THE DESIGN OF STATE FEEDBACK CONTROLLERS FOR REGULATING THE OUTPUT OF THE UNIFIED CHAOTIC SYSTEM

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ABSTRACT

This paper investigates the design problem of constructing state feedback controllers for regulating the output of the unified chaotic system (Lü, Chen, Cheng and Celikovsky, 2002).Explicitly, state feedback controllers have been derived to regulate the output of the unified chaotic system so as to track constant reference signals. The control laws are derived by means of the regulator equations (Byrnes and Isidori, 1990). Numerical simulations are presented to demonstrate the effectiveness of the regulation schemes derived in this paper for the unified chaotic system.

KEYWORDS

Unified Chaotic System, Output Regulation, Nonlinear Control Systems, Feedback Stabilization.

1. INTRODUCTION

The output regulation problem is one of the core problems in control systems theory. For linear control systems, the output regulation problem has been solved by Francis and Wonham ([1], 1975). For nonlinear control systems, the output regulation problem was solved by Byrnes and Isidori ([2], 1990) generalizing the internal model principle obtained by Francis and Wonham [1]. Using Centre Manifold Theory [3], Byrnes and Isidori derived *regulator equations*, which characterize the solution of the output regulation problem of nonlinear control systems satisfying some stability assumptions.

The output regulation problem for nonlinear control systems has been studied extensively by various scholars in the last two decades [4-14]. In [4], Mahmoud and Khalil obtained results on the asymptotic regulation of minimum phase nonlinear systems using output feedback. In [5], Fridman solved the output regulation problem for nonlinear control systems with delay using centre manifold theory. In [6-7], Chen and Huang obtained results on the robust output regulation for output feedback systems with nonlinear exosystems. In [8], Liu and Huang obtained results on the global robust output regulation problem for lower triangular nonlinear systems with unknown control direction.

In [9], Immonen obtained results on the practical output regulation for bounded linear infinite-dimensional state space systems. In [10], Pavlov, Van de Wouw and Nijmeijer obtained results on the global nonlinear output regulation using convergence-based controller design. In [11], Xi and Dong obtained results on the global adaptive output regulation of a class of nonlinear systems with nonlinear exosystems. In [12-14], Serrani, Isidori and Marconi obtained results on the semi-global and global output regulation problem for minimum-phase nonlinear systems.

In this paper, we solve the output regulation problem for the unified chaotic system ([15], 2002). We derive state feedback control laws solving the constant regulation problem of the Shimizu-Morioka chaotic system using the regulator equations of Byrnes and Isidori (1990). The unified chaotic system is an important one-parameter family of three-dimensional chaotic systems discovered by Lü, Chen, Cheng and Celikovsky ([15], 2002). As special cases, the unified chaotic system includes the Lorenz system ([16], 1963), Chen system ([17], 1999) and Lü system ([18], 2002). The unified chaotic system has important applications in Electronics and Communication Engineering ([19], 2002).

This paper is organized as follows. In Section 2, we provide a review the problem statement of output regulation problem for nonlinear control systems and the regulator equations of Byrnes and Isidori [2], which provide a solution to the output regulation problem under some stability assumptions. In Section 3, we describe the unified chaotic system and its special cases. In Section 4, we describe the main results of this paper, namely, the solution of the output regulation problem for the unified chaotic system for the important case of constant reference signals (*set-point signals*). In Section 5, we describe the numerical results illustrating the effectiveness of the regulation schemes derived for the unified chaotic system. In Section 6, we summarize the main results obtained in this paper.

2. Review of the Output Regulation Problem For Nonlinear Control Systems

In this section, we consider a multi-variable nonlinear control system described by

$$\dot{x} = f(x) + g(x)u + p(x)\omega \tag{1a}$$

$$\dot{\omega} = s(\omega) \tag{1b}$$

$$e = h(x) - q(\omega) \tag{2}$$

Here, the differential equation (1a) describes the *plant dynamics* with state x defined in a neighbourhood X of the origin of R^n and the input u takes values in R^m subject to te effect of a disturbance represented by the vector field $p(x)\omega.$ The differential equation (1b) describes an autonomous system, known as the *exosystem*, defined in a neighbourhood W of the origin of $R^k,$ which models the class of disturbance and reference signals taken into consideration. The equation (2) defines the error between the actual plant output $h(x) \in R^p$ and a reference signal $q(\omega),$ which models the class of disturbance and reference signals taken into consideration.

We also assume that all the constituent mappings o the system (1) and the error equation (2), namely, f, g, p, s, h and q are continuously differentiable mappings vanishing at the origin, i.e.

$$f(0) = 0, \; g(0) = 0, \; p(0) = 0, \; s(0) = 0, \; h(0) = 0 \; \text{ and } \; q(0) = 0.$$

Thus, for $u = 0,$ the composite system (1) has an equilibrium $(x, \omega) = (0, 0)$ with zero error (2).

A state feedback controller for the composite system (1) has the form

$$u = \rho\left(x, \omega\right) \tag{3}$$

where ρ is a continuously differentiable mapping defined on $X \times W$ such that $\rho(0,0) = 0.$

Upon substitution of the feedback control law (3) into (1), we get the closed-loop system

$$\begin{aligned} \dot{x} &= f(x) + g(x)\rho(x,\omega) + p(x)\omega \\ \dot{\omega} &= s(\omega) \end{aligned} \tag{4}$$

State Feedback Regulator Problem [2]:

Find, if possible, a state feedback control law $u = \rho(x,\omega)$ such that the following conditions are satisfied.

(OR1) [*Internal Stability*] The equilibrium $x = 0$ of the dynamics

$$\dot{x} = f(x) + g(x)\rho(x,0)$$

is locally exponentially stable.

(OR2) [*Output Regulation*] There exists a neighbourhood $U \subset X \times W$ of $(x,\omega) = (0,0)$ such that for each initial condition $(x(0),\omega(0)) \in U$, the solution $(x(t),\omega(t))$ of the closed-loop system (4) satisfies

$$\lim_{t\to\infty}\left[h(x(t)) - q(\omega(t))\right] = 0. \qquad \blacksquare$$

Byrnes and Isidori [2] solved the output regulation problem stated above under the following two assumptions.

(H1) The exosystem dynamics $\dot{\omega} = s(\omega)$ is neutrally stable at $\omega = 0$, i.e. the exosystem is Lyapunov stable in both forward and backward time at $\omega = 0$.

(H2) The pair $(f(x), g(x))$ has a stabilizable linear approximation at $x = 0$, i.e. if

$$A = \left[\frac{\partial f}{\partial x}\right]_{x=0} \quad \text{and} \quad B = \left[\frac{\partial g}{\partial x}\right]_{x=0},$$

then (A,B) is stabilizable. $\blacksquare$

Next, we recall the solution of the output regulation problem derived by Byrnes and Isidori [2].

Theorem 1. [2] *Under the hypotheses (H1) and (H2), the state feedback regulator problem is solvable if and only if there exist continuously differentiable mappings* $x = \pi(\omega)$ *with* $\pi(0) = 0$ *and* $u = \varphi(\omega)$ *with* $\varphi(0) = 0,$ *both defined in a neighbourhood of* $W^0 \subset W$ *of* $\omega = 0$ *such that the following equations (called the* ***regulator equations****) are satisfied:*

(1) $\dfrac{\partial \pi}{\partial \omega} s(\omega) = f(\pi(\omega)) + g(\pi(\omega))\varphi(\omega) + p(\pi(\omega))\omega$

(2) $h(\pi(\omega)) - q(\omega) = 0$

When the regulator equations (1) and (2) are satisfied, a control law solving the state feedback regulator problem is given by

$$u = \varphi(\omega) + K\left[x - \pi(\omega)\right] \tag{5}$$

where K is any gain matrix such that $A + BK$ is Hurwitz. ■

3. DESCRIPTION OF THE UNIFIED CHAOTIC SYSTEM

The unified chaotic system ([15], 2002) is described by the dynamics

$$\begin{aligned} \dot{x}_1 &= (25\alpha + 10)(x_2 - x_1) \\ \dot{x}_2 &= (28 - 35\alpha)x_1 + (29\alpha - 1)x_2 - x_1 x_3 \\ \dot{x}_3 &= x_1 x_2 - \frac{1}{3}(8 + \alpha)x_3 \end{aligned} \tag{6}$$

where x_1, x_2, x_3 are the state variables and α is real parameter taking values in $[0,1]$.

In ([15], 2002), Lü, Chen, Cheng and Celikovsky showed that the system (5) bridges the gap between the Lorenz system ([16], 1963) and the Chen system ([17], 1999). Obviously, the system (6) reduces to the original Lorenz system for $\alpha = 0$, while the system (6) reduces to the original Chen system for $\alpha = 1$. When $\alpha = 0.8$, the system (6) reduces to the critical system or the Lü system ([18], 2002). Moreover, the system (6) is chaotic for all values of the parameter α in the closed interval $[0,1]$.

The state orbits of the Lorenz chaotic system ($\alpha = 0$), Chen chaotic system ($\alpha = 1$) and the Lü system ($\alpha = 0.8$) are shown in Figures 1, 2 and 3, respectively.

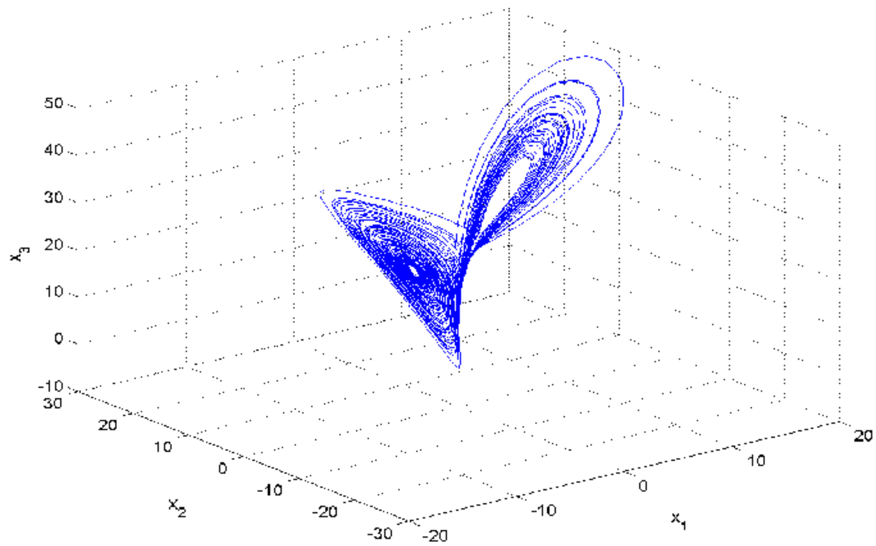

Figure 1. State Orbits of the Lorenz Chaotic System

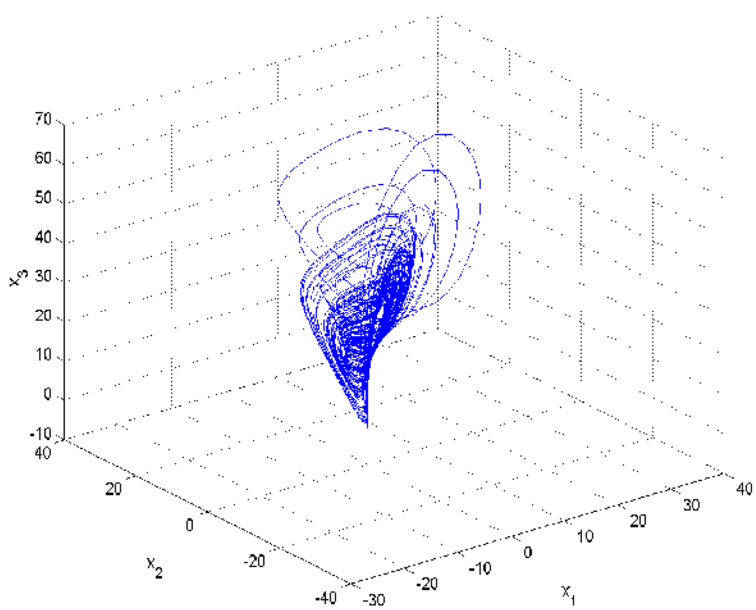

Figure 2. State Orbits of the Chen Chaotic System

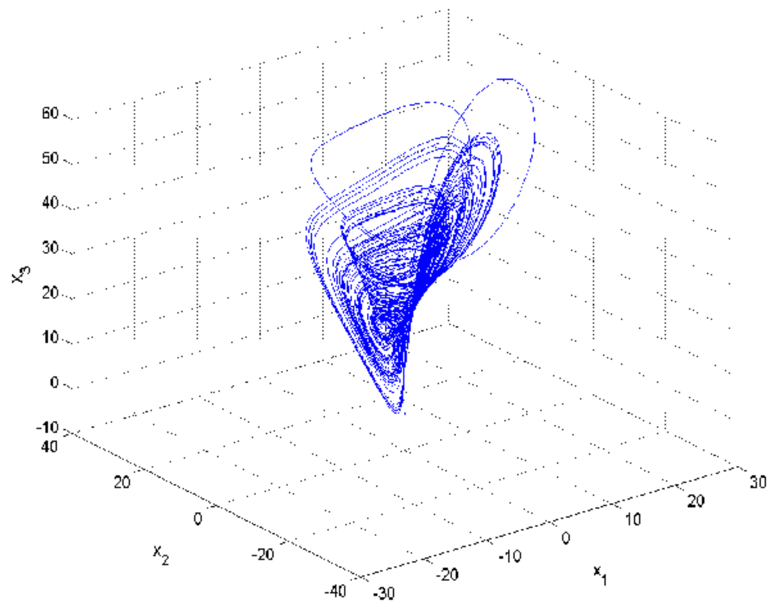

Figure 3. State Orbits of the Lü Chaotic System

4. OUTPUT REGULATION OF THE UNIFIED CHAOTIC SYSTEM

In this paper, we solve the output regulation problem for the unified chaotic system ([15], 2002) for the tracking of the constant reference signals (*set-point signals*).

Thus, we consider the unified chaotic system described by the dynamics

$$\begin{aligned} \dot{x}_1 &= (25\alpha + 10)(x_2 - x_1) \\ \dot{x}_2 &= (28 - 35\alpha)x_1 + (29\alpha - 1)x_2 - x_1 x_3 + u \\ \dot{x}_3 &= x_1 x_2 - \frac{1}{3}(8 + \alpha)x_3 \end{aligned} \tag{7}$$

where x_1, x_2, x_3 are the state variables, u is the control and $\alpha \in [0,1]$.

The constant reference signals are generated by the scalar exosystem dynamics

$$\dot{\omega} = 0 \tag{8}$$

It is important to note that the exosystem given by (8) is neutrally stable, because the exosystem (8) admits only constant solutions. Thus, the assumption (H1) of Theorem 1 holds trivially.

Linearizing the dynamics of the unified chaotic system (7) at $x = 0$, we obtain

$$A = \begin{bmatrix} -(25\alpha+10) & 25\alpha+10 & 0 \\ 28-35\alpha & 29\alpha-1 & 0 \\ 0 & 0 & -(\alpha+8)/3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

The system pair (A, B) can be expressed as

$$A = \begin{bmatrix} A_1 & 0 \\ 0 & \lambda^* \end{bmatrix} \text{ and } B = \begin{bmatrix} B_1 \\ 0 \end{bmatrix},$$

where

$$A_1 = \begin{bmatrix} -(25\alpha+10) & 25\alpha+10 \\ 28-35\alpha & 29\alpha-1 \end{bmatrix} \text{ and } B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

It is easy to see that the system pair (A_1, B_1) is completely controllable and the uncontrollable mode of A is $\lambda^* = -(\alpha+8)/3 < 0$ for all $\alpha \in [0,1]$.

Thus, the system pair (A, B) is stabilizable since we can easily find a gain matrix

$$K = \begin{bmatrix} K_1 & 0 \end{bmatrix} = \begin{bmatrix} k_1 & k_2 & 0 \end{bmatrix}$$

so that $A_1 + B_1 K_1$ is Hurwitz. Thus, the assumption (H2) of Theorem 1 also holds.

4.1 The Constant Tracking Problem for x_1

Here, the tracking problem for the unified chaotic system (7) is given by

$$\begin{aligned} \dot{x}_1 &= (25\alpha+10)(x_2 - x_1) \\ \dot{x}_2 &= (28-35\alpha)x_1 + (29\alpha-1)x_2 - x_1 x_3 + u \\ \dot{x}_3 &= x_1 x_2 - \frac{1}{3}(8+\alpha)x_3 \\ \dot{\omega} &= 0 \\ e &= x_1 - \omega \end{aligned} \tag{9}$$

By Theorem 1, the regulator equations of the system (9) are obtained as

$$\begin{aligned} (25\alpha+10)[\pi_2(\omega) - \pi_1(\omega)] &= 0 \\ (28-35\alpha)\pi_1(\omega) + (29\alpha-1)\pi_2(\omega) - \pi_1(\omega)\pi_3(\omega) + \varphi(\omega) &= 0 \\ \pi_1(\omega)\pi_2(\omega) - [(8+\alpha)\pi_3(\omega)]/3 &= 0 \\ \pi_1(\omega) - \omega &= 0 \end{aligned} \tag{10}$$

Solving the regulator equations (10) for the system (9), we obtain the unique solution as

$$
\begin{aligned}
\pi_1(\omega) &= \omega \\
\pi_2(\omega) &= \omega \\
\pi_3(\omega) &= \frac{3\omega^2}{8+\alpha} \\
\varphi(\omega) &= \frac{3\omega}{8+\alpha}\left[\omega^2 + (2\alpha-9)(8+\alpha)\right]
\end{aligned}
\tag{11}
$$

Using Theorem 1 and the solution (11) of the regulator equations for the system (9), we obtain the following result which provides a solution of the output regulation problem for (9).

Theorem 2. *A state feedback control law solving the output regulation problem for the unified chaotic system (9) is given by*

$$u = \varphi(\omega) + K\left[x - \pi(\omega)\right], \tag{12}$$

where $\varphi(\omega)$, $\pi(\omega)$ *are defined as in (11) and the gain matrix* K *is given by*

$$K = \left[K_1 \quad 0\right]$$

with K_1 *chosen so that* $A_1 + B_1K_1$ *is Hurwitz.* ■

3.2 The constant Tracking Problem for x_2

Here, the tracking problem for the unified chaotic system (7) is given by

$$
\begin{aligned}
\dot{x}_1 &= (25\alpha+10)(x_2 - x_1) \\
\dot{x}_2 &= (28-35\alpha)x_1 + (29\alpha-1)x_2 - x_1x_3 + u \\
\dot{x}_3 &= x_1x_2 - \frac{1}{3}(8+\alpha)x_3 \\
\dot{\omega} &= 0 \\
e &= x_2 - \omega
\end{aligned}
\tag{13}
$$

By Theorem 1, the regulator equations of the system (13) are obtained as

$$
\begin{aligned}
(25\alpha+10)[\pi_2(\omega) - \pi_1(\omega)] &= 0 \\
(28-35\alpha)\pi_1(\omega) + (29\alpha-1)\pi_2(\omega) - \pi_1(\omega)\pi_3(\omega) + \varphi(\omega) &= 0 \\
\pi_1(\omega)\pi_2(\omega) - [(8+\alpha)\pi_3(\omega)]/3 &= 0 \\
\pi_2(\omega) - \omega &= 0
\end{aligned}
\tag{14}
$$

Solving the regulator equations (14) for the system (13), we obtain the unique solution as

$$\begin{aligned}
\pi_1(\omega) &= \omega \\
\pi_2(\omega) &= \omega \\
\pi_3(\omega) &= \frac{3\omega^2}{8+\alpha} \\
\varphi(\omega) &= \frac{3\omega}{8+\alpha}\left[\omega^2 + (2\alpha-9)(8+\alpha)\right]
\end{aligned} \tag{15}$$

Using Theorem 1 and the solution (15) of the regulator equations for the system (13), we obtain the following result which provides a solution of the output regulation problem for (13).

Theorem 3. *A state feedback control law solving the output regulation problem for the unified chaotic system (13) is given by*

$$u = \varphi(\omega) + K\left[x - \pi(\omega)\right], \tag{16}$$

where $\varphi(\omega)$, $\pi(\omega)$ *are defined as in (15) and the gain matrix* K *is given by*

$$K = \left[K_1 \quad 0\right]$$

with K_1 *chosen so that* $A_1 + B_1 K_1$ *is Hurwitz.* ■

3.3 The constant Tracking Problem for x_3

Here, the tracking problem for the unified chaotic system (7) is given by

$$\begin{aligned}
\dot{x}_1 &= (25\alpha+10)(x_2 - x_1) \\
\dot{x}_2 &= (28-35\alpha)x_1 + (29\alpha-1)x_2 - x_1x_3 + u \\
\dot{x}_3 &= x_1x_2 - \frac{1}{3}(8+\alpha)x_3 \\
\dot{\omega} &= 0 \\
e &= x_3 - \omega
\end{aligned} \tag{17}$$

By Theorem 1, the regulator equations of the system (17) are obtained as

$$\begin{aligned}
(25\alpha+10)[\pi_2(\omega) - \pi_1(\omega)] &= 0 \\
(28-35\alpha)\pi_1(\omega) + (29\alpha-1)\pi_2(\omega) - \pi_1(\omega)\pi_3(\omega) + \varphi(\omega) &= 0 \\
\pi_1(\omega)\pi_2(\omega) - [(8+\alpha)\pi_3(\omega)]/3 &= 0 \\
\pi_3(\omega) - \omega &= 0
\end{aligned} \tag{18}$$

Solving the regulator equations (18) for the system (17), we obtain the unique solution as

$$\begin{aligned} \pi_1(\omega) &= \sqrt{\frac{\omega(8+\alpha)}{3}} \\ \pi_2(\omega) &= \sqrt{\frac{\omega(8+\alpha)}{3}} \\ \pi_3(\omega) &= \omega \\ \varphi(\omega) &= (6\alpha - 27 + \omega)\sqrt{\frac{\omega(8+\alpha)}{3}} \end{aligned} \tag{19}$$

Using Theorem 1 and the solution (19) of the regulator equations for the system (17), we obtain the following result which provides a solution of the output regulation problem for (17).

Theorem 4. *A state feedback control law solving the output regulation problem for the unified chaotic system (17) is given by*

$$u = \varphi(\omega) + K\left[x - \pi(\omega)\right], \tag{20}$$

where $\varphi(\omega)$, $\pi(\omega)$ *are defined as in (15) and the gain matrix* K *is given by*

$$K = \left[K_1 \quad 0\right]$$

with K_1 *chosen so that* $A_1 + B_1K_1$ *is Hurwitz.* ■

4. NUMERICAL SIMULATIONS

For simulation, the parameters are chosen as the chaotic case of the unified chaotic system, *viz.* $\alpha \in [0,1]$.

For achieving internal stability of the state feedback regulator problem, a feedback gain matrix K must be chosen so that $A + BK$ is Hurwitz.

As noted in Section 3, $\lambda^* = -(8+\alpha)/3$ is always a stable eigenvalue of $A + BK$.

Suppose we wish to choose a gain matrix such that the other two eigenvalues of $A + BK$ are $-4, -4$.

We choose the constant reference signal as $\omega = 2$.

For the numerical simulations, the fourth order Runge-Kutta method with step-size $h = 10^{-6}$ is deployed to solve the systems of differential equations using MATLAB.

4.1 Constant Tracking Problem for x_1

Here, we take $\alpha = 0$ so that the unified system (7) reduces to the Lorenz system. Then $\lambda^* = -2.6667$. We choose $K = \left[K_1 \quad 0\right]$ so that $A_1 + B_1K_1$ has eigenvalues $\{-4, -4\}$. Using Ackermann's formula (MATLAB), we obtain $K_1 = \left[-31.6 \quad 3\right]$.

The initial conditions are taken as

$$x_1(0)=10,\ \ x_2(0)=12,\ \ x_3(0)=18$$

The simulation graph is depicted in Figure 4 from which it is clear that the state trajectory $x_1(t)$ tracks the constant reference signal $\omega = 2$ in 3 seconds.

4.2 Constant Tracking Problem for x_2

Here, we take $\alpha = 0.8$ so that the unified system (7) reduces to the Lü system. Then $\lambda^* = -2.9333$. We choose $K = [K_1 \quad 0]$ so that $A_1 + B_1 K_1$ has eigenvalues $\{-4, -4\}$. Using Ackermann's formula (MATLAB), we obtain $K_1 = [-22.5333 \quad -0.2]$.

The initial conditions are taken as

$$x_1(0)=15,\ x_2(0)=24,\ x_3(0)=8$$

The simulation graph is depicted in Figure 5 from which it is clear that the state trajectory $x_2(t)$ tracks the constant reference signal $\omega = 2$ in 3 seconds.

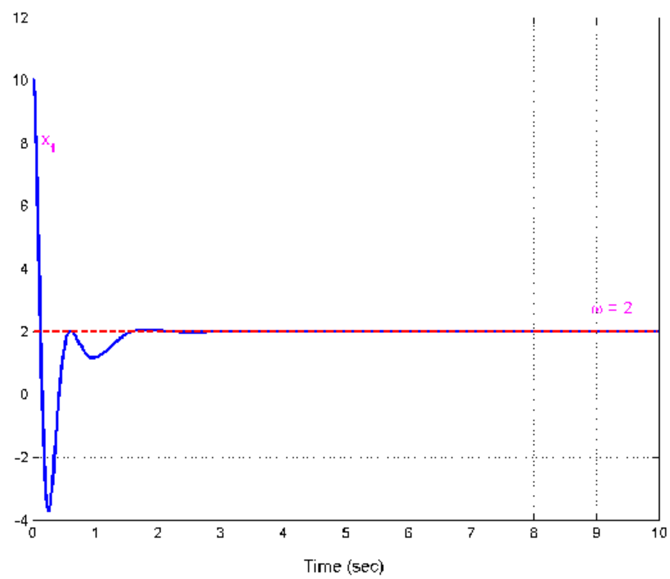

Figure 4. Constant Tracking Problem for x_1

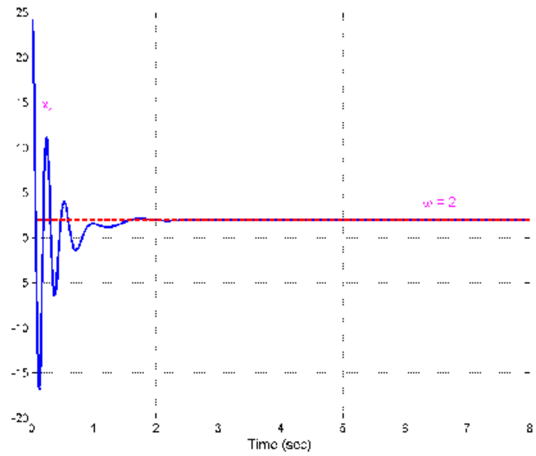

Figure 5. Constant Tracking Problem for x_2

4.3 Constant Tracking Problem for x_3

Here, we take $\alpha = 1$ so that the unified system (7) reduces to the Chen system. Then $\lambda^* = -3$. We choose $K = [K_1 \quad 0]$ so that $A_1 + B_1 K_1$ has eigenvalues $\{-4, -4\}$. Using Ackermann's formula (MATLAB), we obtain $K_1 = [-20.4571 \quad -1.0]$.

The initial conditions are taken as

$$x_1(0) = 3,\ x_2(0) = 7,\ x_3(0) = 5$$

The simulation graph is depicted in Figure 6 from which it is clear that the state trajectory $x_3(t)$ tracks the constant reference signal $\omega = 2$ in 3 seconds.

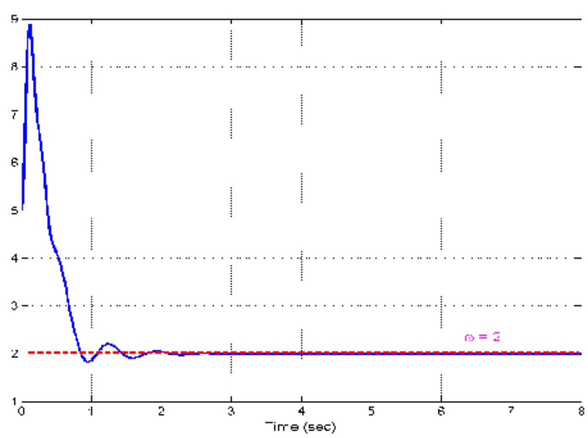

Figure 3. Constant Tracking Problem for x_3

5. CONCLUSIONS

In this paper, new results have been derived for the design of state feedback controllers for solving the output regulation problem for the unified chaotic system (2002) for the tracking of constant reference signals (*set-point signals*). The classical chaotic systems such as the Lorenz system (1963), Chen system (1999) and Lü system are special cases of the unified chaotic system. The state feedback control laws achieving output regulation proposed in this paper were derived using the regulator equations of Byrnes and Isidori (1990). Numerical simulation results were presented in detail to illustrate the effectiveness of the proposed control schemes for the output regulation problem of unified chaotic system to track constant reference signals.

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