ACTIVE CONTROLLER DESIGN FOR REGULATING THE OUTPUT OF THE SPROTT-P SYSTEM

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ABSTRACT

This paper discusses new results derived for regulating the output of the Sprott-P system (1994), which is one of the paradigms of 3-dimensional chaotic systems discovered by J.C. Sprott. For the constant tracking problem, new state feedback control laws have been derived for regulating the output of the Sprott-P system. Numerical simulations are shown to illustrate the effectiveness of the control schemes derived in this paper for the output regulation of the Sprott-P system.

KEYWORDS

Active Control; Chaos; Sprott-P system; Nonlinear Control Systems; Output Regulation.

1. INTRODUCTION

Output regulation of control systems is one of the core research problems in Control Systems Engineering, which have important applications in Science and Engineering. Basically, the output regulation problem is to control a fixed linear or nonlinear plant so that the output of the plant tracks reference signals produced by the exo-system. For linear control systems, the output regulation problem has been solved by Francis and Wonham ([1], 1975). For nonlinear control systems, the output regulation problem was solved by Byrnes and Isidori ([2], 1990) generalizing the internal model principle [1] and using centre manifold theory [3].

The output regulation of nonlinear control systems has been studied well in the last two decades [4-14]. In [4], Mahmoud and Khalil obtained results on the asymptotic regulation of minimum phase nonlinear systems using output feedback. In [5], Fridman solved the output regulation problem for nonlinear control systems with delay using centre manifold theory. In [6-7], Chen and Huang obtained results on the robust output regulation for output feedback systems with nonlinear exosystems. In [8], Liu and Huang obtained results on the global robust output regulation problem for lower triangular nonlinear systems with unknown control direction.

In [9], Immonen obtained results on the practical output regulation for bounded linear infinite-dimensional state space systems. In [10], Pavlov, Van de Wouw and Nijmeijer obtained results on the global nonlinear output regulation using convergence-based controller design. In [11], Xi and Dong obtained results on the global adaptive output regulation of a class of nonlinear systems with nonlinear exosystems. In [12-14], Serrani, Isidori and Marconi obtained results on the semi-global and global output regulation problem for minimum-phase nonlinear systems.

In this paper, new results have been derived on the active controller design for regulating the output of the Sprott-P system ([15], 1994). Explicitly, we find state feedback control laws solving the constant regulation problem of the Sprott-P chaotic system (1994).

2. OUTPUT REGULATION PROBLEM FOR NONLINEAR CONTROL SYSTEMS

In this section, we consider a multi-variable nonlinear control system described by

$$\dot{x} = f(x) + g(x)u + p(x)\omega \tag{1a}$$

$$\dot{\omega} = s(\omega) \tag{1b}$$

$$e = h(x) - q(\omega) \tag{2}$$

Here, Eq. (1a) describes the *plant dynamics* with state x defined in a neighbourhood X of the origin of R^n and the input u takes values in R^m subject to te effect of a disturbance represented by the vector field $p(x)\omega.$ Eq. (1b) describes an autonomous system, known as the *exosystem*, defined in a neighbourhood W of the origin of R^k, which models the class of disturbance and reference signals taken into consideration. Eq. (2) defines the error between the actual plant output $h(x) \in R^p$ and a reference signal $q(\omega),$ which models the class of disturbance and reference signals taken into consideration.

We also assume that all the constituent mappings of the system (1) and the error equation (2), namely, f, g, p, s, h and q are continuously differentiable mappings vanishing at the origin, i.e.

$$f(0) = 0,\ g(0) = 0,\ p(0) = 0,\ s(0) = 0,\ h(0) = 0 \text{ and } q(0) = 0.$$

Thus, for $u = 0$, the composite system (1) has an equilibrium $(x, \omega) = (0, 0)$ with zero error (2).

A state feedback controller for the composite system (1) has the form

$$u = \rho(x, \omega) \tag{3}$$

where ρ is a continuously differentiable mapping defined on $X \times W$ such that $\rho(0, 0) = 0.$

Upon substitution of the feedback control law (3) into (1), we get the closed-loop system

$$\begin{aligned} \dot{x} &= f(x) + g(x)\rho(x, \omega) + p(x)\omega \\ \dot{\omega} &= s(\omega) \end{aligned} \tag{4}$$

The purpose of designing the state feedback controller (3) is to achieve both *internal stability* and *output regulation* of the given nonlinear control system (1).

State Feedback Regulator Problem [2]:

Find, if possible, a state feedback control law $u = \rho(x, \omega)$ such that the following conditions are satisfied.

(OR1) [*Internal Stability*] The equilibrium $x = 0$ of the dynamics

$$\dot{x} = f(x) + g(x)\rho(x, 0) \tag{5}$$

is locally exponentially stable.

(OR2) [*Output Regulation*] There exists a neighbourhood $U \subset X \times W$ of $(x, \omega) = (0, 0)$ such that for each initial condition $(x(0), \omega(0)) \in U$, the solution $(x(t), \omega(t))$ of the closed-loop system (4) satisfies

$$\lim_{t \to \infty} \left[h(x(t)) - q(\omega(t)) \right] = 0. \qquad \blacksquare$$

Byrnes and Isidori [2] solved the output regulation problem stated above under the following two assumptions.

(H1) The exosystem dynamics $\dot{\omega} = s(\omega)$ is neutrally stable at $\omega = 0$, i.e. the exosystem is Lyapunov stable in both forward and backward time at $\omega = 0$.

(H2) The pair $\left(f(x), g(x) \right)$ has a stabilizable linear approximation at $x = 0$, i.e. if

$$A = \left[\frac{\partial f}{\partial x} \right]_{x=0} \quad \text{and} \quad B = \left[\frac{\partial g}{\partial x} \right]_{x=0},$$

then (A, B) is stabilizable. ■

Next, we recall the solution of the output regulation problem derived by Byrnes and Isidori [2].

Theorem 1. [2] *Under the hypotheses (H1) and (H2), the state feedback regulator problem is solvable if and only if there exist continuously differentiable mappings* $x = \pi(\omega)$ *with* $\pi(0) = 0$ *and* $u = \varphi(\omega)$ *with* $\varphi(0) = 0$, *both defined in a neighbourhood of* $W^0 \subset W$ *of* $\omega = 0$ *such that the following equations (called the* ***regulator equations****) are satisfied:*

(1) $\dfrac{\partial \pi}{\partial \omega} s(\omega) = f(\pi(\omega)) + g(\pi(\omega))\varphi(\omega) + p(\pi(\omega))\omega$

(2) $h\left(\pi(\omega)\right) - q(\omega) = 0$

When the regulator equations (1) and (2) are satisfied, a control law solving the state feedback regulator problem is given by

$$u = \varphi(\omega) + K\left[x - \pi\left(\omega\right) \right] \tag{6}$$

where K *is any gain matrix such that* $A + BK$ *is Hurwitz.* ■

3. Regulating the Output of the Sprott-P Chaotic System

In this section, we solve the output regulation problem for the Sprott-P system ([15], 1994). Sprott-P system is one of the paradigms of the 3-dimensional chaotic systems discovered by J.C. Sprott in 1994.

We consider the Sprott-P system described by the dynamics

$$\begin{aligned} \dot{x}_1 &= a x_2 + x_3 \\ \dot{x}_2 &= -b x_1 + x_2^2 + u \\ \dot{x}_3 &= x_1 + x_2 \end{aligned} \tag{7}$$

where x_1, x_2, x_3 are the states of the system, a, b are positive, constant parameters of the system and u is the scalar control.

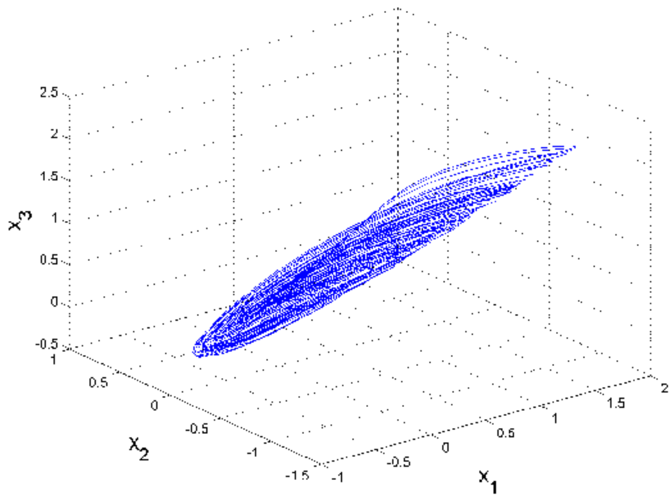

Figure 1. Strange Attractor of the Sprott-P Chaotic System

J.C. Sprott showed that the system (7) has chaotic behaviour when $a = 2.7$, $b = 1$ and $u = 0$.

The strange attractor of the Sprott-P chaotic system is illustrated in Figure 1.

In this paper, we consider the output regulation problem for the tracking of constant reference signals (*set-point signals*).

In this case, the exosystem is given by the scalar dynamics

$$\dot{\omega} = 0 \tag{8}$$

We note that the assumption (H1) of Theorem 1 holds trivially.

Linearizing the dynamics of the Sprott-P chaotic system (7) at $x = 0$, we obtain

$$A = \begin{bmatrix} 0 & a & 1 \\ -b & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{9}$$

By Kalman's rank test for controllability [14], we see that the system pair (A, B) is completely controllable. Thus, it follows that we can find a gain K matrix so that the closed-loop system matrix $A + BK$ will have stable eigenvalues $\{\lambda_1, \lambda_2, \lambda_3\}$. This calculation shows that the pair (A, B) is stabilizable.

Hence, the assumption (H2) of Theorem 1 also holds.

Hence, Theorem 1 can be applied to solve the constant regulation problem for the Sprott-P chaotic system (7).

3.1 The Constant Tracking Problem for x_1

Here, the tracking problem for the Sprott-P chaotic system (7) is given by

$$
\begin{aligned}
\dot{x}_1 &= ax_2 + x_3 \\
\dot{x}_2 &= -bx_1 + x_2^2 + u \\
\dot{x}_3 &= x_1 + x_2 \\
e &= x_1 - \omega
\end{aligned}
\tag{10}
$$

By Theorem 1, the regulator equations of the system (8) are obtained as

$$
\begin{aligned}
a\pi_2(\omega) + \pi_3(\omega) &= 0 \\
-b\pi_1(\omega) + \pi_2^2(\omega) + \varphi(\omega) &= 0 \\
\pi_1(\omega) + \pi_2(\omega) &= 0 \\
\pi_1(\omega) - \omega &= 0
\end{aligned}
\tag{11}
$$

Solving the regulator equations (11) for the system (10), we obtain the unique solution as

$$
\pi_1(\omega) = \omega,\ \ \pi_2(\omega) = -\omega,\ \ \pi_3(\omega) = a\omega,\ \ \varphi(\omega) = b\omega - \omega^2 \tag{12}
$$

Using Theorem 1 and the solution (12) of the regulator equations for the system (10), we obtain the following result which provides a solution of the output regulation problem for (10).

Theorem 2. *A state feedback control law solving the output regulation problem for the Sprott-P chaotic system (10) is given by*

$$
u = \varphi(\omega) + K\left[x - \pi(\omega)\right], \tag{13}
$$

where $\varphi(\omega),\ \pi(\omega)$ *are defined as in (10) and* K *is chosen so that* $A + BK$ *is Hurwitz.* ■

3.2 The constant Tracking Problem for x_2

Here, the tracking problem for the Sprott-P chaotic system (7) is given by

$$
\begin{aligned}
\dot{x}_1 &= ax_2 + x_3 \\
\dot{x}_2 &= -bx_1 + x_2^2 + u \\
\dot{x}_3 &= x_1 + x_2 \\
e &= x_2 - \omega
\end{aligned}
\tag{14}
$$

By Theorem 1, the regulator equations of the system (14) are obtained as

$$
\begin{aligned}
a\pi_2(\omega) + \pi_3(\omega) &= 0 \\
-b\pi_1(\omega) + \pi_2^2(\omega) + \varphi(\omega) &= 0 \\
\pi_1(\omega) + \pi_2(\omega) &= 0 \\
\pi_2(\omega) - \omega &= 0
\end{aligned}
\tag{15}
$$

Solving the regulator equations (15) for the system (14), we obtain the unique solution as

$$
\pi_1(\omega) = -\omega,\ \ \pi_2(\omega) = \omega,\ \ \pi_3(\omega) = -a\omega,\ \ \varphi(\omega) = -b\omega - \omega^2 \tag{16}
$$

Using Theorem 1 and the solution (16) of the regulator equations for the system (14), we obtain the following result which provides a solution of the output regulation problem for (14).

Theorem 3. *A state feedback control law solving the output regulation problem for the Sprott-P chaotic system (12) is given by*

$$u = \varphi(\omega) + K\left[x - \pi(\omega)\right], \tag{17}$$

where $\varphi(\omega),\ \pi(\omega)$ *are defined as in (16) and* K *is chosen so that* $A + BK$ *is Hurwitz.* ■

3.3 The Constant Tracking Problem for x_3

Here, the tracking problem for the Sprott-P chaotic system (7) is given by

$$\begin{aligned} \dot{x}_1 &= ax_2 + x_3 \\ \dot{x}_2 &= -bx_1 + x_2^2 + u \\ \dot{x}_3 &= x_1 + x_2 \\ e &= x_3 - \omega \end{aligned} \tag{18}$$

By Theorem 1, the regulator equations of the system (18) are obtained as

$$\begin{aligned} a\pi_2(\omega) + \pi_3(\omega) &= 0 \\ -b\pi_1(\omega) + \pi_2^2(\omega) + \varphi(\omega) &= 0 \\ \pi_1(\omega) + \pi_2(\omega) &= 0 \\ \pi_3(\omega) - \omega &= 0 \end{aligned} \tag{19}$$

Solving the regulator equations (19) for the system (18), we obtain the unique solution as

$$\pi_1(\omega) = \frac{\omega}{a},\ \pi_2(\omega) = -\frac{\omega}{a},\ \pi_3(\omega) = \omega,\ \varphi(\omega) = \frac{\omega}{a^2}(ab - \omega) \tag{20}$$

Using Theorem 1 and the solution (20) of the regulator equations for the system (18), we obtain the following result which provides a solution of the output regulation problem for (18).

Theorem 4. *A state feedback control law solving the output regulation problem for the Sprott-P chaotic system (18) is given by*

$$u = \varphi(\omega) + K\left[x - \pi(\omega)\right], \tag{21}$$

where $\varphi(\omega),\ \pi(\omega)$ *are defined as in (20) and* K *is chosen so that* $A + BK$ *is Hurwitz.* ■

4. NUMERICAL SIMULATIONS

For the numerical simulations, the fourth order Runge-Kutta method with step-size $h = 10^{-8}$ is deployed to solve the systems of differential equations using MATLAB.

For simulation, the parameters are chosen as the chaotic case of the Sprott-P system, *viz.*

$$a = 2.7 \text{ and } b = 1$$

For achieving internal stability of the state feedback regulator problem, a feedback gain matrix K is chosen so that $A + BK$ is Hurwitz with the stable eigenvalues

$$\lambda_1 = -4,\ \lambda_2 = -4,\ \lambda_3 = -4$$

Using Ackermann's formula [16], we find that

$$K = [-7.9507 \quad -12 \quad -24.8331].$$

In the following, we describe the simulations for the following three cases:

(A) Constant tracking problem for x_1

(B) Constant tracking problem for x_2

(C) Constant tracking problem for x_3

4.1 Constant Tracking Problem for x_1

Here, the initial conditions are taken as $x_1(0) = 9,\ x_2(0) = -6,\ x_3(0) = 7$ and $\omega = 2.$

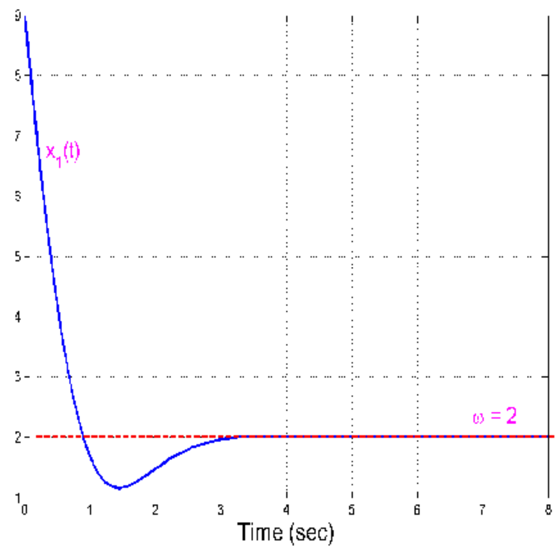

Figure 2. Constant Tracking Problem for x_1

The simulation graph is depicted in Figure 2 from which it is clear that the state trajectory $x_1(t)$ tracks the constant reference signal $\omega = 2$ in 4 seconds.

4.2 Constant Tracking Problem for x_2

Here, the initial conditions are taken as $x_1(0) = 5,\ x_2(0) = 9,\ x_3(0) = 4$ and $\omega = 2.$

The simulation graph is depicted in Figure 3 from which it is clear that the state trajectory $x_2(t)$ tracks the constant reference signal $\omega = 2$ in 4 seconds.

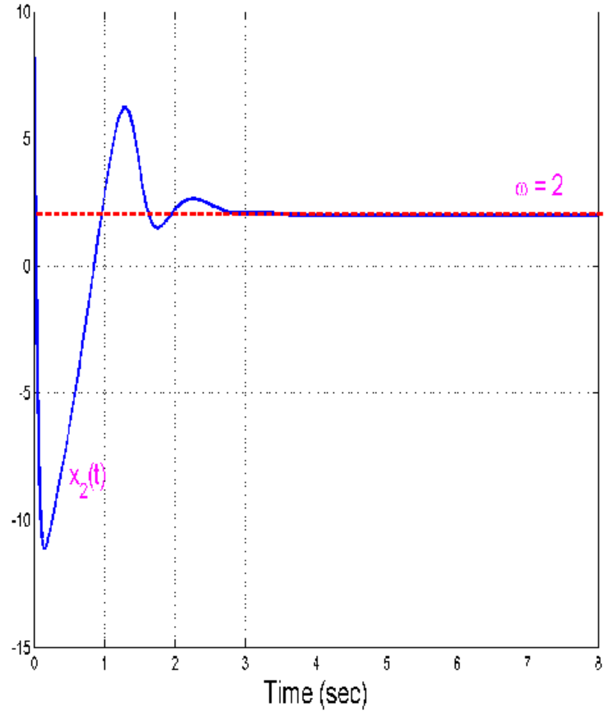

Figure 3. Constant Tracking Problem for x_2

4.3 Constant Tracking Problem for x_3

Here, the initial conditions are taken as $x_1(0) = 7,\ \ x_2(0) = -8,\ x_3(0) = -5$ and $\omega = 2.$

The simulation graph is depicted in Figure 4 from which it is clear that the state trajectory $x_3(t)$ tracks the constant reference signal $\omega = 2$ in 4 seconds.

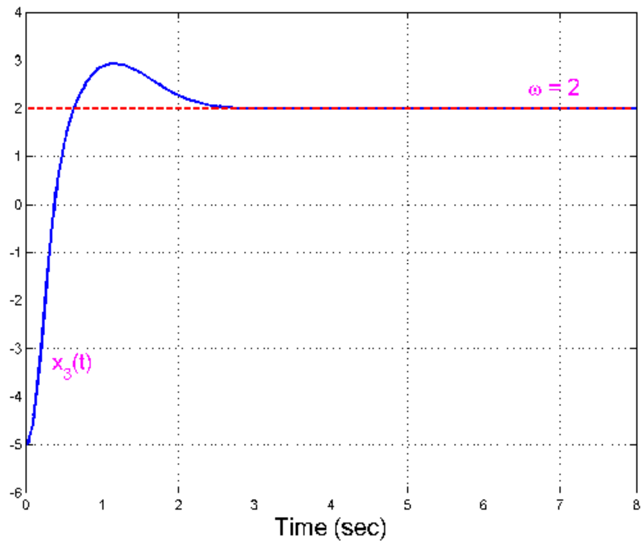

Figure 4. Constant Tracking Problem for x_3

5. CONCLUSIONS

In this paper, active controller has been designed to solve the output regulation problem for the Sprott-P chaotic system (1994) and a complete solution for the tracking of constant reference signals (*set-point signals*). The state feedback control laws achieving output regulation proposed in this paper were derived using the *regulator equations* of Byrnes and Isidori (1990). Numerical simulation results were presented in detail to illustrate the effectiveness of the proposed control schemes for the output regulation problem of Sprott-P chaotic system to track constant reference signals.

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