LEAD TIME REDUCTION IN AN INTEGRATED INVENTORY MODEL FOR NON-DEFECTIVE ITEMS UNDER A SUPPLY CHAIN SYSTEM

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ABSTRACT

A supply Chain (SC) with single vendor and single buyer is considered. In this research is presents lead time reduction system in an integrated inventory model for non-defective items in order to minimizing the sum of the ordering cost/setup cost, holding cost, transportation cost, production cost and lead time crashing cost of simultaneously optimizing the optimal order quantity, lead time and number of deliveries. The major role of this model is an efficient iterative algorithm has been developed to minimize the integrated total cost for the single vendor and the single buyer integrated system.

Furthermore, a numerical example and sensitivity analysis is presented to illustrate the chief issues related to the proposed models. Lead time is a chief part in any inventory management system. In numerous realistic circumstances, lead time can be concentrated by a bonus crashing cost, that is, lead time is controllable. At this point, the buyer lead time can be reduced by paying a bonus crashing cost which is measured as a piecewise linear function. Graphical representation is also presented to illustrate the proposed model.

Lastly, a statistical example will be offered to express the integrated inventory model and solution method and provide administrator a useful decision discussion. Sensitivity analyses are used to exemplify the decision-making insights of the model.

KEYWORDS

Inventory control, integrated inventory model, lead time crashing cost, transportation cost, production cost, supply chain management.

SUBJECT CLASSIFICATION CODE

90B05, 90C25, 90C30.

1. INTRODUCTION

Operations Research (OR), which began as an interdisciplinary activity to solve complex problems in the military during World War II, has grown in the past 50 years to a full-fledged academic discipline. At the present OR is viewed as a body of established mathematical models and methods to solve complex management problems. OR provides a quantitative analysis of the problem from which the management can make an objective decision. Inventory is the main stream of OR. The problem of inventory is one of the most important in organizational management. Inventory is a stock or store of goods or services, kept for use or sale in the future. The most main goal or inventory control is to determine and keep up an optimum level of investment in the inventory.
Generally businesses have now productively installed one or the other system of inventory preparation and organize. The inventory control models range from very simple methods to extremely complicated arithmetical inventory models. Inventory control is such a significant piece of an organization’s operations and foundation queue that it is too imperative to leave to person error or old-fashioned systems. That’s why so many companies opt to invest in inventory control systems, so that all the components of inventory control are managed by one integrated system. The term of Inventory Control (IC) is used to cover functions which are quite different and are related to one another only in that they both required the maintenance of adequate records of inventory and receipt and issue corresponding to these two functions. It is interpreted to these two functions. It is interpreted as accounting control operating control.

The problem of IC is one of the most important in organizational management. Because a rule, there is no normal solution the conditions at each corporation or solid are sole and include many skin texture and limitations. A going on task of the arithmetic models growth and influential the optimal inventory control plan is associated with this problem. Features of inventory management models are that the resulting optimal solutions can be implemented in speedy altering circumstances where, for example, the conditions are changed daily. There is a need for new and efficient methods for modelling systems associated with inventory management, in the face of doubt. Uncertainty exists regarding the control object, as the process of obtaining the required information about the object is not forever probable. The answer of such intricate tasks requires the use of systems analysis, growth of a systematic approach to the problem of administration in general. Inventory models are famed by the assumptions made about the key variables: demand, the cost structure, physical characteristics of the system. The aim of holding inventories is to agree to the solid to part the procedure of purchasing, manufacturing, and marketing of its main products. Inventories are a module of the firm’s working capital and as such correspond to a current explanation. Inventories are also viewed as a basis of near all cash. The reason is to attain inefficiencies in areas where costs are concerned. The technical inventory manages results in the decrease of stocks on the one hand and considerable refuse in serious shortages on the other. Hot researches have shown the consequence of civilizing the supply chain competitiveness by means of premeditated alliances.

The majority of the actions in use by one SC associate affect other members as well. One SC member may make decisions associated with production planning, inventory control, replenishment strategy, advertising, and purchase strategy and so on. Approximately all of these decisions can power other associates. There is a lack of system view in the field of supply chain management (Agrell & Hatami-Marbini [1]). A Supply Chain (SC) is lively systems that comprise all activities mixed up in delivering a creation from the period of raw material to the purchaser. A SC is a complex network consisting of multiple interdependent members having different aims and objectives that often conflict with each other (Hou et al. [19]). Then, supply chain coordination is indispensable for well-organized and efficiently running big business processes and increasing commercial achievement. Freshly, integrated vendor–buyer models have become extra important because both the vendor and buyer can enlarge their mutual benefits by establishing a long-term strategic partnership and by coordinating production and order quantities. Most integrated inventory models have been developed based on the assumption that the items received by the buyer from the vendor are of perfect quality. These activities include manufacturing, inventory control, distribution, warehousing, and customer service. Supply chain management coordinates and integrates all of these activities into a smooth process. The main objective of the supply chain management is to minimize system-wide costs while satisfying service-level requirements. With increasing global competition and emergence of e-business, supply chain management is viewed as a major solution to cost reduction and profitability [39]. Stock has been measured one of the most important drivers of the supply chain and its supervision [9]. There is nil more main within the land of supply chain management than the
Management of inventory [23]. As such, decisions regarding inventory replacement have a straight effect on supply chain act.

Sustainable supply chains indicate the relation among the supply chain players is forever to obtain maximum profit together and individual profit. Many researchers studied supply chain without its sustainability. The aim of this model is to make sustainable supply chain to reduce the total supply chain cost. Supply chain management (SCM) is the collaboration among suppliers, manufacturers, retailers, and customers. Practically, the aim of the SCM model is to minimize the total cost or to maximize the total profit throughout the channel. In this direction, the idea of integrated vendor-buyer inventory management has successfully considered since last few decades. In a few sensible states of affairs, lead time and setup cost can be controlled and concentrated in diverse ways. It’s a trend by decrease the lead time and reducing setup cost; the safety stock can be minimized. Thus, it is the target always to decrease the stockout loss, and improve the service level for the customer as to increase the competitive edge in business within the sustainable SCM environment. Thus, the controllable lead time and setup cost reduction are the key concepts to obtain successful business and have attracted extensive research attention [40]. Reduced setup in the basic inventory model was investigated by Porteus [34] which is the key research idea for cost reduction policy in a sustainable supply chain. In addition, the objective of effective supply chain management is the reduction of costs, improvement of cash flow and increased operational efficiency across the entire business through connecting inventory control, purchasing coordination and sales order processing with market demand [11].

In this competitive marketing environment, one of the ways to attract customers is to allow the shortest lead time and quick services. Lead time is the beyond point between insertion the command and delivery it. If a manufacturing can deliver goods within very short duration comparing to others, then it may obtain another chance to receive future order. Hence, lead time reduction plays an important role in every industry sector. By reducing lead time, one can decrease the stockout loss as well as improve the customer’s satisfaction level.

In this direction, different models are considered with constant lead time by several researchers. But in reality as Just-in-Time (JIT) cannot be implemented in all situations, thus researchers should consider variable lead time. In preparation, the distributional in sequence about the demand is often inadequate. There is a propensity to utilize the usual sharing under these conditions by quite a lot of researchers. However, the normal distribution does not offer the best shield against the incident of other distributions with the same mean and same variance. Consequently, it is confronted to the managers to take a conclusion without having the proposal of the distribution of the lead time demand. In order to explain this problem, Scarf [35] first developed the solution of distribution free newsboy problem with known mean and standard deviation of the lead time. The objective of this proposed model is to find out an optimal inventory strategy that can minimize the value of the integrated total cost for the single vendor and the single buyer. An algorithm is developed to determine the optimal strategy and numerical examples are taken to illustrate the solution procedure in model development. A graphical representation of the algorithm is characterized by a flowchart. To conclude, the graphical representation is presented to illustrate the model. Furthermore, the sensitivity analysis is incorporated and the numerical examples are given to illustrate the results.

2. Literature Review

Inventories are raw materials, work-in-process goods and completely finished goods that are considered to be the portion of business’s assets that are ready or will be ready for sale. Inventory management is one of the key components of any business that can be controlled by a business manager to efficiently and successfully operate in the fiercely competitive modern global market.
Therefore, it is very important to build inventory models that are sensitive and responsive to the dynamic real life market situations.

The economic order quantity (EOQ), first proposed by Harris [16], is the most fundamental result which has generated whole new directions of research in inventory management since its inception. Inventory control is a vital field in supply chain management, and an enormous deal of research efforts have been devoted to it over past few decades. Lead time show business an important responsibility in today’s logistics administration. In order to gratify client demands in today’s spirited markets, serious in sequence needs to be shared along the supply chain. A supercilious level of synchronization involving vendors’ and buyers’ judgment making is also required. The conception of Joint Economic Lot Sizing (JELS) has been introduced to treat conventional methods for sovereign inventory control. At the present time, companies can no longer compete solely as individual entities in the constantly changing business world. Globalization of market and increased competition force organizations to rely on effective supply chains to improve their overall performance. Victorious supply chain management requires from organization distinct purpose to integrating actions into key supply chain processes.

Incorporation between two dissimilar business entities is a significant way to increase competitive compensation as it lowers the joint total cost of the system. Therefore, in modern enterprises, the integration of vendor–buyer inventory system is a major issue. The thought of optimizing the joint total cost in a single-vendor, single-buyer model was measured early on on by Goyal [10]. Banerjee [3] developed the model by incorporating a limited production rate and following a lot-for-lot policy for the vendor. By relaxing Banerjee’s lot-for-lot hypothesis, Goyal [14] proposed a more universal joint economic lot sizing model. Lu [25] specified the optimal creation and shipment policies when the shipment sizes are equal. He relaxed the assumption of Goyal [14] about completing a whole batch before starting shipments.

Goyal [12] then developed a model where consecutive shipment sizes increase by a ratio equal to the production rate divided by the demand rate. He found an expression for the optimal first shipment size as a function of the number of shipments. Afterward, Hill [17] took this suggestion one step further by allowing for the geometric growth factor as a decision variable. He optional a solution method based on an exhaustive search for both the growth factor and the number of shipments within certain ranges. Finally, Hill [18] determined the form of the overall optimal policy. This turns out to be a combination of the policy suggested by Goyal [12] used initially and an equal shipment’s policy used later. However, since the policy with equal-sized shipments linking the vendor and the buyer are straightforward to implement in practice, this shipment policy is usually employed in the JELS modeling literature.

Lead time lessening has been individual of the major reimbursement of winning completion of the fashionable Just-In-Time (JIT) inventory system. In this competitive marketing environment, one of the ways to attract customers is to allow the shortest lead time and quick services. If a manufacturing can bring products inside very short period comparing to others, then it may get another chance to obtain prospect order. Thus, lead time reduction plays an essential role in each business division. By reducing lead time, one can decrease the stock out loss as well as get better the customer’s satisfaction level. In this way, different models are considered with constant lead time by several researchers. But in reality as JIT cannot be implemented in all situations, thus researchers should consider variable lead time. In practice, the distributional in sequence about the demand is often limited. There is a tendency to use the normal distribution under these conditions by several researchers. Lead time reduction is another important production activity in an integrated inventory control. Lead time consists of order training, order transmittal, order dispensation and meeting, additional stock acquisition time and release time Ballou [2]. Recently, Ouyang et al. [27] comprehensive the (Q, r) model by Ben-Daya and Raouf [5] to regard as the
lead time consequence and slot in the partial backordering in the list model. Hariga [15] deliberate the affiliation between lot size and lead time in the procedure time feature. Pan and Yang [31] obtain an integrated supplier-purchaser model with convenient lead time. The model has a considerable cost saving when lead time is controllable. Chen et al. [7] presented a continuous review inventory model when ordering cost is dependent on lead time. Ben-Daya and Hariga [3] urbanized a continuous review inventory model where lead time is well thought-out as a controllable variable. Lead time is decayed into all its components: set-up time, processing time and non-productive time. Later, Ouyang et al. [26] extended Pan and Yang’s model [31] by allowing shortages.

Powerful competition, short product life cycles, and finely tuned concentration to customer call for an additional efficient supply chain management through better harmonization and cooperation among all members. A number of studies have examined coordination issues in a supply chain (Goyal[13]; Cachon and Zipkin [6]; Chen and Kang [8]. Most of them, however, viewed lead time as either a prescribed constant or a stochastic variable, which therefore is not subject to be controlled. In practical situations, however, this may not be realistic. Performance of lean thinking and Japanese JIT system has guide to substantial efforts in reduction of lead time and inventory-related costs simultaneously (Ben-Daya and Hariga [4]). The success of JIT formation exposed the significance of lead time reduction in output improvement. Time-based resistance focusing on the reduction of overall system lead time has been one of the favourite topics for both researchers and practitioners. As meaningful out by Tersine [37], lead time frequently consists of five components: order preparation, order transit, supplier lead time, delivery time and setup time, where each part can be reduced by an added crashing cost. In other words, lead time is controllable.

Lead time reduction can get better forecasting accuracy, reduce excess inventory and stock outs and improve the speed of response to market changes. In this increasingly intense competitive environment, lead time reduction has become an effective approach to realize supply chain quick response and one of the most important sources of competitive advantage (Tersine and Hummingbird [38]). Recently, the value of lead time reduction in inventory management has been extensively standard. Liao and Shyu [24] initial offered a continuous review model where order quantity was predetermined and lead time was a unique decision variable. Ben-Daya and Raouf [5] extended Liao and Shyu [24] model by bearing in mind both lead time and order quantity as decision variables. Ouyang et al. [27] advance treated stock out to be a combination of backorders and lost sales. Pan et al. [33] considered lead time crashing cost to be a function of order quantity and reduced lead time. Pan and Hsiao [32] studied inventory problem with variable lead time and backorder rate by contribution a price discount to stock out items.

Ouyang et al. [26] comprehensive Pan and Yang [31] model by further considering the reorder point as one of assessment variables and shortages permitted. Yang and Pan [49] presented an integrated inventory model to diminish the total of the ordering/setup cost, holding cost, quality improvement investment and crashing cost by simultaneously optimizing the order quantity, lead time, process quality and number of deliveries. Ouyang et al. [29] formulated an integrated inventory mold involving defective making procedure with controllable lead time in which the order quantity, reorder point, process quality level, lead time and number of deliveries are decision variables. Jha and Shanker ([20], [21]) planned two-echelon integrated supply chain inventory models with controllable lead time and service level constraint for regular items and decaying items, respectively. Vijayashree and Uthayakumar [44] have measured integrated inventory model with controllable lead time involving investment for quality improvement in supply chain system. Vijayashree and Uthayakumar [47] have offered inventory models involving lead time crashing cost as an exponential function.
Vijayashree and Uthayakumar [42] have to be had a two stage supply chain model with selling price dependent demand and investment for quality improvement. Vijayashree and Uthayakumar [41] have talk about vendor-buyer integrated inventory model with quality improvement and negative exponential lead time crashing cost. Vijayashree and Uthayakumar [45] have urbanized two-echelon supply chain inventory model with controllable lead time. Vijayashree and Uthayakumar [46] have developed an integrated vendor and buyer inventory model with investment for quality improvement and setup cost reduction. Vijayashree and Uthayakumar [43] have urbanized two-echelon supply chain inventory model with controllable lead time. Vijayashree and Uthayakumar [46] have developed an integrated vendor and buyer inventory model with investment for quality improvement and setup cost reduction. Vijayashree and Uthayakumar [48] have measured an optimizing integrated inventory model with investment for quality improvement and setup cost reduction.

To the best our knowledge, no author have developed a lead time reduction system in an integrated inventory model for non-defective items under supply chain system. Here the proposed model we have assumed that lead time crashing described by a piecewise linear function. In this proposed we have included the transportation and production cost. The purpose of this paper is to find out an optimal inventory strategy that can minimize the value of the integrated total cost for the single vendor and the single buyer. The contribution of this study is to develop an effective iterative solution procedure to determine the optimal solution. Finally, numerical examples are presented to illustrate the proposed model. A solution algorithm is designed and illustrated through numerical example. Furthermore, a sensitivity analysis is carried out to study the influence of the model-parameters on the optimal solution.

The rest of the paper is organized as follows. The most important assumption and notations are provided in section 3. Section 4 describes the model development. In section 5, the solution procedure is given. In section 6, a well-organized algorithm is developed to obtain the optimal solution. A numerical pattern is provided in section 7, to illustrate the results. In section 8 sensitivity analysis of the parameters is provided. Managerial implications are also included in section 9. Finally, in Section 10 the conclusions and future work are drawn.

3. NOTATIONS AND ASSUMPTIONS

In this paper, the subsequent notation and assumptions which are similar to those used in Pan and Yang [31]

3.1. NOTATIONS

The following notations are used in the proposed Model

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$D$</td>
<td>Average demand per unit time on the buyer</td>
</tr>
<tr>
<td>$P$</td>
<td>Production rate of the vendor ($P &gt; D$)</td>
</tr>
<tr>
<td>$Q$</td>
<td>Order quantity for the buyer</td>
</tr>
<tr>
<td>$A$</td>
<td>Buyer ordering cost per order</td>
</tr>
<tr>
<td>$S$</td>
<td>Vendor’s setup cost per setup</td>
</tr>
<tr>
<td>$F$</td>
<td>Transportation costs per deliveries</td>
</tr>
<tr>
<td>$t_1$</td>
<td>Fixed component of the transportation cost, ($/shipment)</td>
</tr>
<tr>
<td>$t_2$</td>
<td>Variable component of the transportation cost, ($/unit item/shipment)</td>
</tr>
<tr>
<td>$PC$</td>
<td>Unit production cost function</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of lead time (Decision Variable)</td>
</tr>
<tr>
<td>$R(L)$</td>
<td>The lead time crashing cost per cycle</td>
</tr>
</tbody>
</table>
$c_v$  Unit production cost paid by the vendor  

$c_b$  Unit purchase cost paid by the buyer  

$m$  A digit in place of the number of deliveries in which the items are delivered from the vendor to the buyer  

$r$  Annual inventory holding cost per dollar invested in stocks  

ITC  Integrated total cost for the single vendor and the single buyer

### 3.2. Assumptions

The assumptions through in the manuscript are as follows:

1. A two-echelon supply chain consisting of single vendor and single buyer is considered.
2. The buyer orders a lot of size $Q$ and the vendor manufactures $mQ$ with a finite production rate $P \ (P > D)$ at one set-up but ship in quantity $Q$ to the buyer over $m$ times. The vendor incurs a set-up cost $S$ for each production run and the buyer incurs an ordering cost $A$ for each order of quantity $Q$.
3. The buyer orders a lot of size $Q$ and the vendor manufactures a lot of size $mQ$, on average every $Q/D$ time units, the vendor transfer a shipment of size $Q$ to the buyer.
4. The transportation cost is $F = t_1 + t_2Q$, where $t_1$ the fixed cost per shipment is, and $t_2$ is the unit transportation cost per item (Yang et al. [50]).
5. The vendor obtain unit production costs as a function of $P$. We visualize this functional from to be $PC = c_1/P + c_2P$ (See Khouja and Mehrez [22]).
6. The buyer adopts a continuous review $(Q)$ system. When the on hand inventory reaches a predetermined reorder point $R$, a fixed quantity $Q$ is ordered by the buyer. The reorder point $R = $ Average demand during lead time + Safety stock. The demand during lead time is assumed to be normally distributed with mean $\mu L$ and standard deviation $\sigma L$

That is, $R = \mu L + k\sigma L$, where $k$ is a predetermined safety factor, also named cycle-service level.
7. The lead time $L$ consists of $n$ mutually independent components. The $i$th component has a normal duration $b_i$, minimum duration $a_i$, and crashing cost per unit time $c_i$. For convenience, we rearrange $c_i$ such that $c_1 < c_2 < c_3 < \ldots < c_n$.
8. The mechanism of lead time are crashed one at a time starting from the first component because it has the minimum unit crashing cost and then the second component, and so on.

9. Let $L_0 = \sum_{j=1}^{n} b_j$, and $L_i$ be the length of lead time with components $1, 2, 3, \ldots, i$ crashed to their minimum duration, then $L_i$ can be expressed as $L_i = L_0 - \sum_{j=1}^{i} (b_j - a_j), i = 1, 2, \ldots, n$.
10. and the lead time crashing cost per cycle $R(L)$ is given by $R(L) = c_1(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j), L \in [L_i, L_{i-1}]$. In addition, the length of lead time is equal for all shipping cycles, and the lead time crashing costs occur in each shipping
cycle. (Liao and Shyu [24], Pan and Yang [31] Vijayashree and Uthayakumar [43] and Yang and Pan [49]).

11. The further cost incurred by the vendor resolve be transferred to the buyer if shortened lead time is requested.

4. MODEL DEVELOPMENT

4.1. TOTAL COST FOR BUYER (TCB)

The buyer places an order after every \( Q \) demand; therefore for average cycle time of \( \frac{Q}{D} \), ordering cost \( \frac{A}{Q} = \frac{AD}{Q} \). The expected net inventory level just before arrival of a procurement is the safety stock \( s = R - DL \). The anticipated mesh inventory level directly after arrival of procurement is \( Q + s \). Hence the standard inventory over the cycle can be approximated by \( \left( \frac{Q}{2} \right) + s \), i.e. \( \left( \frac{Q}{2} \right) + k\sigma\sqrt{L} \) (Assumption 4),

So the buyer’s holding cost per unit time is \( rc_b \left( \frac{Q}{2} + k\sigma\sqrt{L} \right) \) \( (1) \)

Lead time crashing cost per unit time is \( \frac{R(L)}{Q} = \frac{DRL(L)}{Q} \) \( (2) \)

Transportation cost per unit time is \( \frac{F}{Q} = \frac{(t_1 + t_2 Q)D}{Q} \) \( (3) \)

The total cost for the buyer \( (TCB) \) from the equations (1) to (3) is given by

\[
TCB = \text{Ordering cost} + \text{holding cost} + \text{lead time crashing cost} + \text{transportation cost}
\]

\[
= \frac{DA}{Q} + \left( \frac{Q}{2} + k\sigma\sqrt{L} \right)rc_b + \frac{DRL(L)}{Q} + \frac{(t_1 + t_2 Q)D}{Q}
\]

(4)

4.2. TOTAL COST FOR VENDOR (TCV)

The vendor-buyer integrated system is designed for a vendor’s production situation in which once the buyer orders a lot of size \( Q \) the buyer begins production with a constant production rate \( P \) and a finite number of units are added to inventory until the production run has been completed. The vendor produces the item in lot of size \( mQ \) in each production cycle of length \( \frac{mQ}{D} \), and the buyer will receive the supply in \( m \) lots each of size \( Q \). The first lot of size \( Q \) is ready for deliveries after time \( \frac{Q}{P} \) just after the start of the production, and then the vendor continues making the delivery on average every \( \frac{Q}{D} \) units of time until the inventory level falls to zero (Figures 1 to 3).
So the vendor’s average inventory is evaluated as the difference of the vendor’s accumulated inventory and the buyer’s accumulated inventory (see figures 1 to 3). Hence the vendor total cost is

That is

\[
\left[ mQ \left( \frac{Q}{P} + (m-1) \frac{Q}{D} \right) - \frac{m^2Q^2}{2P} \right] - \left[ \frac{Q^2}{D} \left( 1 + 2 + \ldots + (m-1) \right) \right] / mQ \frac{D}{D}
\]

\[
= \frac{Q}{2} \left[ m \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right]
\]

So the vendor’s holding cost per unit time is

\[
r_{cv} \frac{Q}{2} \left[ m \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] \tag{4}
\]

On the other hand for the vendor, since is the vendor set-up cost per set-up and the production quantity for a vendor in a lot is \( mQ \) units, the vendor total cost per unit time is given by

\[
\frac{SD}{mQ} \tag{5}
\]

Considering production costs is given by

\[
PC = c_1/P + c_2P \tag{6}
\]

Hence, the total cost for the vendor’s \( TCV \) from the equations (4) to (6) is given by

\[
\]
\[ TCV(Q,m) = \text{Setup cost} + \text{holding cost} + \text{production cost} \]
\[ = \left( \frac{D}{mQ} \right) S + rc_i \left( \frac{Q}{2} \left( m \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right) \right) + D \left( \frac{c_1}{P} + c_2P \right) \]  

(7)

4.3. Integrated Total Cost (ITC)

The integrated total cost \( ITC \) of the system are now given as the sum of (4) and (7)
\[ \text{ITC} = \text{TCB} + TCV \]
\[ = \frac{DA}{Q} \left( \frac{Q}{2} + k\sigma \sqrt{L} \right) rc_i + \left( t_i + t_3Q \right) D + \frac{DR(L)}{Q} + \frac{DS}{mQ} + rc_i \left( \frac{Q}{2} \left( m \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right) \right) \]
\[ + D \left( \frac{c_1}{P} + c_2P \right) \]
\[ = \frac{D}{Q} \left( A + \frac{S}{m} + R(L) + t_i \right) + \frac{Q}{2} \left( m \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right) c_i + c_b \]
\[ + rc_i k\sigma \sqrt{L} + D \left( \frac{c_1}{P} + c_2P \right) \]

(8)

5. Solution Procedure

In order to solve the above non linear problem, we take the first order partial derivatives of \( ITC \) with respect to \( Q \) and \( L \in [L_i, L_{i-1}] \), respectively. We obtain
\[ \frac{\partial ITC(Q,L,m)}{\partial Q} = - \frac{D}{Q^2} \left( A + \frac{S}{m} + R(L) + t_t \right) + \frac{r}{2} \left( m \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right) c_i + c_b \]
\[ \frac{\partial ITC(Q,L,m)}{\partial L} = - \frac{Dc_i}{Q} + \frac{rc_i k\sigma L^{\frac{1}{2}}}{2} \]

(9)

(10)

Hence for fixed \( (Q,L,m) \), the minimum total integrated cost per unit time will occur at the end points of the interval \( L \in [L_i, L_{i-1}] \), \( i = 1,2,\ldots,n \). On the other hand, for given \( L \in [L_i, L_{i-1}] \), the minimum value of (4) will occur at the point \( Q \) satisfying the equation (4), equal to zero, we obtain the resulting solution is given below
\[ Q = \frac{2D}{r} \left( A + \frac{S}{m} + R(L) + t_t \right) \]
\[ \sqrt{\frac{1}{m} \left( m \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right) c_i + c_b} \]

(11)

For fixed \( m \) and \( L \in [L_i, L_{i-1}] \), by solving equation (11), we obtain the values of \( Q \) (denote the value by \( Q^* \)). The following proposition asserts that for fixed \( m \) and \( L \in [L_i, L_{i-1}] \), the point \( Q^* \) is the optimal solution such that the integrated total cost has minimum value.
Proposition 1
For fixed $m$ and $L \in [L_i, L_{i-1}]$, the integrated total cost $ITC(Q, L, m)$ is convex at point $Q$
\[
\frac{\partial ITC(Q, L, m)}{\partial Q^2} = 2D \left( \frac{A + S}{m} + R(L) + t_i \right) > 0
\]
(12)

Therefore, $ITC(Q, L, m)$ is convex in $Q$, for fixed $m$ and $L \in [L_i, L_{i-1}]$. This completes the proof of proposition 1.

Proposition 2
For fixed $m$ and $Q$, the integrated total cost $ITC(Q, L, m)$ is concave at point $L \in [L_i, L_{i-1}]$
\[
\frac{\partial ITC(Q, L, m)}{\partial L^2} = -\frac{rc_h k \sigma L^{-\frac{1}{2}}}{4} < 0
\]
(13)

Therefore, $ITC(Q, L, m)$ is concave in $L \in [L_i, L_{i-1}]$, for fixed $m$ and $Q$. This completes the proof of proposition 1.

Proposition 3
For fixed $Q$ and $L \in [L_i, L_{i-1}]$, the integrated total cost $ITC(Q, L, m)$ is convex at point $m$.
\[
\frac{\partial ITC(Q, L, m)}{\partial m} = -\frac{DS}{Qm^2} + Qr \left[ c_v \left( 1 - \frac{D}{P} \right) \right]
\]
(14)

and
\[
\frac{\partial^2 ITC(Q, L, m)}{\partial^2 m} = \frac{2DS}{Qm^3} > 0
\]
(15)

Therefore, $ITC(Q, L, m)$ is convex in $m$, for fixed $Q$ and $L \in [L_i, L_{i-1}]$. This completes the proof of proposition 3.

Further, based on the convexity performance of the purpose function with respect to the decision variable. Then, the following algorithm is designed to find the optimal values of order quantity $Q$, transport cost $F$, production cost $PC$, lead time $L$ and total number of deliveries $m$ which minimizes the integrated total cost $ITC(Q, L, m)$. Computer flow chart to describe in figure (4).

5. ALGORITHM

Step 1. Let $m = 1$. Since $m$ is integer
Step 2. Perform step (2.1)-(2.2) for all integer values of $L$ in this interval $L \in [L_i, L_{i-1}]$.

2.1. Compute $Q$ from equation (11).
2.2. Compute the corresponding $ITC(Q, L, m)$, by putting $Q$ in equation (8).
2.3. Choose the optimal pair of $Q$ and $L$ that result in the minimum integrated total cost $ITC(Q, L, m)$ in step (2.2).
Step 3. Let $\text{ITC}(Q^*, L^*, m) = \text{minimum of } \text{ITC}(Q, L, m)$, then $(Q^*, L^*)$ is an optimal solution for fixed $m$.

Step 4. Set $m = m + 1$ repeat steps (2)-(3) to get $\text{ITC}(Q^*, L^*, m)$.

Step 5. If $\text{ITC}(Q^*, L^*, m) \leq \text{ITC}(Q^*, L^*, m - 1)$; go to step 4, otherwise go to step 6.

Step 6. $\text{ITC}(Q^*, L^*, m^*) = \text{ITC}(Q^*, L^*, m - 1)$, then $(Q^*, L^*, m^*)$ is the set of optimal solutions.

**Figure (4) Computer flowchart of the algorithm**

6. **Numerical Example**

In this section, a numerical example is given to illustrate the above solution procedure. The solution to this example is obtained by using MatLab software. Let us consider the Study of an integrated inventory with controllable lead time with the following data used in Pan and Yang [31]:

- $D = 1000$ unit/year,
- $P = 3200$ unit/year,
- $A = $25 / order,
- $S = $400 / setup,
- $c_h = 25/\text{unit},$
- $r = 0.2, c_v = 20/\text{unit},$
- $k = 2.33, t_1 = 5, t_2 = 0.2, c_1 = 800, c_2 = 1/800, \sigma = 7 \text{ unit/week}.$

Lead time has three components with data shown in Table (1) and summarized lead time data shown in Table (2).
Table (1) Lead time components with data

<table>
<thead>
<tr>
<th>Lead time component (i)</th>
<th>Normal duration ( b_i ) (days)</th>
<th>Minimum duration ( a_i ) (days)</th>
<th>Unit crashing cost ( c_i ) (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>6</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>6</td>
<td>1.2</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td>9</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Table (2) summarized lead time data

<table>
<thead>
<tr>
<th>Lead time in (weeks)</th>
<th>( R(L) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1.4</td>
</tr>
<tr>
<td>4</td>
<td>18.2</td>
</tr>
<tr>
<td>3</td>
<td>53.2</td>
</tr>
</tbody>
</table>

Applying the solution procedure of the above algorithm, the computational results are presented in table (3). The optimal solutions from table (3) can be read off as lead time \( L^* = 6 \) weeks, order quantity \( Q^* = 167 \) units, number of deliveries \( m^* = 3 \), transportation cost \( F^* = 38 \), production cost \( PC^* = 4 \) and the corresponding integrated total cost \( ITC^* = 6617 \).

A graphical representation is presented to show the convexity of \( ITC(Q^*, L^*, m) \) in figure (5) and the graphical representation of the integrated total cost for different number of deliveries \( m \) and lead time \( L \) are shown in figure (6).

Table (3) optimal solution for different values of lead time

<table>
<thead>
<tr>
<th>m</th>
<th>L=8</th>
<th>L=6</th>
<th>L=4</th>
<th>L=3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q</td>
<td>F</td>
<td>PC</td>
<td>ITC</td>
</tr>
<tr>
<td>1</td>
<td>371</td>
<td>79</td>
<td>4</td>
<td>699</td>
</tr>
<tr>
<td>2</td>
<td>226</td>
<td>50</td>
<td>4</td>
<td>6715</td>
</tr>
<tr>
<td>3</td>
<td>167</td>
<td>38</td>
<td>4</td>
<td>6640</td>
</tr>
<tr>
<td>4</td>
<td>134</td>
<td>32</td>
<td>4</td>
<td>6622</td>
</tr>
<tr>
<td>5</td>
<td>113</td>
<td>27</td>
<td>4</td>
<td>6629</td>
</tr>
<tr>
<td>6</td>
<td>98</td>
<td>25</td>
<td>4</td>
<td>6647</td>
</tr>
</tbody>
</table>
7. Sensitivity Analysis

This part performs sensitivity analysis of this proposed model. This examination gives plain scheme about the performance of parameters over the cost function. It can also be establish out which parameter is more responsive to the integrated total cost. We now study the effects of changes in the system parameters Demand, Production rate, Vendor’s setup cost on the order quantity $Q$, lead time $L$, and the total number of deliveries $m$ transportation cost $F$, production cost $PC$ in order to minimize the integrated total cost $ITC$ of the given example.

7.1. Effects of Demand on Optimal Solution

In order to study how various demand $D$ affect the optimal solution of the proposed model, the demand sensitivity analysis is performed by changing the parameter of $D$ by $+50\%$, $+25\%$, $-50\%$, $-25\%$ and keeping the remaining parameters unchanged. The results of the demand analysis are shown in table (4) and the corresponding curves of the minimum integrated total cost are plotted in figure (7) as well.
Table (4) Effects of demand on optimal solution

<table>
<thead>
<tr>
<th>Demand</th>
<th>m</th>
<th>L weeks</th>
<th>Q</th>
<th>F</th>
<th>PC</th>
<th>ITC</th>
</tr>
</thead>
<tbody>
<tr>
<td>+50%</td>
<td>5</td>
<td>6</td>
<td>147</td>
<td>34</td>
<td>4</td>
<td>9142</td>
</tr>
<tr>
<td>+25%</td>
<td>5</td>
<td>6</td>
<td>131</td>
<td>31</td>
<td>4</td>
<td>7894</td>
</tr>
<tr>
<td>-25%</td>
<td>4</td>
<td>6</td>
<td>114</td>
<td>28</td>
<td>4</td>
<td>5264</td>
</tr>
<tr>
<td>-50%</td>
<td>3</td>
<td>6</td>
<td>115</td>
<td>28</td>
<td>4</td>
<td>3852</td>
</tr>
</tbody>
</table>

Table (5) Effects of production cost on optimal solution

<table>
<thead>
<tr>
<th>Production Rate</th>
<th>m</th>
<th>L weeks</th>
<th>Q</th>
<th>F</th>
<th>PC</th>
<th>ITC</th>
</tr>
</thead>
<tbody>
<tr>
<td>+50% (4800)</td>
<td>3</td>
<td>6</td>
<td>164</td>
<td>38</td>
<td>6</td>
<td>8569</td>
</tr>
<tr>
<td>+25% (4000)</td>
<td>4</td>
<td>6</td>
<td>132</td>
<td>31</td>
<td>5</td>
<td>7585</td>
</tr>
<tr>
<td>-25% (2400)</td>
<td>4</td>
<td>6</td>
<td>118</td>
<td>29</td>
<td>3</td>
<td>5621</td>
</tr>
<tr>
<td>-50% (1600)</td>
<td>7</td>
<td>6</td>
<td>104</td>
<td>26</td>
<td>2</td>
<td>4609</td>
</tr>
</tbody>
</table>

7.2. Effects of Production Rate on Optimal Solution

In order to study how various production rate $P$ affect the optimal solution of the proposed model, the production rate sensitivity analysis is performed by changing the parameter of $P$ by +50%, +25%, -50%, -25% and keeping the remaining parameters unchanged. The results of the production rate analysis are shown in table (5) and the corresponding curves of the minimum integrated joint total cost are plotted in figure (8) as well.

Figure (7) Curves representing minimum $ITC$ for various demand

Figure (8) Curves representing minimum $ITC$ for various production rate
7.3. Effects of Setup Cost on Optimal Solution

In order to study how various setup cost $S$ affect the optimal solution of the proposed model, the setup cost sensitivity analysis is performed by changing the parameter of $S$ by +50%, +25%, -50%, -25% and keeping the remaining parameters unchanged. The results of the setup cost analysis are shown in table (6) and the corresponding curves of the minimum integrated joint total cost are plotted in figure (9) as well.

Table (6) Effects of setup cost on optimal solution

<table>
<thead>
<tr>
<th>Setup Cost</th>
<th>m</th>
<th>L weeks</th>
<th>Q</th>
<th>F</th>
<th>PC</th>
<th>ITC</th>
</tr>
</thead>
<tbody>
<tr>
<td>+50% (600)</td>
<td>5</td>
<td>6</td>
<td>132</td>
<td>31</td>
<td>4</td>
<td>6935</td>
</tr>
<tr>
<td>+25% (500)</td>
<td>5</td>
<td>6</td>
<td>123</td>
<td>30</td>
<td>4</td>
<td>6779</td>
</tr>
<tr>
<td>-25% (300)</td>
<td>4</td>
<td>6</td>
<td>121</td>
<td>29</td>
<td>4</td>
<td>6406</td>
</tr>
<tr>
<td>-50% (200)</td>
<td>3</td>
<td>6</td>
<td>129</td>
<td>31</td>
<td>4</td>
<td>6168</td>
</tr>
</tbody>
</table>

Figure (9) Curves representing minimum $ITC$ for various setup cost

8. Managerial Implications

There is some good-looking managerial insinuation in the beyond analyses. We make the following commentary

1. From table 4, it is interesting to observe that in our results show that when the demand decrease, at that time the number of deliveries, order quantity, transportation cost and integrated total cost is also decreased without affecting lead time and production cost.
2. From table 5, it is motivating to watch that in our results show that when the production rate decrease, at that place order quantity, transportation cost, production cost and integrated total cost is also decreasing as well as a number of deliveries is increased without affecting lead time.
3. From table 6, it is fascinating to monitor that in our results show that when the setup cost decrease, at that condition the number of deliveries, order quantity, transportation cost and integrated total cost is also decreased at that situation the lead time and production cost is same.
4. In addition, the proposed model can be used in industries like airplane, healthcare, automobiles, computers, textiles, footwear, printers, refrigerators, mobile phones,
televisions, air conditioners, washing machines, tyres and bulky products such as
in print circuit boards, etc.

5. The proposed integrated inventory model is more valid for the supply chain
manufacturing system and vendor and buyer management.

6. The proposed integrated inventory model is helpful particularly for Just-in-time
(JIT) inventory systems where the vendor and the buyer form a planned grouping
for profit sharing.

9. CONCLUSION

Lead time reduction has been one of the favorite topics for both investigators and managers. The
primary purpose of this paper is an integrated inventory model with lead time reduction in non-
defective items. The majorities of the companies recognizes the implication of answer time as an
aggressive stick and have used time as means of discriminate themselves in the market place.

An integrated inventory model for single vendor and single buyer is presented in this paper. Based on the integrated expected total relevant costs of both single vendor and single buyer, we
obtain the optimal solution.

Mathematical modelling is employed in this study for optimizing the order quantity, lead time,
transportation, production cost and total number of deliveries from the vendor to the buyer in one
production run with the objective of minimizing integrated expected total cost of the system.

A model with the integrated total cost as an objective function is constructed and analyzed and
the lead time crashing cost \( R(L) \) is assumed to be piecewise linear function.

A numerical example is given to demonstrate the application and the performance of the
proposed methodology. In addition, sensitivity analysis has been carried out to illustrate the
behaviors of the proposed model and some managerial implications are also included.

Our model considers a single vendor and a single buyer situation. There are a number of
extensions of this work that could constitute future research related to this field.

One immediate probable extension could be to discuss the effect of defective items under
investment for quality. In future researches on this problem, on the one hand we could consider
multi items in this model. Another possible extension of this work can be done by assuming a
discrete investment to reduce the vendor’s setup cost, buyer’s ordering cost, investment for
quality improvement instead of continuous investment.

As a scope of future research, the model can be extended by considering fuzzy demand and
permissible delay-in-payments concepts in this model.

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