

# MANEUVERING TARGET TRACK PREDICTION MODEL

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## ABSTRACT

*The issues about maneuvering target track prediction were discussed in this paper. Firstly, using Kalman filter which based on current statistical model describes the state of maneuvering target motion, thereby analyzing time range of the target maneuvering occurred. Then, predict the target trajectory in real time by the improved gray prediction model. Finally, residual test and posterior variance test model accuracy, model accuracy is accurate.*

## KEYWORDS

*Current Statistical Model, Maneuvering Target, Kalman Filter, Gray Theory*

## 1. THE STATE DESCRIPTION OF MANEUVERING TARGET

To build model of maneuvering target motion by current statistical model, to use Kalman filter process tracking.

### 1.1 To build current statistical model for maneuvering target

Current statistical model is a nonzero mean time-dependent model. Current acceleration probability density is described by the modified Rayleigh distribution. It is assumed that the target acceleration  $a(t)$  satisfies the following relationship:

$$a(t) = \bar{a}(t) + a_1(t) \quad (1)$$

$$a_1'(t) = -\alpha a_1(t) + \omega(t) \quad (2)$$

In the above formula:  $\bar{a}$  is the mean acceleration, and it is an estimate (a constant in each sampling period) about previous time acceleration;  $a_1(t)$  is a first order Markov process of zero mean,  $\alpha$  is the reciprocal of the maneuvering time constant;  $\omega(t)$  is zero mean Gaussian white noise.

The discrete state equation of current statistical model is:

$$X(k+1) = F(k)X(k) + U(k)\bar{a}(k+1) + W(k) \quad (3)$$

$X(k)$  is state vector,  $F(k)$  is state transition matrix,  $U(k)$  is Disturbance transfer matrix,  $W(k)$  is zero mean white noise with variance  $Q(k)$ , radar scan for 1s, so the sampling period  $T=1$ , take  $\alpha=1$ , the following expression:

$$X(k) = \begin{bmatrix} x(k) & x'(k) & x''(k) \end{bmatrix} \quad (4)$$

$$F(k) = \begin{bmatrix} 1 & T & (-1 + \alpha T + e^{-\alpha T}) / \alpha^2 \\ 0 & 1 & (1 - e^{-\alpha T}) / \alpha \\ 0 & 0 & e^{-\alpha T} \end{bmatrix} \quad (5)$$

$$U(k) = \begin{bmatrix} (-T + \alpha T^2 / 2 + 1 - (e^{-\alpha T}) / \alpha) / \alpha \\ T - (1 - e^{-\alpha T}) / \alpha \\ 1 - e^{-\alpha T} \end{bmatrix} \quad (6)$$

In the above formula,  $T$  is the sampling period.

Variance adaptive algorithm based on current statistical model, that is, the following equation adaptive variance:

$$Q(k) = E[W(k)W^T(k)] = 2\alpha\sigma_a^2 \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{12} & q_{22} & q_{23} \\ q_{13} & q_{23} & q_{33} \end{bmatrix} \quad (7)$$

The variables are as follows:

$$q_{11} = [1 - e^{-2\alpha T} + 2\alpha T + 2\alpha^3 T^3 / 3 - 2\alpha^2 T^2 - 4\alpha T e^{-\alpha T}] / 2\alpha^5 \quad (8)$$

$$q_{12} = [1 + e^{-2\alpha T} - 2e^{-\alpha T} + 2\alpha T e^{-\alpha T} - 2\alpha T + \alpha^2 T^2] / 2\alpha^4 \quad (9)$$

$$q_{13} = [1 - e^{-2\alpha T} - 2\alpha T e^{-\alpha T}] / 2\alpha^3 \quad (10)$$

$$q_{22} = [4e^{-\alpha T} - 3 - e^{-2\alpha T} + 2\alpha T] / 2\alpha^3 \quad (11)$$

$$q_{23} = [e^{-2\alpha T} + 1 - 2e^{-\alpha T}] / 2\alpha^2 \quad (12)$$

$$q_{33} = [1 - e^{-2\alpha T}] / 2\alpha \quad (13)$$

$$\sigma_a^2 = \begin{cases} \frac{4-\pi}{\pi} [a_{\max} - \bar{a}(k)]^2 & \bar{a}(k) > 0 \\ \frac{4-\pi}{\pi} [a_{-\max} - \bar{a}(k)]^2 & \bar{a}(k) < 0 \end{cases} \quad (14)$$

$a_{\max}$  is the maximum positive acceleration ,  $a_{-\max}$  is the maximum negative acceleration.

Measurement equation :

$$Z(k) = H(k)X(k) + V(k) \quad (15)$$

In the above formula ,  $H(k)$  is measurement matrix,  $V(k)$  is white noise that it's zero mean and variance is  $R(k)$  .

## 1.2. Kalman filter based on current statistical model

Kalman filter based on current statistical model smooth the state of the target on the past and present time, while predicting the target movement in the future time, including the location of the target, velocity and acceleration parameters.

State variable method is a valuable method to describe a dynamic system, using this methods, the system input-output relationship is described in the time domain by the state transition model and output observation model. Input can be determined by dynamic model consisted of a function of time and unpredictable variables or random noise process to describe. Output is a function of the state, which often disturbed by random observation errors, can be described by measurement equation.

Dynamic equations of discrete-time systems (state equations) can be expressed as :

$$X(k) = F(k)X(k-1) + \bar{a}(k)U(k) + W(k) \quad (16)$$

The measurement equation of discrete-time systems :

$$Z(k) = H(k)X(k) + V(k) \quad (17)$$

$H(k)$  is the measurement matrix ,  $V(k)$  is zero mean Gaussian white noise sequence. Different time measurement noise is independent

The formulas of Kalman filter algorithm based on the current statistical model are :

$$X(k|k-1) = F(k)X(k-1|k-1) + \bar{a}(k)B(k) \quad (18)$$

$$P(k|k-1) = F(k)P(k-1|k-1)F^T(k) + Q \quad (19)$$

$$X(k|k) = X(k|k-1) + K_g(k)[Z(k) - H(k)X(k|k-1)] \quad (20)$$

$$K_g(k) = P(k|k-1)H^T(k)[(H(k)P(k|k-1)H^T(k) + R(k))]^{-1} \quad (21)$$

$$P(k|k) = [I - K_g(k)H(k)]P(k|k-1) \quad (22)$$

Wherein,  $W(k) \sim N(0, Q)$ ,  $V(k) \sim N(0, R)$ .

Kalman filter does not require saving measurement data in the past, when new data measured, according to new data and various valuation in previous time, by means of state transition equation of the system, according to the above recurrence formula, thereby calculating the amount of all the new valuation.

## 2. TRACK PREDICTION BASED ON THE GRAY THEORY

To solve track forecast problem of aerial target, an aerial targets track prediction method based on gray theory is proposed in this paper. Grey system theory is based on new understanding about objective system. Although insufficient information on some systems, but as a system must have specific functions and order, but its inherent laws are not fully exposed. Some random amount, no rules interference component and chaotic data columns, from gray system point of view, is not considered to be elusive. Conversely, in gray system theory, a random amount is regard as a gray amount within a certain range, according to the proper way to deal with the raw data, grey number is converted to generation number, and then get strong regularity generation functions from generation number.

To achieve a real-time online track prediction and improve the prediction accuracy, an improved GM forecasting model is established. In the case of limited data, the model is used to predict the track effectively. The basic steps of aerial target track grey prediction: 1) the raw data accumulated generating 1; 2) the establishment of GM (1, 1) prediction model; 3) to test the model; 4) establishment of a residual model; 5) target track forecast.

### 2.1. Target track prediction

The current track graphics, coordinate information is known.

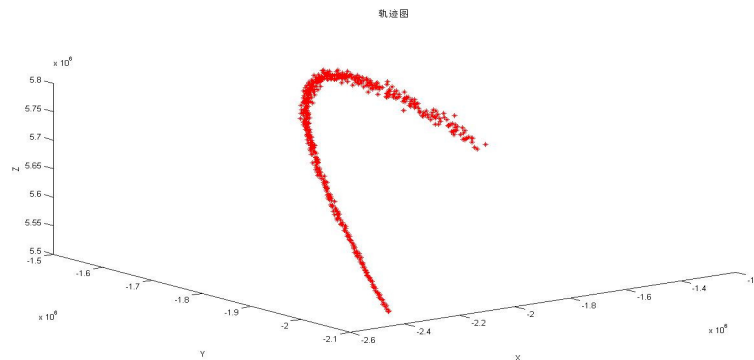


Figure 1. The current track data

Provided the original data sequence :

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)), Y^{(0)} = (y^{(0)}(1), y^{(0)}(2), \dots, y^{(0)}(n))$$

$Z^{(0)} = (z^{(0)}(1), z^{(0)}(2), \dots, z^{(0)}(n))$  ,  $(x^{(0)}(i), y^{(0)}(i), z^{(0)}(i))$  is the coordinates of the target (Figure1 data) in rectangular coordinate system with the center of earth.  $x^{(0)}(i) > 0, i = 1, 2, \dots, m$  ,  $y^{(0)}(i) > 0, i = 1, 2, \dots, m$  ,  $z^{(0)}(i) > 0, i = 1, 2, \dots, m$  , using the data sequence, general procedures to establish GM (1,1) model is :

The first step: to do the first order accumulated generating of the original data series  $X^{(0)}$  (i.e., 1-AGO), get accumulated generating sequence:

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)) \tag{23}$$

$$x^{(1)}(1) = x^{(0)}(1), x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i), (k = 2, 3, \dots, n)$$

Wherein,

The second step: the first order accumulated generating sequence  $X^{(1)}$  established GM (1, 1) model, getting related albino differential equations:

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b \tag{24}$$

Where, a is the development factor, reflecting the development trend between  $\hat{x}^{(1)}$  and  $\hat{x}^{(0)}$  . B is gray action , the gray action of GM (1,1) model is mined from the background data, it reflects the relationship to data changes, so the exact meaning is gray.

The related gray differential equation form as:

$$x^{(0)}(k) + ad^{(1)}(k) = b, k = 2, 3, \dots \tag{25}$$

$$B = \begin{bmatrix} -d^{(1)}(2) & 1 \\ -d^{(1)}(3) & 1 \\ \vdots & \vdots \\ -d^{(1)}(n) & 1 \end{bmatrix}$$

The third step: Solving the parameters a, b. Parameter sequence determined by the least squares method :  $\Phi = [B^T, B]^{-1} B^T A$  ,

$$B = \begin{bmatrix} -d^{(1)}(2) & 1 \\ -d^{(1)}(3) & 1 \\ \vdots & \vdots \\ -d^{(1)}(n) & 1 \end{bmatrix}, \quad d^{(1)}(k) = \frac{1}{2}[x^{(1)}(k) + x^{(1)}(k-1)], \quad A = (x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n))^T.$$

The fourth step: In the initial conditions  $\hat{x}^{(1)}(1) = x^{(1)}(1) = x^{(0)}(1)$ , Generate data series models are available:

$$\hat{x}^{(1)}(k) = (x^{(0)}(1) - \frac{\hat{b}}{\hat{a}})e^{-\hat{a}(k-1)} + \frac{\hat{b}}{\hat{a}} \quad (k = 2, 3, \dots, n) \quad (26)$$

The fifth step: In the initial conditions  $\hat{x}^{(1)}(1) = x^{(1)}(1) = x^{(0)}(1)$ , Generate data series models are available:

$$\hat{x}^{(0)}(1) = x^{(0)}(1), \hat{x}^{(0)}(k) = (1 - e^{\hat{a}})(x^{(0)}(1) - \frac{\hat{b}}{\hat{a}})e^{-\hat{a}(k-1)}, \quad k = 2, 3, \dots, n \quad (27)$$

Namely,  $\hat{x}^{(0)}(1) = x^{(0)}(1)$ ,  $\hat{x}^{(0)}(k) = (1 - e^{\hat{a}})(x^{(0)}(1) - \frac{\hat{b}}{\hat{a}})e^{-\hat{a}(k-1)}$ ,  $k = 2, 3, \dots, n$

So,  $k = 2, 3, \dots, n$  is substituted into the formula, getting the fitted values of the initial data; when  $k > n$ , to get the gray model predictive value for the future.

The sixth step:  $Y^{(0)} = (y^{(0)}(1), y^{(0)}(2), \dots, y^{(0)}(n))$ ,  $Z^{(0)} = (z^{(0)}(1), z^{(0)}(2), \dots, z^{(0)}(n))$  repeat steps one to step five.

## 2.2. Model accuracy test

Gray model prediction test are generally residual test and posterior variance test.

First, the relative size of the error test method, it is a straightforward arithmetic test methods comparing point by point. In this method, prediction data is compared with the actual data, observing the relative error whether satisfies the practical requirements.

The actual data set is used for modeling:

$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n))$ , according to GM (1,1) modeling method to obtain

$\hat{X}^{(1)} = (\hat{x}^{(1)}(1), \hat{x}^{(1)}(2), \hat{x}^{(1)}(3), \dots, \hat{x}^{(1)}(n))$ ,  $\hat{X}^{(1)}$  do a regressive generate transformed into  $\hat{X}^{(0)}$

, Model value of actual data :  $\hat{X}^{(0)} = (\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \hat{x}^{(0)}(3), \dots, \hat{x}^{(0)}(n))$ .

Computing residuals, residual sequence is:

$$E = (e(1), e(2), \dots, e(n)) = X^{(0)} - \hat{X}^{(0)} \quad (28)$$

where ,  $e(i) = x^{(0)}(i) - \hat{x}^{(0)}(i)$  ,  $i = 1, 2, \dots, n$

Calculate the relative error, the relative error is :

$$\varepsilon(i) = \frac{e(i)}{x^{(0)}(i)} \times 100\% = \frac{x(i) - \hat{x}(i)}{x^{(0)}(i)} \times 100\% \quad (29)$$

$\varepsilon(i) = \frac{e(i)}{x^{(0)}(i)} \times 100\%$  is the origin of the error ,  $\bar{\varepsilon}(i) = \frac{1}{m} \sum_{i=1}^m |\varepsilon(i)|$  is the average relative error of GM ( 1,1 ) model;  $P^o = (\varepsilon(k) - \bar{\varepsilon}) \times 100\%$  is the model accuracy of GM ( 1,1 ) model , i.e., minimum error probability, and it generally requires  $P^o > 80\%$  , best  $P^o > 90\%$  .

Second, posterior deviation test belongs to the statistical concept, it is according to the probability distribution of residual test.

If the actual model data used was:  $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n))$  , getting model value of actual data with the GM (1, 1) modeling method:  $\hat{X}^{(1)} = (\hat{x}^{(1)}(1), \hat{x}^{(1)}(2), \hat{x}^{(1)}(3), \dots, \hat{x}^{(1)}(n))$  , Assuming variances of the actual data sequence  $X^{(0)}$  and residuals sequence E separately are  $S_1^2$  and  $S_2^2$  ,

$$S_1^2 = \frac{1}{n} \sum_{i=1}^n (x^{(0)}(i) - \bar{x}^{(0)})^2 \quad , \text{ and } \quad \bar{x}^{(0)} = \frac{1}{n} \sum_{i=1}^n x^{(0)}(i)$$

$$S_2^2 = \frac{1}{n} \sum_{i=1}^n (e(i) - \bar{e})^2 \quad , \text{ and } \quad \bar{e} = \frac{1}{n} \sum_{i=1}^n e(i)$$

calculating Posterior variance ratio :

$$C = S_2 / S_1 \quad (30)$$

Determine the model level, indicators such as Table 1:

Table 1. Model accuracy class

Model accuracy class	Small Error probability P	Posterior inferior C
I	>0.95	<0.35
II	>0.8	<0.5
III	>0.7	.0.65
IV	≤ 0.7	≥ 0.65

Level Description: C values as small as possible, i.e.,  $S_1$  is much smaller than the  $S_0$ . raw date discrete is large, prediction error discrete is small, and prediction accuracy is high; P bigger the better, equally error probability is small, fitting accuracy high. If the residual test, posterior variance test can be passed, it can be predicted by their model, otherwise corrected residuals.

Table 2. Target prediction location

Target prediction	Small error probability	Posterior variance ratio	Model accuracy
X	1	0.188	I
Y	1	0.107	I
Z	1	0.115	I

### 2.3. Track forecast results and analysis

The radius, velocity and range angle  $\beta$  of observation point D are known, the plane parallel to the direction of velocity and passed point D can be approximated as missile flight plane, plane  $Q_2$  that through geocentric  $O$ , point D and perpendicular to the direction of the bullet speed, plane  $Q_3$  that through geocentric  $O$ , placement B and perpendicular to the direction of the bullet speed. Angle between plane  $Q_2$  and plane  $Q_3$  is the range angle  $\beta$ . Release impact coordinate, shown in FIG 1.

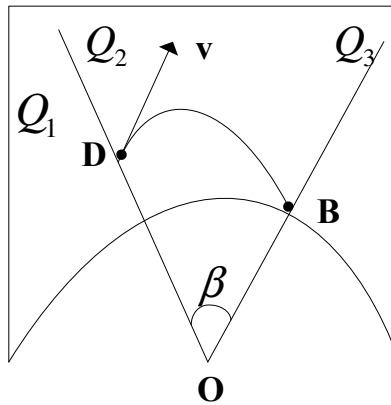


Figure 2. Solving placement schematic

If plane  $Q_1$  is  $A_1x + B_1y + C_1z = 0$ , normal vector is  $n_1 = A_1i + B_1j + C_1k$ , and

$n_1 = r_{OD} \times v = \begin{bmatrix} i & j & k \\ x_0 & y_0 & z_0 \\ v_x & v_y & v_z \end{bmatrix}$ ,  $i, j, k$  are unit vector for each axis, the flat  $Q_1$  can be solved by the equation.



If plane  $Q_2$  is  $A_2x + B_2y + C_2z = 0$ , normal vector is  $n_2 = A_2i + B_2j + C_2k$ , and  $n_2 = r_{OD} \times n_1 = \begin{bmatrix} i & j & k \\ x_0 & y_0 & z_0 \\ A_1 & B_1 & C_1 \end{bmatrix}$ , the flat  $Q_2$  can be solved by the equation.

If plane  $Q_3$  is  $A_3x + B_3y + C_3z = 0$ , normal vector is  $n_3 = A_3i + B_3j + C_3k$ , and  $\begin{cases} n_2 \cdot n_3 = |n_2||n_3|\cos\beta \\ n_3 \cdot n_1 = 0 \\ |n_3| = 1 \end{cases}$ , the flat  $Q_3$  can be solved by the equation, intersection of the plane  $Q_1$ ,

the plane  $Q_3$  and the earth's surface is the placement of B,  $B = (x, y, z)$ , and  $\begin{cases} A_1x + B_1y + C_1z = 0 \\ A_3x + B_3y + C_3z = 0 \\ x^2 + y^2 + z^2 = R^2 \end{cases}$

Placement coordinates can be obtained by solve formulas, after forecast, Placement coordinates is  $(-2423400, -2103400, 5488000)$ , error radius 1000m, algorithm complexity  $O(n)$ .

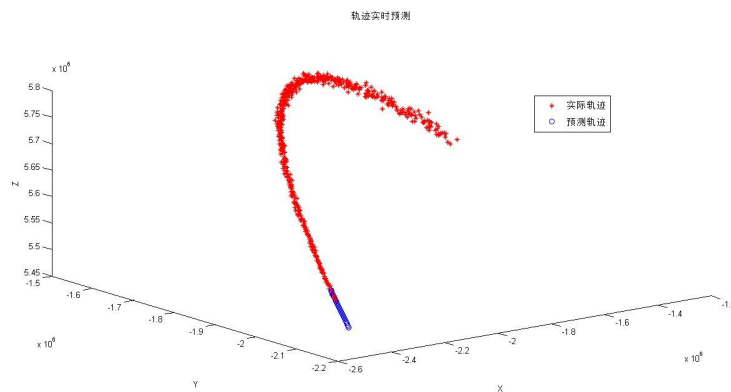


Figure 3. Forecasting model predicted target track

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