Synchronization and Inverse Synchronization of Some Different Dimensional Discrete-time Chaotic Dynamical Systems via Scaling Matrices

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Abstract

In this paper, new types of synchronization and inverse synchro-nization are proposed for some di¤erent dimensional chaotic dynamical systems in discrete-time using scaling matrices. Based on Lyapunov stability theory and nonlinear controllers, new synchronization results are derived. Numerical simulations are used to verify the e¤ectiveness of the proposed schemes.

Keyword

Synchronization, inverse synchronization, chaotic dynamical sys-tems, discrete-time, Lyapunov stability

1.Introduction

Dynamical systems in discrete-time play an important role in chaos theory and mathematical modelisation of many scienti.c problems [1, 2, 3, 4]. Re-cently, more and more attention has been paid to the synchronization of chaos(hyperchaos) in discrete-time dynamical systems, due it.s applications in se- cure communication and cryptology [5, 6]. Many synchronization types have been found [7, 8, 9] and di¤erent methods are used to study synchronization of discrete-time chaotic systems [10, 11, 12].

In this paper, the proplems of synchronization with scaling matrix and it.s inverse type are studied between drive-response chaotic systems in discrete-time. Based on Lyapunov stability theory, we would like to present a con-structive schemes to investigate synchronization and inverse synchronization between some typical chaotic dynamical systems with respect to scaling matri-ces in discrete-time with di¤erent dimensions. Because in real world all chaotic maps are described by plane equations or space systems, we restrict our study about the new chaos synchronization types to 2D and 3D discrete chaotic sys-tems and this restriction does .n lose the generality of our main results. Firstly, anew schemes are proposed to study synchronization and inverse synchronization between the drive 2D Lorenz discrete-time system and the response 3D Wang map. Secondly, the 3D generalized Hénon map is considered as the drive system and the controlled Fold map as the response system to achieve synchronization and inverse synchronizationThe remainder of this paper is organized as follows. In Section 2, de.n-itions of synchronization and inverse synchronization for discrete systems via scaling matrices are introduced. In section 3; synchronization and inverse syn-chronization are applied to 2D drive system and 3D response system and new synchronization results are derived. In Section 4, synchronization and inverse synchronization are studied between 3D drive system and 2D response system.Finally, the paper is concluded in Section 5.

2.Definitions of synchronization and inverse syn-chronization via scaling matrices

Consider the following drive chaotic system described by

Consider the following drive chaotic system described by

$$X(k+1) = f(X(k)), \tag{1}$$

where $X(k) = (x_1(k), ..., x_n(k))^T \in \mathbb{R}^n$ is the state vector of the drive system and $f : \mathbb{R}^n \to \mathbb{R}^n$. As the response system, we consider the following chaotic system described by

$$Y(k+1) = g(Y(k)) + U, \tag{2}$$

where $Y(t) = (y_1(k), ..., y_m(k))^T \in \mathbb{R}^m$ is the state vector of the response system, $g : \mathbb{R}^m \to \mathbb{R}^m$ and $U = (u_i)_{1 \le i \le m} \in \mathbb{R}^m$ is the vector controller to be determined.

We present the definition of synchronization via scaling matrix for coupled chaotic systems given in Eqs. (1) and (2).

Definition 1 *The drive system (1) and the rsponse system (2) are said to be synchronized, with respect to the scaling matrix* Λ, *if there exists a controller* $U = (u_i)_{1 \le i \le m} \in \mathbb{R}^m$ *and a given matrix* Λ, $m \times n$, *such that the synchronization error*

$$e(k) = Y(k) - \Lambda X(k), \tag{3}$$

satisfies that $\lim_{k \longrightarrow +\infty} \|e(k)\| = 0$.

The definition of inverse synchronization via scaling matrix for coupled chaotic systems given in Eqs. (1) and (2).is given next.

Definition 2 *The drive system (1) and the response system (2) are said to be inverse synchronized, with respect to the scaling matrix* θ, *if there exists a controller* $U = (u_i)_{1 \le i \le m} \in \mathbb{R}^m$ *and a given matrix* θ, $n \times m$, *such that the synchronization error*

$$e(k) = X(k) - \theta Y(k), \tag{4}$$

satisfies that $\lim_{k \longrightarrow +\infty} \|e(k)\| = 0$.

3.Synchronization and inverse synchronization of 2D drive system and 3D response system

In this section, we consider Lorenz discrete-time system and as the drive system the controlled Wang system. Lorenz discrete-time system can be described as

$$\begin{cases} x_1(k+1) = (1+ab)\,x_1(k) - bx_1(k)\,x_2(k), \\ x_2(k+1) = (1-b)\,x_2(k) + bx_1^2(k), \end{cases} \quad (5)$$

which has a chaotic attractor, for example, when $(a, b) = (1.25,\ 0.75)$ [12]. The Lorenz discrete-time chaotic attractor is shown in Fig. 1.

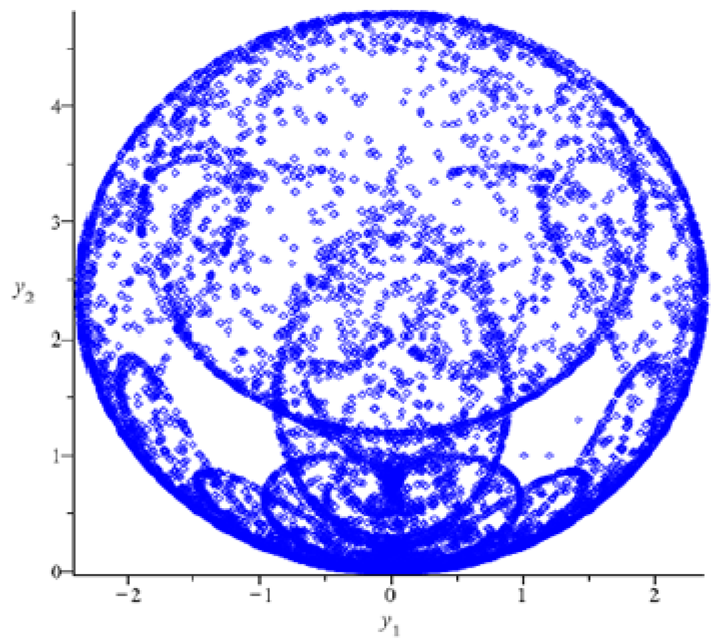

Fig. 1 The chaotic attractor of Lorenz discrete-time system.

The controlled Wang system can be described as

$$\begin{cases} y_1(k+1) = a_3\delta y_2(k) + (a_4\delta + 1)\,y_1(k) + u_1, \\ y_2(k+1) = a_1\delta y_1(k) + y_2(k) + a_2\delta y_3(k) + u_2, \\ y_3(k+1) = (a_7\delta + 1)\,y_3(k) + a_6\delta y_2(k)\,y_3(k) + a_5\delta + u_3, \end{cases} \quad (6)$$

where $U = (u_1,\ u_2,\ u_3)^T$ is the vector controller. The Wang discrete-time system has a chaotic attractor, for example, when $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, \delta) = (-1.9, 0.2, 0.5, -2.3, 2, -0.6, -1.9, 1)$ [12] . the chaotic attractor of Wang discrete-

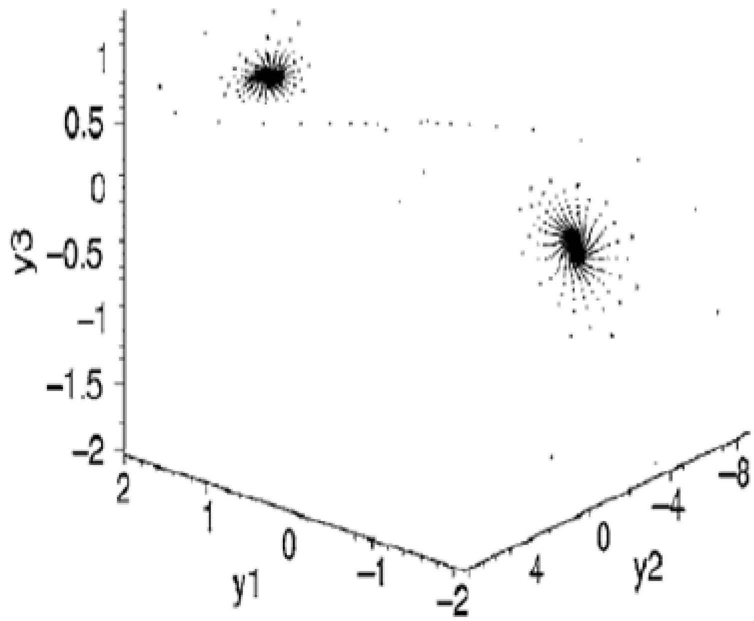

Fig. 2 The chaotic attractor of Wang discrete-time system .

3.1.Synchronization of Lorenz discrete-time system and Wang system

According to de ntion 1, the synchronization errors between the drive system (5) and the response system (6) can be derived as

$$\begin{cases} e_1(k+1) = (a_4\delta + 1 - l_1)\, e_1(k) + L_2 + N_1 + u_1, \\ e_2(k+1) = e_2(k) + L_2 + N_2 + u_2, \\ e_3(k+1) = (a_7\delta + 1 - l_2)\, e_3(k) + L_3 + N_3 + u_3, \end{cases} \tag{7}$$

where l_1, l_2 are real control constants to be detrmined,

$$\begin{cases} N_1 = \Lambda_{11} b x_1(k)\, x_2(k) - \Lambda_{12} b x_1^2(k), \\ N_2 = \Lambda_{21} b x_1(k)\, x_2(k) - \Lambda_{22} b x_1^2(k), \\ N_3 = a_6\delta y_2(k)\, y_3(k) + a_7\delta + \Lambda_{31} b x_1(k)\, x_2(k) - \Lambda_{32} b x_1^2(k), \end{cases} \tag{8}$$

$$\begin{cases} L_1 = l_1 y_1(k) + a_3\delta y_2(k) + \sum_{j=1}^{2} \omega_{1j} x_j(k), \\ L_2 = a_1\delta y_1(k) + a_3\delta y_3(k) + \sum_{j=1}^{2} \omega_{2j} x_j(k), \\ L_3 = \sum_{j=1}^{2} \omega_{3j} x_j(k), \end{cases} \tag{9}$$

$$\begin{cases} \omega_{11} = \Lambda_{11}\left(a_4\delta - ab - l_1\right), \\ \omega_{12} = \Lambda_{12}\left(a_4\delta + b - l_1\right), \\ \omega_{21} = -\Lambda_{21}ab, \\ \omega_{22} = \Lambda_{22}b, \\ \omega_{31} = \Lambda_{31}\left(a_7\delta - ab - l_2\right), \\ \omega_{32} = \Lambda_{32}\left(a_7\delta + b - l_2\right), \end{cases} \tag{10}$$

and $\Lambda = (\Lambda_{ij}) \in \mathbb{R}^{3\times 2}$ is the scaling matrix.

Theorem 3 *If l_1 and l_2 are chosen such that*

$$|a_4\delta + 1 - l_1| < 1 \text{ and } |a_7\delta + 1 - l_2| < 1. \tag{11}$$

Then, the drive system (5) and the response system (6) are globally synchronized, with respect to the arbitrary scaling matrix Λ, under the following controllers

$$u_i = -L_i - N_i, \quad 1 \leq i \leq 3. \tag{12}$$

Proof. By substituting the control law (12) into (7), the synchronization error can be written as

$$\begin{cases} e_1\left(k+1\right) = \left(a_4\delta + 1 - l_1\right)e_1\left(k\right). \\ e_2\left(k+1\right) = e_2\left(k\right). \\ e_3\left(k+1\right) = \left(a_7\delta + 1 - l_3\right)e_3\left(k\right). \end{cases} \tag{13}$$

We take as a candidate Lyapunov function:

$$V\left(e\left(k\right)\right) = \sum_{j=1}^{3} e_j^2\left(k\right), \tag{14}$$

we get:

$$\begin{aligned} \Delta V\left(e(k)\right) &= V\left(e(k+1)\right) - V\left(e(k)\right) \\ &= \sum_{j=1}^{3} e_i^2\left(k+1\right) - \sum_{j=1}^{3} e_i^2\left(k\right) \\ &= \left(\left(a_4\delta + 1 - l_1\right)^2 - 1\right)e_1^2\left(k\right) + \left(\left(a_7\delta + 1 - l_2\right)^2 - 1\right)e_3^2\left(k\right), \end{aligned}$$

and by using (11), we obtain: $\Delta V\left(e\left(k\right)\right) < 0$. Thus, by Lyapunov stability it is immediate that $\lim_{k\to\infty} e_i(k) = 0$, $(i = 1, 2)$, and from the fact $\lim_{k\to\infty} \|e(k)\| = 0$. We conclude that the systems (5) and (6) are globally generalized synchronized. ■

The error functions evolution are shown in Fig. 3.

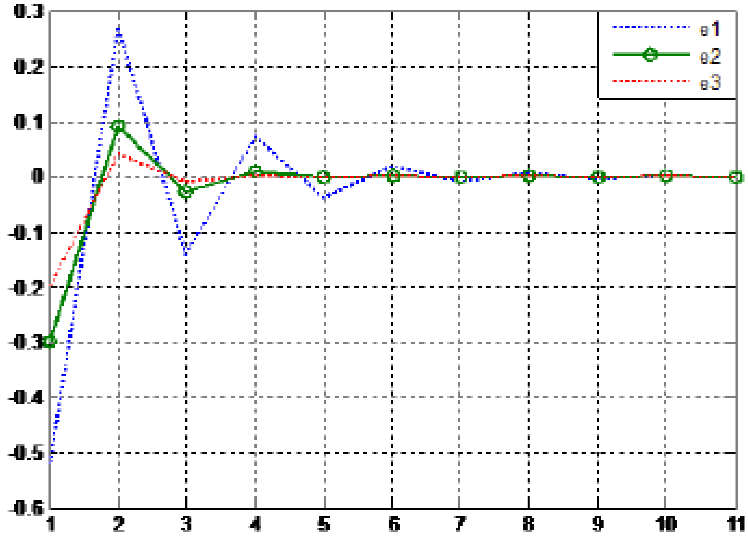

Fig. 3 Time evolution of synchronization errors between the drive Lorenz discrete-time system and the response Wang system.

3.2.Inverse synchronization between Lorenz discrete-time system and Wang system

According to de   ntion 2, the synchronization errors between the drive system (5) and the response system (6)), can be derived as

$$\begin{cases} e_1(k+1) = (1+ab-l_1)\,e_1(k) + R_1 - \sum_{j=1}^{3}\theta_{1j}u_j, \\ e_2(k+1) = (1-b-l_2)\,e_2(k) + R_2 - \sum_{j=1}^{3}\theta_{2j}u_j, \end{cases} \tag{15}$$

where l_1, l_2 are real constants to be detrmined,

$$R_i = L_i + N_i, \quad i = 1, 2, \tag{16}$$

where

$$\begin{cases} N_1 = -bx_1(k)\,x_2(k) - \theta_{13}a_6\delta y_2(k)\,y_3(k) - \theta_{13}a_5\delta, \\ N_2 = bx_1^2(k) - \theta_{23}a_6\delta y_2(k)\,y_3(k) - \theta_{23}a_5\delta, \end{cases} \tag{17}$$

$$\begin{cases} L_1 = \sum_{j=1}^{3}\omega_{1j}y_j(k), \\ L_2 = \sum_{j=1}^{3}\omega_{2j}y_j(k), \end{cases} \tag{18}$$

where

$$\begin{cases} \omega_{11} = \theta_{11}(ab - a_4\delta - l_1) - \theta_{12}a_1\delta, \\ \omega_{12} = \theta_{12}(ab - l_1) + \theta_{11}a_3\delta, \\ \omega_{13} = \theta_{13}(ab - a_7\delta - l_1) + \theta_{12}a_2\delta, \\ \omega_{21} = -\theta_{21}(a_4\delta + b + l_2) - \theta_{22}a_1\delta, \\ \omega_{22} = -\theta_{22}(b + l_2) + \theta_{21}a_3\delta, \\ \omega_{23} = -\theta_{23}(a_7\delta + b + l_2) + \theta_{22}a_2\delta, \end{cases} \tag{19}$$

and $\theta = (\theta_{ij}) \in \mathbb{R}^{2\times 3}$ is the scaling matrix.

To achieve synchonization between the drive system (5) and the response system (6), we assume that

$$\theta_{11}\theta_{22} \neq \theta_{12}\theta_{21}, \tag{20}$$

and we choose the controllers u_i, $(1 \leq i \leq 3)$, as follow

$$u_1 = \frac{\theta_{12}R_2 - \theta_{22}R_1}{\theta_{11}\theta_{22} - \theta_{12}\theta_{21}},\ u_2 = \frac{\theta_{21}R_1 - \theta_{11}R_2}{\theta_{11}\theta_{22} - \theta_{12}\theta_{21}} \text{ and } u_3 = 0. \tag{21}$$

Theorem 4 *If l_1 and l_2 are chosen such that*

$$|1 + ab - l_1| < 1 \text{ and } |1 - b - l_2| < 1. \tag{22}$$

Then, the drive system (5) and the response system (6) are globally inverse synchronized,.with respect to the scaling matrix θ which verifies (20), under the controllers.(21).

Proof. By substituting the controllers (21) into (15), the synchronization errors can be written as

$$\begin{cases} e_1(k+1) = (1 + ab - l_1)\, e_1(k). \\ e_2(k+1) = (1 - b - l_2)\, e_2(k). \end{cases} \tag{23}$$

We take as a candidate Lyapunov function:

$$V(e(k)) = \sum_{j=1}^{2} e_j^2(k), \tag{24}$$

we get:

$$\begin{aligned} \Delta V(e(k)) &= V(e(k+1)) - V(e(k)) \\ &= \sum_{j=1}^{2} e_i^2(k+1) - \sum_{j=1}^{2} e_i^2(k) \\ &= \left((1 + ab - l_1)^2 - 1\right) e_1^2(k) + \left((1 - b - l_2)^2 - 1\right) e_1^2(k), \end{aligned}$$

and by using (22), we obtain: $\Delta V(e(k)) < 0$. Thus, by Lyapunov stability it is immediate that $\lim_{k\to\infty} e_i(k) = 0$, $(i = 1, 2)$, and from the fact $\lim_{k\to\infty} \|e(k)\| =$

0. We conclude that the systems (5) and (6) are globally inverse synchronized. ∎

The error functions evolution are shown in Fig. 4.

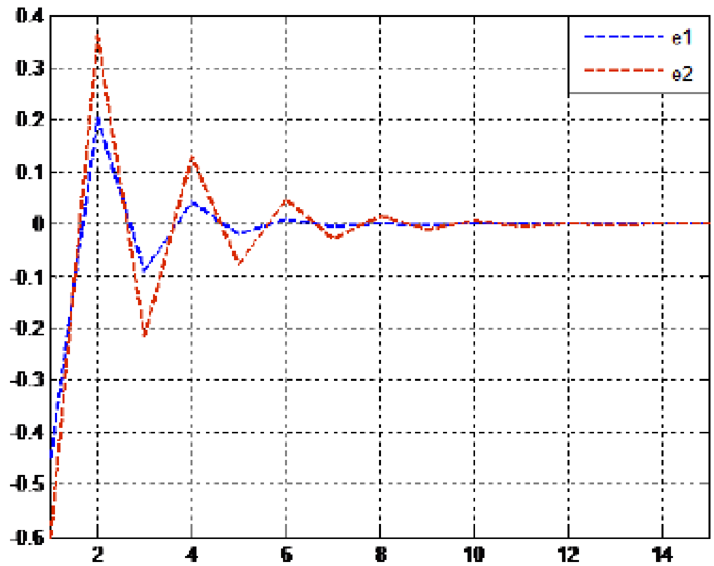

Fig. 4 Time evolution of inverse synchronization errors between the drive Lorenz discrete-time system and the response Wang system.

4.Synchronization and inverse synchronization of 3D drive system and 2D response system

Now, we consider 3D generalized Hénon map as the drive system and the con- trolled Fold map as the response system. The 3D generalized Hénon map can be described as

$$\begin{cases} x_1(k+1) = -\beta x_2(k), \\ x_2(k+1) = x_3(k) + 1 - \alpha x_2^2(k), \\ x_3(k+1) = x_1(k) + \beta x_2(k), \end{cases} \tag{25}$$

which has a chaotic attractor, for example, when $(\alpha,\ \beta) = (1.07,\ 0.3)$ [11]. The 3D generalized Hénon chaotic attractor is shown in Fig. 4.

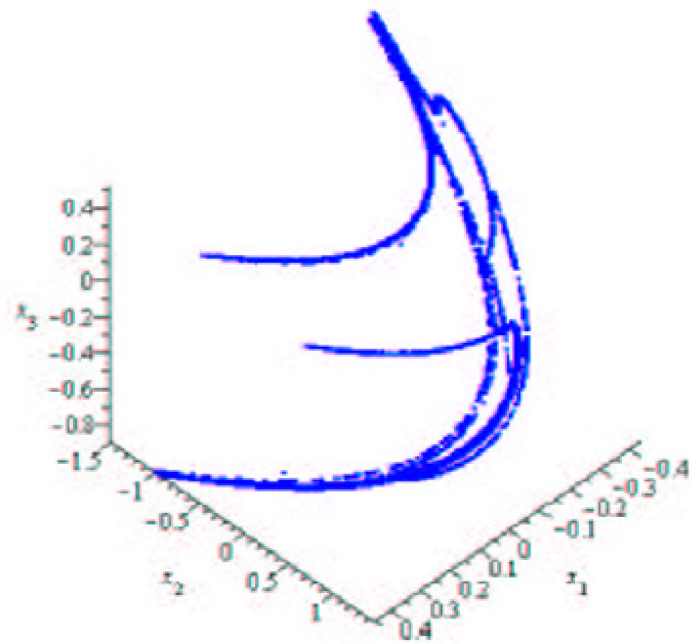

Fig. 5 The chaotic attractor of 3D generalized Henon.

The controlled Fold map can be described as

$$\begin{cases} y_1(k+1) = ay_1(k) + y_2(k) + u_1, \\ y_2(k+1) = y_1^2(k) + b + u_2, \end{cases} \tag{26}$$

where $U = (u_1,\ u_2)^T$ is the vector controller. The Fold map has a chaotic attractor, for example, when $(a, b) = (-0.1, -1.7)$ [12] . the chaotic attractor of

Fold map is shown in Fig. 6.

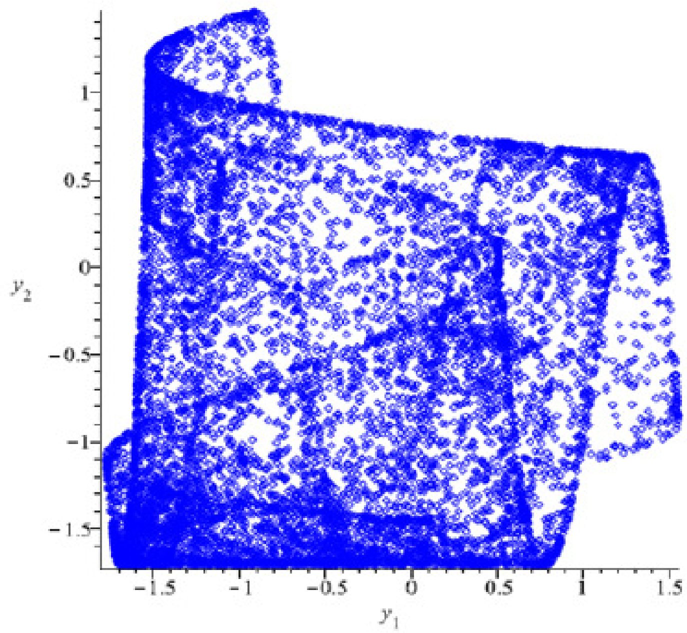

Fig. 6 The chaotic attractor of Fold map.

4.1.Synchronization between 3D generalized Hénon map and Fold map

According to de ntion 1, the synchronization errors between systems (25) and (26), can be derived as

$$\begin{cases} e_1(k+1) = (a - l_1)e_1(k) + N_i + L_i + u_1, \\ e_2(k+1) = e_2(k) + N_i + L_i + u_2, \end{cases} \tag{27}$$

where l is a real constant to be detrmined,

$$\begin{cases} N_1 = -\Lambda_{12}\left(1 - \alpha x_2^2(k)\right), \\ N_2 = y_1^2(k) + b - \Lambda_{22}\left(1 - \alpha x_2^2(k)\right), \end{cases} \tag{28}$$

$$\begin{cases} L_1 = y_2(k) + \sum_{j=1}^{3} \gamma_{1j} x_j(k), \\ L_2 = -y_2(k) + \sum_{j=1}^{3} \gamma_{2j} x_j(k), \end{cases} \tag{29}$$

where

$$\begin{cases} \gamma_{11} = \Lambda_{11}\left(a - l_1\right) - \Lambda_{13}, \\ \gamma_{12} = \Lambda_{12}\left(a + \beta - l_1\right) - \Lambda_{13}\beta, \\ \gamma_{13} = \Lambda_{13}\left(a - l_1\right) - \Lambda_{12}, \\ \gamma_{21} = \Lambda_{21} - \Lambda_{23}, \\ \gamma_{22} = \Lambda_{22} + \Lambda_{21}\beta - \Lambda_{23}\beta, \\ \gamma_{23} = \Lambda_{23} - \Lambda_{22}, \end{cases} \tag{30}$$

and $\Lambda = (\Lambda_{ij}) \in \mathbb{R}^{2\times 3}$ is the scaling matrix.

Theorem 5 *If the control constant l is chosen such that*

$$|b - l| < 1. \tag{31}$$

Then, the drive system (25) and the response system (26) are globally synchronized,.respect to the arbitrary scaling matrix Λ,.under the following controllers

$$u_i = -N_i - L_i, \quad 1 \leq i \leq 2. \tag{32}$$

Proof. By substituting Eq. (32) into (27), the synchronization errors can be written as

$$\begin{cases} e_1\left(k+1\right) = \left(a - l\right) e_1\left(k\right). \\ e_2\left(k+1\right) = e_2\left(k\right). \end{cases} \tag{33}$$

We take as a candidate Lyapunov function:

$$V\left(e\left(k\right)\right) = \sum_{j=1}^{2} e_j^2\left(k\right), \tag{34}$$

we get:

$$\begin{aligned} \Delta V\left(e(k)\right) &= V\left(e(k+1)\right) - V\left(e(k)\right) \\ &= \sum_{j=1}^{2} e_i^2\left(k+1\right) - \sum_{j=1}^{2} e_i^2\left(k\right) \\ &= \left(\left(b - l\right)^2 - 1\right) e_1^2\left(k\right), \end{aligned}$$

and by using (31), we obtain: $\Delta V\left(e\left(k\right)\right) < 0$. Thus, by Lyapunov stability it is immediate that $\lim_{k\to\infty} e_i(k) = 0$, $(i = 1, 2)$, and from the fact $\lim_{k\to\infty} \|e(k)\| = 0$. We conclude that the systems (25) and (26) are globally synchronized. ■

We get the numeric result that is shown in Fig. 7.

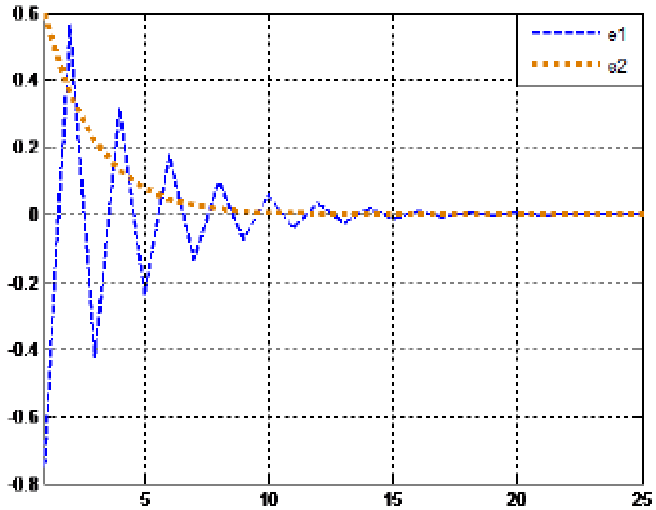

Fig. 7 Time evolution of synchronization errors between the drive 3D generalized Henon map and the response Fold map.

4.2.Inverse synchronization between 3D generalized Hénon map and Fold map

In this case, the synchronization errors between the drive system (25) and the response system (26) can be derived as

$$\begin{cases} e_1(k+1) = (-\beta + l)\, e_2(k) + R_1 - \sum_{j=1}^{2} \theta_{1j} u_j, \\ e_2(k+1) = e_3(k) + R_2 - \sum_{j=1}^{2} \theta_{2j} u_j, \\ e_3(k+1) = e_1(k) + R_3 - \sum_{j=1}^{2} \theta_{3j} u_j, \end{cases} \tag{35}$$

where l is a real control constant to be determined,

$$R_i = N_i + L_i, \quad i = 1, 2, 3, \tag{36}$$

where

$$\begin{cases} N_1 = -\theta_{12}\left(y_1^2(k) + b\right), \\ N_2 = 1 - \alpha x_2^2(k) - \theta_{22}\left(y_1^2(k) + b\right), \\ N_3 = -\theta_{32}\left(y_1^2(k) + b\right), \end{cases} \tag{37}$$

$$\begin{cases} L_1 = \sum_{j=1}^{3} \mu_{3j} x_j(k), \\ L_2 = \sum_{j=1}^{3} \mu_{3j} x_j(k), \\ L_3 = \beta x_2(k) + \sum_{j=1}^{3} \mu_{3j} x_j(k), \end{cases} \tag{38}$$

where

$$\begin{cases} \mu_{11} = \theta_{11}(l - \beta - a), \\ \mu_{12} = (-\beta + l)\,\theta_{12} - \theta_{11}, \\ \mu_{21} = \theta_{31} - \theta_{21} a, \\ \mu_{22} = \theta_{32} - \theta_{21}, \\ \mu_{31} = \theta_{11} - \theta_{31} a, \\ \mu_{32} = \theta_{12} - \theta_{31}, \end{cases} \tag{39}$$

and $\theta = (\theta_{ij}) \in \mathbb{R}^{2\times 3}$ is the scaling matrix.

To achieve synchonization between the drive system (25) and the response system (26), we assume that

$$\theta_{12}\theta_{21}\theta_{31} - 2\theta_{11}\theta_{31}\theta_{22} + \theta_{11}\theta_{21}\theta_{32} \neq 0, \tag{40}$$

and we choose the controllers u_i, $(1 \leq i \leq 2)$, as follow

$$u_1 = \frac{-\theta_{12}\theta_{31}R_1 + 2\theta_{22}\theta_{31}R_2 - \theta_{21}\theta_{32}R_3}{\theta_{12}\theta_{21}\theta_{31} - 2\theta_{11}\theta_{31}\theta_{22} + \theta_{11}\theta_{21}\theta_{32}}, \tag{41}$$

and

$$u_2 = \frac{-\theta_{21}\theta_{31}R_1 + 2\theta_{11}\theta_{31}R_2 - \theta_{11}\theta_{21}R_3}{\theta_{12}\theta_{21}\theta_{31} - 2\theta_{11}\theta_{31}\theta_{22} + \theta_{11}\theta_{21}\theta_{32}}. \tag{42}$$

Theorem 6 *If the control constant l is chosen such that*

$$|\beta - l| < 1. \tag{43}$$

Then, the drive system (25) and the response system (26) are globally inverse synchronized,.with respect to the scaling matrix Λ wich verifies (40), under the control laws (41) and (42).

Proof. By substituting Eqs. (41) and (42) into (35), the synchronization errors can be written as

$$\begin{cases} e_1(k+1) = (-\beta + l)\, e_2(k). \\ e_2(k+1) = e_3(k). \\ e_3(k+1) = e_1(k). \end{cases} \tag{44}$$

We take as a candidate Lyapunov function:

$$V(e(k)) = \sum_{j=1}^{3} e_j^2(k), \tag{45}$$

we get:

$$\begin{aligned} \Delta V(e(k)) &= V(e(k+1)) - V(e(k)) \\ &= \sum_{j=1}^{3} e_i^2(k+1) - \sum_{j=1}^{3} e_i^2(k) \\ &= \left((\beta - l)^2 - 1\right) e_2^2(k) \end{aligned}$$

and by using (41), we obtain: $\Delta V(e(k)) < 0$. Thus, by Lyapunov stability it is immediate that $\lim_{k\to\infty} e_i(k) = 0$, $(i = 1, 2)$, and from the fact $\lim_{k\to\infty} \|e(k)\| = 0$. We conclude that the systems (25) and (26) are globally inverse generalized synchronized. ■

Finally, we get the numeric result that is shown in Fig. 8.

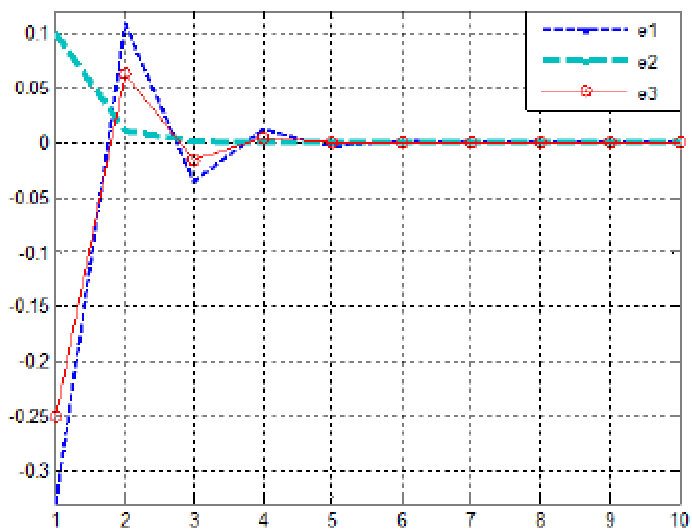

Fig. 8 Time evolution of inverse synchronization errors between the drive 3D generalized Henon map and the response Fold map.

5.Conclusion

In this paper, we analysed the synchronization and the inverse synchronization problems using scaling matrices for some typical di¤erent dimensional chaotic systems in discrete-time. A new control schemes are derived and new synchro-nization controllers are proposed. Numerical simulations are used to verify the e¤ectiveness of the derived results.

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