_j}[f_i(X) + g_i(X)\eta] \\ -\sum_{i=1}^{n} x_i g_i(X) - \sum_{i=1}^{n} k_i[\eta - \varphi_i(X)] - f_0(X,\eta) \end{array} \right\}, k_i > 0\ , i = 1,2, \cdots, n \tag{3}$$

$$V_t(X,\eta) = \frac{1}{2}\sum_{i=1}^{n} x_i^2 + \frac{1}{2}\sum_{i=1}^{n}[\eta - \varphi_i(X)]^2 \tag{4}$$

3. SYSTEM DESCRIPTION

The hyperchaotic Lorenz system [16] is described by

$$\begin{aligned} \dot{x} &= a(y - x) + w \\ \dot{y} &= -xz + cx - y \\ \dot{z} &= xy - bz \\ \dot{w} &= -xz + dw \end{aligned} \tag{5}$$

Where a, b, c and d are constants. When parameters $a = 10, b = \frac{8}{3}, c = 28$ and $d = 1.3$, the system (5) shows hyperchaotic behavior. See figure 1.

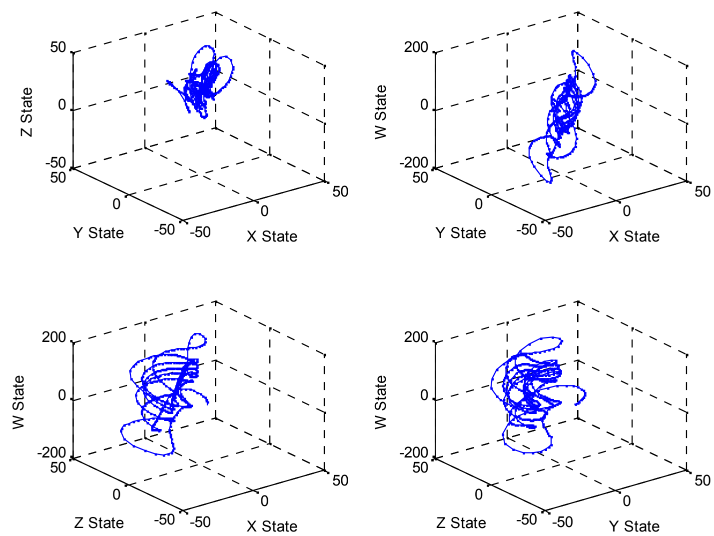

Figure 1. Phase Plane of hyperchaotic Lorenz system.

The hyperchaotic Lu system [18] is described by

$$\begin{aligned}
\dot{x} &= a(y - x) + w \\
\dot{y} &= -xz + cy \\
\dot{z} &= xy - bz \\
\dot{w} &= xz + dw
\end{aligned} \tag{6}$$

Where x, y, z and w are state variables and a, b, c and d are real constants. When $a = 36, b = 3, c = 20, -0.35 < d \leq 1.3$, system (6) has hyperchaotic attractor. See fig 2.

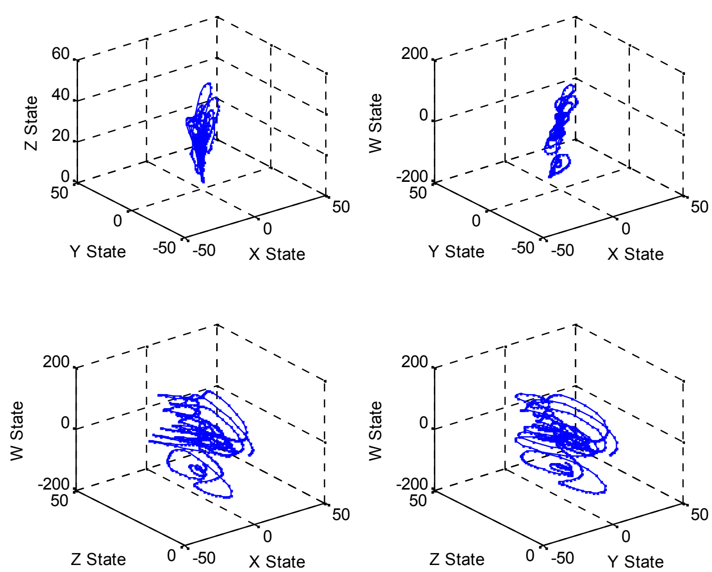

Figure 2. Phase Plane of hyperchaotic Lu system.

4. ANTI-SYNCHRONIZATION OF TWO DIFFERENT HYPERCHAOTIC SYSTEMS

In this section, the hybrid active control method and generalized backstepping method is applied to anti-synchronize between the hyperchaotic lorenz system and the hyperchaotic lu system.
Suppose the drive system takes the following from

$$\begin{aligned}
\dot{x}_1 &= a_1(y_1 - x_1) + w_1 \\
\dot{y}_1 &= -x_1 z_1 + c_1 x_1 - y_1 \\
\dot{z}_1 &= x_1 y_1 - b_1 z_1 \\
\dot{w}_1 &= -x_1 z_1 + d_1 w_1
\end{aligned} \tag{7}$$

And the response system is given as follows

$$\begin{aligned}
\dot{x}_2 &= a_2(y_2 - x_2) + w_2 + u_1(t) \\
\dot{y}_2 &= -x_2 z_2 + c_2 y_2 + u_2(t) \\
\dot{z}_2 &= x_2 y_2 - b_2 z_2 + u_3(t) \\
\dot{w}_2 &= x_2 z_2 + d_2 w_2 + u_4(t)
\end{aligned} \tag{8}$$

Where $u_1(t), u_2(t), u_3(t)$ and $u_4(t)$ are control functions to be determined for achieving anti-synchronization between the two systems (7) and (8).
Define state errors between system (7) and (8) as follows

$$\begin{aligned}
e_x &= x_1 + x_2 \\
e_y &= y_1 + y_2 \\
e_z &= z_1 + z_2 \\
e_w &= w_1 + w_2
\end{aligned} \tag{9}$$

We obtain the following error dynamical system by adding the drive system (7) with the response system (8).

$$\begin{aligned}
\dot{e}_x &= a_2(e_y - e_x) + e_w + (a_1 - a_2)(y_1 - x_1) + u_1(t) \\
\dot{e}_y &= c_2 e_y - (1 + c_2) y_1 + c_1 x_1 - x_1 z_1 - x_2 z_2 + u_2(t) \\
\dot{e}_z &= -b_2 e_z + (b_2 - b_1) z_1 + x_1 y_1 + x_2 y_2 + u_3(t) \\
\dot{e}_w &= d_2 e_w + (d_1 - d_2) w_1 + x_2 z_2 - x_1 z_1 + u_4(t)
\end{aligned} \tag{10}$$

Define the following active control functions $u_1(t), u_2(t), u_3(t)$ and $u_4(t)$.

$$\begin{aligned}
u_1(t) &= -(a_1 - a_2)(y_1 - x_1) + v_1(t) \\
u_2(t) &= (1 + c_2) y_1 - c_1 x_1 + x_1 z_1 + x_2 z_2 + v_2(t) \\
u_3(t) &= -(b_2 - b_1) z_1 - x_1 y_1 - x_2 y_2 + v_3(t) \\
u_4(t) &= -(d_1 - d_2) w_1 + x_1 z_1 - x_2 z_2 + v_4(t)
\end{aligned} \tag{11}$$

Where $v_1(t), v_2(t), v_3(t)$ and $v_4(t)$ are control inputs.
Substituting equation (11) into equation (10) yields.

$$\begin{aligned}
\dot{e}_x &= a_2(e_y - e_x) + e_w + v_1(t) \\
\dot{e}_y &= c_2 e_y + v_2(t) \\
\dot{e}_z &= -b_2 e_z + v_3(t) \\
\dot{e}_w &= d_2 e_w + v_4(t)
\end{aligned} \tag{12}$$

Thus, the error system (12) to be controlled with control inputs $v_1(t), v_2(t), v_3(t)$ and $v_4(t)$ as functions of error states e_x, e_y, e_z and e_w. When system (12) is stabilized by control inputs $v_1(t), v_2(t), v_3(t)$ and $v_4(t)$, e_x, e_y, e_z and e_w will converage to zeroes as time t tends to infinity. Which implies that system (5) and (6) are anti-synchronized.
To achieve this purpose, we choose control inputs by using generalized backstepping method such that

$$\begin{aligned} v_1(t) &= 0 \\ v_2(t) &= -a_2 e_x - (k_1 + c_2) e_y \\ v_3(t) &= -k_2 e_z \\ v_4(t) &= -e_x - (k_3 + d_2) e_w \end{aligned} \tag{13}$$

Now, using the gradient optimization of neural network coefficients controllers suitable for use relationship (13) let's examine. In this case, the coefficients $k_i; i = 1,2,3$ benefit obtained from the following relationship will come.

$$\begin{aligned} k_1(t+1) &= |k_1(t) - \alpha e_y| \\ k_2(t+1) &= |k_2(t) - \alpha e_z| \\ k_3(t+1) &= |k_3(t) - \alpha e_w| \end{aligned} \tag{14}$$

Where α the learning rate is would be equal to 0.01. Initial value of $k_i; i = 1,2,3$ are equal to 50.

5. Numerical Simulation

This section presents numerical simulations anti-synchronization of hyperchaotic lorenz system and hyperchaotic lu system. The Active Generalized Backstepping Method (AGBM) is used as an approach to anti-synchronize hyperchaotic lorenz and lu system, eventually the result of this method would be compared with the anti-synchronization result of Nonlinear Control Method (NCM) [32]. We select the parameters of the hyperchaotic lorenz system as $a = 10, b = \frac{8}{3}, c = 28, d = 1.3$ and for the hyperchaotic lu as $a = 36, b = 3, c = 20, d = 1.3$, so that these systems exhibits a hyperchaotic behavior. The initial values of the the drive and response systems are $x_1(0) = 5, y_1(0) = 8, z_1(0) = -1, w_1(0) = -3$ and $x_2(0) = 3, y_2(0) = 4, z_2(0) = 5, w_2(0) = 5$ respectively. The time response of x, y, z, w states for drive system (hyperchaotic Lorenz) and the response system (hyperchaotic Lu) via active generalized backstepping method shown in order figure 3 until figure 6. Anti-Synchronization errors (e_x, e_y, e_z, e_w) in hyperchaotic lorenz system and hyperchaotic lu system shown in order figure 7 until figure 10.

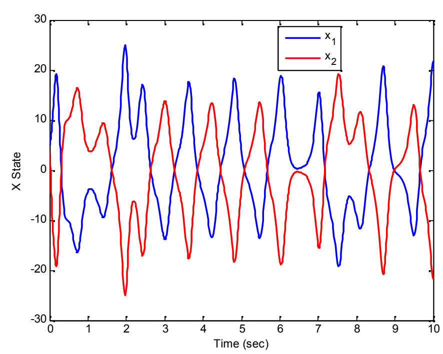

Figure 3. The time response of signals x_1 and x_2 for hyperchaotic Lorenz and Lu systems.

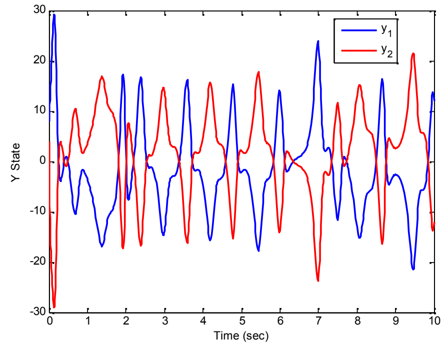

Figure 4. The time response of signals y_1 and y_2 for hyperchaotic Lorenz and Lu systems.

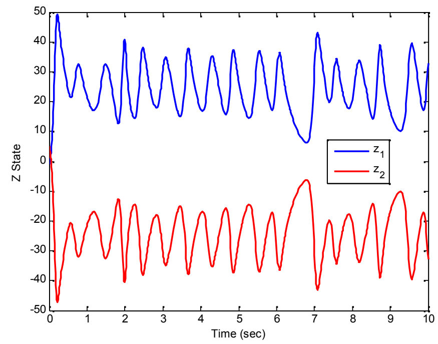

Figure 5. The time response of signals z_1 and z_2 for hyperchaotic Lorenz and Lu systems.

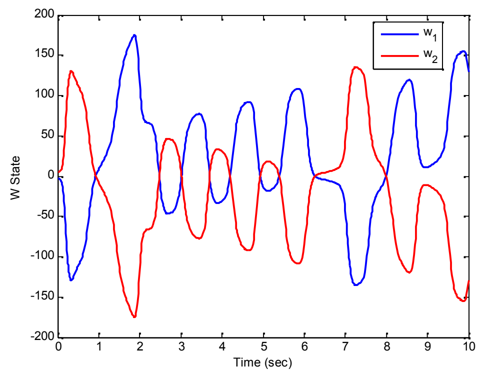

Figure 6. The time response of signals w_1 and w_2 for hyperchaotic Lorenz and Lu systems.

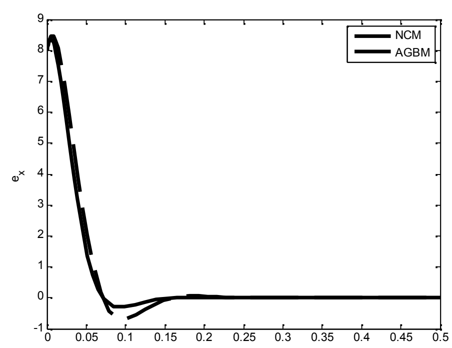

Figure 7. Anti-Synchronization e_x in hyperchaotic Lorenz and Lu systems.

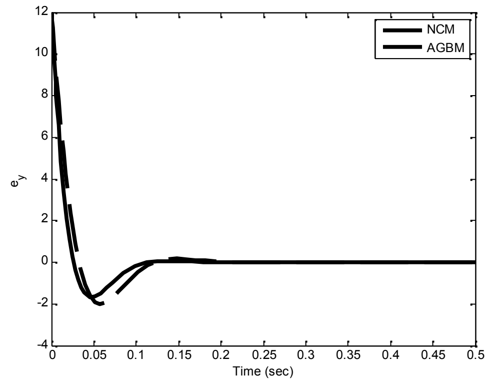

Figure 8. Anti-Synchronization e_y in hyperchaotic Lorenz and Lu systems.

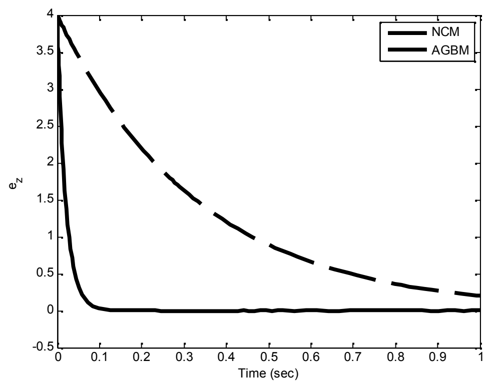

Figure 9. Anti-Synchronization e_z in hyperchaotic Lorenz and Lu systems.

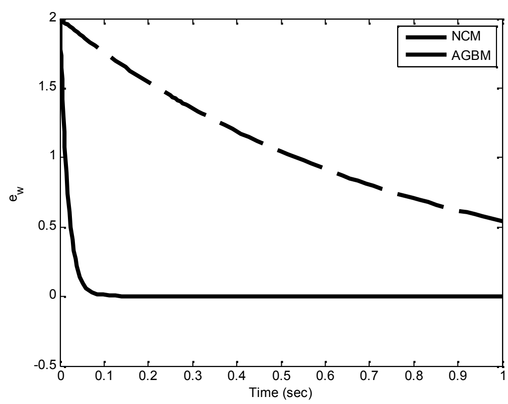

Figure 10. Anti-Synchronization e_w in hyperchaotic Lorenz and Lu systems.

6. CONCLUSIONS

This study demonstrated that anti-synchronization can coexist in two different hyperchaotic systems ratchets moving in different asymmetric potentials by active generalized backstepping method. Hyperchaotic Lu system is controlled to be anti-synchronized with hyperchaotic Lorenz system. In the Active Generalized Backstepping Method in relation to the Nonlinear Control Method [32], control will be accomplished in a much shorter time and overshoot. The simulations confirm that Anti-Synchronization of two systems operates satisfactorily in presence of the proposed control method.

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