Some Results on Fuzzy Soft Multi Sets

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ABSTRACT

In this paper we define some new operations in fuzzy soft multi set theory and show that the De Morgan's type of results hold in fuzzy soft multi set theory with respect to these newly defined operations in our way. Also some new results along with illustrating examples have been put forward in our work.

KEYWORDS

Fuzzy set, soft set, soft multi set, fuzzy soft multi set.

1. INTRODUCTION

Theory of fuzzy sets, soft sets and soft multi sets are powerful mathematical tools for modeling various types of vagueness and uncertainty. In 1999, Molodtsov [9] initiated soft set theory as a completely generic mathematical tool for modelling uncertainty and vague concepts. Later on Maji et al. [8] presented some new definitions on soft sets and discussed in details the application of soft set in decision making problem. Based on the analysis of several operations on soft sets introduced in [9], Ali et al. [2] presented some new algebraic operations for soft sets and proved that certain De Morgan's law holds in soft set theory with respect to these new definitions. Combining soft sets [9] with fuzzy sets [13], Maji et al. [7] defined fuzzy soft sets, which are rich potential for solving decision making problems. Alkhazaleh and others [1, 3, 5, 6, 12] as a generalization of Molodtsov's soft set, presented the definition of a soft multi set and its basic operations such as complement, union, and intersection etc. In 2012, Alkhazaleh and Salleh [4] introduced the concept of fuzzy soft multi set theory and studied the application of these sets and recently, Mukherjee and Das [10, 11] constructed the fundamental theory on soft multi topological spaces and fuzzy soft multi topological spaces.

In this paper we define some new notions in fuzzy soft multi set theory and show that the De Morgan's type of results holds in fuzzy soft multi set theory for the newly defined operations in our way. Also some new results along with illustrating examples have been put forward in our work.

2. PRELIMINARY NOTES

In this section, we recall some basic notions in soft set theory, and fuzzy soft multi set theory. Molodstov defined soft set in the following way. Let U be an initial universe and E be a set of parameters. Let *P(U)* denotes the power set of U and $A \subseteq E$.

Definition 2.1[9]

A pair (*F, A*) is called a soft set over U, where *F* is a mapping given by $F: A \rightarrow P(U)$. In other words, soft set over U is a parameterized family of subsets of the universe U.

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Definition 2.2 [7]

Let U be an initial universe and E be a set of parameters. Let F(U) be the set of all fuzzy subsets of U and $A \subseteq E$. Then the pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by F: A →F(U).

Definition 2.3[4]

Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \phi$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} FS(U_i)$ where $FS(U_i)$ denotes the set of all fuzzy subsets of U_i, $E = \prod_{i\in I} E_{U_i}$ and $A \subseteq E$. A pair (F, A) is called a fuzzy soft multi set over U, where $F : A \to U$ is a mapping given by $\forall e \in A$,

$$F(e) = \left(\left\{ \frac{u}{\mu_{F(e)}(u)} : u \in U_i \right\} : i \in I \right)$$

Definition 2.4[12]

The complement of a fuzzy soft multi set *(F, A)* over U is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $\forall e \in A$

$$F^c(e) = \left(\left\{ \frac{u}{1-\mu_{F(e)}(u)} : u \in U_i \right\} : i \in I \right).$$

3. A STUDY ON SOME OPERATIONS IN FUZZY SOFT MULTI SETS

Definition 3.1

A fuzzy soft multi set (F, A) over U is called fuzzy soft multi subset of a fuzzy soft multi set (G, B) if

(a) $A \subseteq B$ and

(b) $\forall e \in A,\ F(e) \subseteq G(e) \Leftrightarrow \mu_{F(e)}(u) \leq \mu_{G(e)}(u),\ \forall u \in U_i, i \in I.$

This relationship is denoted by (F, A) $\tilde{\subseteq}$ (G, B).

Definition 3.2

Two fuzzy soft multiset (*F, A) and (G, B*) over U is called equal if (*F, A*) is fuzzy soft multi subset of (*G, B) and (G, B*) is fuzzy soft multi subset of (*F, A*).

Definition 3.3

A fuzzy soft multiset (*F, A*) over U is called a null fuzzy soft multiset**,** denoted by $(F, A)_\varphi$, if all the fuzzy soft multiset parts of (*F,A*) equals ϕ.

Definition 3.4

A fuzzy soft multiset (F, A) over U is called an absolute fuzzy soft multiset, denoted by $(F,A)_U$, if $\left(e_{U_i,j}, F_{e_{U_i,j}}\right) = U_i,\ \forall i$.

Definition 3.5

Union of two fuzzy soft multisets *(F, A) and (G, B*) over *U* is a fuzzy soft multiset (*H, D*) where $D = A \cup B$ and $\forall e \in D$,

$$H(e) = \begin{cases} F(e), & if\ e \in A - B \\ G(e), & if\ e \in B - A \\ \bigcup\left(F(e), G(e)\right), & if\ e \in A \cap B \end{cases}$$

Where $\bigcup\left(F(e), G(e)\right) = s\left(F_{e_{U_i,j}}, G_{e_{U_i,j}}\right)\ \forall i \in \{1,2,3,..,n\}$ and $j \in \{1,2,3,..,n\}$ with s as an s-norm and is written as $(F,A)\tilde{\cup}(G,B) = (H,D)$

Proposition 3.6

If (F, A), (G, B) and (H, D) are three fuzzy soft multisets over U, then

(a) $\left(F,A\right)\tilde{\cup}\left(\left(G,B\right)\tilde{\cup}\left(H,D\right)\right) = \left(\left(F,A\right)\tilde{\cup}\left(G,B\right)\right)\tilde{\cup}\left(H,D\right)$,

(b) $\left(F,A\right)\tilde{\cup}\left(F,A\right) = \left(F,A\right)$,

(c) $\left(F,A\right)\tilde{\cup}\left(G,A\right)_{\varphi} = \left(F,A\right)$,

(d) $\left(F,A\right)\tilde{\cup}\left(G,A\right)_U = \left(G,A\right)_U$,

Definition 3.7

Intersection of two fuzzy soft multisets (F, *A*) and (G, *B*) over *U* is a fuzzy soft multiset (H, D) where $D = A \cap B$ and $\forall e \in D$,

$$H(e) = \begin{cases} F(e), & if\ e \in A - B \\ G(e), & if\ e \in B - A \\ \bigcap\left(F(e), G(e)\right), & if\ e \in A \cap B \end{cases}$$

where $\bigcap\left(F(e), G(e)\right) = t\left(F_{e_{U_i,j}}, G_{e_{U_i,j}}\right)\ \forall i \in \{1,2,3,..,n\}$ and $j \in \{1,2,3,..,n\}$ with t as an t-norm and is written as $(F,\ A)\tilde{\cup}(G,\ B) = (H,C)$

Proposition 3.8

If *(F, A), (G, B) and (H, D*) are three fuzzy soft multisets over U, then

(a) $\left(F,A\right)\tilde{\cap}\left(\left(G,B\right)\tilde{\cap}\left(H,D\right)\right) = \left(\left(F,A\right)\tilde{\cap}\left(G,B\right)\right)\tilde{\cap}\left(H,D\right)$,

(b) $\left(F,A\right)\tilde{\cap}\left(F,A\right) = \left(F,A\right)$,

(c) $\left(F,A\right)\tilde{\cap}\left(G,A\right)_{\varphi} = \left(G,A\right)_{\varphi}$,

(d) $\left(F,A\right)\tilde{\cap}\left(G,A\right)_U = \left(F,A\right)$,

$(e)\ (F,A)\tilde{\cap}(G,A)_U=(F,A),$

Definition3.9

The complement of fuzzy soft multiset (*F, A*) over U is denoted by $(F,\ A)^c$ and is defined by $(F,A)^c=(F^c,A)$, where $F^c:A\to U$ is a mapping given by $F^c(\alpha)=c(F(\alpha)),\forall\alpha\in A$, where c is fuzzy complement.

Proposition 3.10

For a fuzzy soft multiset (*F, A*) over U,

$(a)\ \left((F,A)^c\right)^c=(F,A),$

$(b)\ (F,A)^c_{\approx\varphi_i}=(F,A)_{\approx U_i},$

$(c)\ (F,A)^c_{\varphi}=(F,A)_U,$

$(d)\ (F,A)^c_{\approx U_i}=(F,A)_{\approx\varphi_i},$

$(e)\ (F,A)^c_U=(F,A)_{\varphi},$

Now we introduce some new types of operations

Definition 3.11

The restricted union of two fuzzy soft multi sets (F, A) and (G, B) over U is a fuzzy soft multi set (H, C) where $C=A\cap B$ and $\forall e\in C$, $H(e)=\cup(F(e),G(e))$, $=\left(\left\{\frac{u}{\max\{\mu_{F(e)}(u),\mu_{G(e)}(u)\}}:u\in U_i\right\}:i\in I\right)$ and is written as $(F,\ A)\tilde{\cup}_R(G,\ B)=(H,C)$.

Definition 3.12

The extended union of two fuzzy soft multi sets (*F, A*) and (*G, B*) over U is a fuzzy soft multi set (*H, D*), where $D=A\cup B$ and $\forall e\in D$,

$$H(e)=\begin{cases}F(e), & if\ \ e\in A-B\\ G(e), & if\ \ e\in B-A\\ \cup(F(e),G(e)), & if\ \ e\in A\cap B,\end{cases}$$

where $\cup(F(e),G(e))=\left(\left\{\frac{u}{\max\{\mu_{F(e)}(u),\mu_{G(e)}(u)\}}:u\in U_i\right\}:i\in I\right)$ and is written as $(F,\ A)\tilde{\cup}_E(G,\ B)=(H,D)$.

Definition 3.13

The restricted intersection of two intuitionistic fuzzy soft multi sets (*F, A*) and (*G, B*) over U is a fuzzy soft multi set (*H, D*) where $D = A \cap B$ and $\forall e \in D$, $H(e) = \bigcap\left(F(e), G(e)\right)$ $= \left(\left\{ \frac{u}{\min\left\{\mu_{F(e)}(u), \mu_{G(e)}(u)\right\}} : u \in U_i \right\} : i \in I \right)$ and is written as $(F,\ A)\tilde{\cap}_R (G,\ B) = (H, D).$

Definition 3.14

The extended intersection of two fuzzy soft multi sets (*F, A*) and (*G, B*) over U is a fuzzy soft multi set (*H, D*), where $D = A \cup B$ and $\forall e \in D$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ \bigcap\left(F(e), G(e)\right), & \text{if } e \in A \cap B \end{cases}$$

where $\bigcap\left(F(e), G(e)\right) = \left(\left\{ \frac{u}{\min\left\{\mu_{F(e)}(u), \mu_{G(e)}(u)\right\}} : u \in U_i \right\} : i \in I \right)$ and is written as $(F,\ A)\tilde{\cap}_E (G,\ B) = (H, D).$

Example 3.15

Let us consider there are two universes $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c_2\}$ and let $\{E_{U_1}, E_{U_2}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \left\{ e_{U_1,1} = \text{exp}\,ensive,\ e_{U_1,2} = cheap,\ e_{U_1,3} = wooden \right\}$$

$$E_{U_1} = \left\{ e_{U_2,1} = \text{exp}\,ensive,\ e_{U_2,2} = cheap,\ e_{U_2,3} = sporty \right\},$$

Let $U = \prod_{i=1}^{2} FS\left(U_i\right)$, $E = \prod_{i=1}^{2} E_{U_i}$ and

$$A = \left\{ e_1 = (e_{U_1,1}, e_{U_2,1}), e_2 = (e_{U_1,1}, e_{U_2,2}), e_3 = (e_{U_1,2}, e_{U_2,1}) \right\},$$

$$B = \left\{ e_1 = (e_{U_1,1}, e_{U_2,1}), e_2 = (e_{U_1,1}, e_{U_2,2}), e_4 = (e_{U_1,3}, e_{U_2,2}) \right\}.$$

Suppose that

$$\left(F, A\right) = \left\{ \left(e_1, \left(\left\{ \frac{h_1}{0.2}, \frac{h_2}{0.4}, \frac{h_3}{0.8} \right\}, \left\{ \frac{c_1}{0.8}, \frac{c_2}{0.5} \right\} \right) \right), \left(e_2, \left(\left\{ \frac{h_1}{0.7}, \frac{h_2}{0.7}, \frac{h_3}{1} \right\}, \left\{ \frac{c_1}{0.8}, \frac{c_2}{0.6} \right\} \right) \right), \right.$$

$$\left. \left(e_3, \left(\left\{ \frac{h_1}{0.9}, \frac{h_2}{0.7}, \frac{h_3}{1} \right\}, \left\{ \frac{c_1}{0.3}, \frac{c_2}{0.5} \right\} \right) \right) \right\},$$

$$\left(G, B\right) = \left\{ \left(e_1, \left(\left\{ \frac{h_1}{0.3}, \frac{h_2}{0.3}, \frac{h_3}{0.7} \right\}, \left\{ \frac{c_1}{0.7}, \frac{c_2}{0.6} \right\} \right) \right), \left(e_2, \left(\left\{ \frac{h_1}{0.5}, \frac{h_2}{0.8}, \frac{h_3}{1} \right\}, \left\{ \frac{c_1}{0.4}, \frac{c_2}{0.6} \right\} \right) \right), \right.$$

$$\left. \left(e_4, \left(\left\{ \frac{h_1}{0.8}, \frac{h_2}{0.9}, \frac{h_3}{1} \right\}, \left\{ \frac{c_1}{0.5}, \frac{c_2}{0.2} \right\} \right) \right) \right\},$$

then we have,

$$(i).\ (F,\ A)\tilde{\cup}_R\,(G,\ B)=\left\{\left(e_1,\left(\left\{\frac{h_1}{0.3},\frac{h_2}{0.4},\frac{h_3}{0.8}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.6}\right\}\right)\right),\left(e_2,\left(\left\{\frac{h_1}{0.7},\frac{h_2}{0.8},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.6}\right\}\right)\right)\right\}$$

$$(ii).\ (F,\ A)\tilde{\cup}_E\,(G,\ B)=\left\{\left(e_1,\left(\left\{\frac{h_1}{0.3},\frac{h_2}{0.4},\frac{h_3}{0.8}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.6}\right\}\right)\right),\left(e_2,\left(\left\{\frac{h_1}{0.7},\frac{h_2}{0.8},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.6}\right\}\right)\right),\right.$$
$$\left.\left(e_3,\left(\left\{\frac{h_1}{0.9},\frac{h_2}{0.7},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.3},\frac{c_2}{0.5}\right\}\right)\right),\left(e_4,\left(\left\{\frac{h_1}{0.8},\frac{h_2}{0.9},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.5},\frac{c_2}{0.2}\right\}\right)\right)\right\},$$

$$(iii).\ (F,\ A)\tilde{\cap}_R\,(G,\ B)=\left\{\left(e_1,\left(\left\{\frac{h_1}{0.2},\frac{h_2}{0.3},\frac{h_3}{0.7}\right\},\left\{\frac{c_1}{0.7},\frac{c_2}{0.5}\right\}\right)\right),\left(e_2,\left(\left\{\frac{h_1}{0.5},\frac{h_2}{0.7},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.4},\frac{c_2}{0.6}\right\}\right)\right)\right\}$$

$$(iv).\ (F,\ A)\tilde{\cap}_E\,(G,\ B)=\left\{\left(e_1,\left(\left\{\frac{h_1}{0.2},\frac{h_2}{0.3},\frac{h_3}{0.7}\right\},\left\{\frac{c_1}{0.7},\frac{c_2}{0.5}\right\}\right)\right),\left(e_2,\left(\left\{\frac{h_1}{0.5},\frac{h_2}{0.7},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.4},\frac{c_2}{0.6}\right\}\right)\right)\right.$$
$$\left.\left(e_3,\left(\left\{\frac{h_1}{0.9},\frac{h_2}{0.7},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.3},\frac{c_2}{0.5}\right\}\right)\right),\left(e_4,\left(\left\{\frac{h_1}{0.8},\frac{h_2}{0.9},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.5},\frac{c_2}{0.2}\right\}\right)\right)\right\},$$

Proposition 3.16

For two fuzzy soft multi sets *(F, A)* and *(G, B)* over U, then we have

1. $\left((F,\ A)\tilde{\cup}_E\,(G,\ B)\right)^c\,\tilde{\subseteq}\,(F,\ A)^c\,\tilde{\cup}_E\,(G,\ B)^c$
2. $(F,\ A)^c\,\tilde{\cap}_E\,(G,\ B)^c\,\tilde{\subseteq}\left((F,\ A)\tilde{\cap}_E\,(G,\ B)\right)^c$

Proof.

1. Let $(F,\ A)\tilde{\cup}_E\,(G,\ B)=(H,C),$ where $C=A\cup B$ and $\forall e\in C,$

$$H(e)=\begin{cases}F(e), & if\ \ e\in A-B\\ G(e), & if\ \ e\in B-A\\ \bigcup\big(F(e),G(e)\big), & if\ \ e\in A\cap B,\end{cases}$$

$$where\ \bigcup\big(F(e),G(e)\big)=\left(\left\{\frac{u}{\max\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Thus $\left((F,\ A)\tilde{\cup}_E\,(G,\ B)\right)^c=(H,C)^c=(H^c,C),$ where $C=A\cup B$ and $\forall e\in C,$

$$H^c(e)=\begin{cases}F^c(e), & if\ \ e\in A-B\\ G^c(e), & if\ \ e\in B-A\\ \big(\bigcup\big(F(e),G(e)\big)\big)^c, & if\ \ e\in A\cap B,\end{cases}$$

$$where\ \big(\bigcup\big(F(e),G(e)\big)\big)^c=\left(\left\{\frac{u}{1-\max\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$
$$=\left(\left\{\frac{u}{\min\left\{1-\mu_{F(e)}(u),1-\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Again, $(F,\ A)^c\,\tilde{\cup}_E\,(G,\ B)^c=(F^c,\ A)\tilde{\cup}_E\,(G^c,\ B)=\big(K,D\big)$. Where $D{=}A{\cup}B$ and $\forall e\in D,$

$$K(e)=\begin{cases}F^c(e), & if\ \ e\in A-B\\ G^c(e), & if\ \ e\in B-A\\ \bigcup\big(F^c(e),G^c(e)\big), & if\ \ e\in A\cap B,\end{cases}$$

$$where\ \bigcup\left(F^c(e),G^c(e)\right)=\left(\left\{\frac{u}{\max\left\{1-\mu_{F(e)}(u),1-\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right).$$

We see that $C=D$ and $\forall e\in C,\ H^c(e)\subseteq K(e)$.

Thus $\left((F,\ A)\tilde{\cup}_E\,(G,\ B)\right)^c\ \tilde{\subseteq}\ (F,\ A)^c\,\tilde{\cup}_E\,(G,\ B)^c$.

2. Let $(F,\ A)\tilde{\cap}_E\,(G,\ B)=(H,C)$, where $C=A\cup B$ and $\forall e\in C$,

$$H(e)=\begin{cases}F(e), & if\ e\in A-B\\ G(e), & if\ e\in B-A\\ \bigcap\left(F(e),G(e)\right), & if\ e\in A\cap B,\end{cases}$$

$$where\ \bigcap\left(F(e),G(e)\right)=\left(\left\{\frac{u}{\min\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Thus $\left((F,\ A)\tilde{\cap}_E\,(G,\ B)\right)^c=(H,C)^c=(H^c,C)$, where $C=A\cup B$ and $\forall e\in C$,

$$H^c(e)=\begin{cases}F^c(e), & if\ e\in A-B\\ G^c(e), & if\ e\in B-A\\ \left(\bigcap\left(F(e),G(e)\right)\right)^c, & if\ e\in A\cap B,\end{cases}$$

$$where\ \left(\bigcap\left(F(e),G(e)\right)\right)^c=\left(\left\{\frac{u}{1-\min\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

$$=\left(\left\{\frac{u}{\max\left\{1-\mu_{F(e)}(u),1-\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Again, $(F,\ A)^c\,\tilde{\cap}_E\,(G,\ B)^c=(F^c,\ A)\tilde{\cap}_E\,(G^c,\ B)=(K,D)$. Where $D=A\cup B$ and $\forall e\in D$,

$$K(e)=\begin{cases}F^c(e), & if\ e\in A-B\\ G^c(e), & if\ e\in B-A\\ \bigcap\left(F^c(e),G^c(e)\right), & if\ e\in A\cap B,\end{cases}$$

$$where\ \bigcap\left(F^c(e),G^c(e)\right)=\left(\left\{\frac{u}{\min\left\{1-\mu_{F(e)}(u),1-\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right).$$

We see that $C=D$ and $\forall e\in C,\ K(e)\subseteq H^c(e)$.

Thus $(F,\ A)^c\,\tilde{\cap}_E\,(G,\ B)^c\ \tilde{\subseteq}\ \left((F,\ A)\tilde{\cap}_E\,(G,\ B)\right)^c$.

Proposition 3.17

For two fuzzy soft multi sets *(F, A)* and *(G, B)* over U, then we have

1. $\left((F,\ A)\tilde{\cup}_R\,(G,\ B)\right)^c\ \tilde{\subseteq}\ (F,\ A)^c\,\tilde{\cup}_R\,(G,\ B)^c$
2. $(F,\ A)^c\,\tilde{\cap}_R\,(G,\ B)^c\ \tilde{\subseteq}\ \left((F,\ A)\tilde{\cap}_R\,(G,\ B)\right)^c$

Proof.

1. Let $(F, A)\tilde{\cup}_R(G, B)=(H,C)$, where $C=A\cap B$ and $\forall e\in C$,

$$H(e)=\bigcup\left(F(e),G(e)\right)=\left(\left\{\frac{u}{\max\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right).$$

Thus $\left((F, A)\tilde{\cup}_R(G, B)\right)^c=(H,C)^c=(H^c,C)$, where $C=A\cap B$ and $\forall e\in C$,

$$H^c(e)=\left(\bigcup\left(F(e),G(e)\right)\right)^c=\left(\left\{\frac{u}{1-\max\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$
$$=\left(\left\{\frac{u}{\min\left\{1-\mu_{F(e)}(u),1-\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right).$$

Again, $(F, A)^c\tilde{\cup}_R(G, B)^c=(F^c, A)\tilde{\cup}_R(G^c, B)=(K,D)$. Where $D=A\cap B$ and $\forall e\in D$,

$$K(e)=\bigcup\left(F^c(e),G^c(e)\right)=\left(\left\{\frac{u}{\max\left\{1-\mu_{F(e)}(u),1-\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right).$$

We see that $C=D$ and $\forall e\in C$, $H^c(e)\subseteq K(e)$.

Thus $\left((F, A)\tilde{\cup}_R(G, B)\right)^c\tilde{\subseteq}(F, A)^c\tilde{\cup}_R(G, B)^c$.

2. Let $(F, A)\tilde{\cap}_R(G, B)=(H,C)$, where $C=A\cap B$ and $\forall e\in C$,

$$H(e)=\bigcap\left(F(e),G(e)\right)=\left(\left\{\frac{u}{\min\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right).$$

Thus $\left((F, A)\tilde{\cap}_R(G, B)\right)^c=(H,C)^c=(H^c,C)$, where $C=A\cap B$ and $\forall e\in C$,

$$H^c(e)=\left(\bigcap\left(F(e),G(e)\right)\right)^c=\left(\left\{\frac{u}{1-\min\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$
$$=\left(\left\{\frac{u}{\max\left\{1-\mu_{F(e)}(u),1-\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right).$$

Again, $(F, A)^c\tilde{\cap}_R(G, B)^c=(F^c, A)\tilde{\cap}_R(G^c, B)=(K,D)$. Where $D=A\cap B$ and $\forall e\in D$,

$$K(e)=\bigcap\left(F^c(e),G^c(e)\right)=\left(\left\{\frac{u}{\min\left\{1-\mu_{F(e)}(u),1-\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right).$$

We see that $C=D$ and $\forall e\in C$, $K(e)\subseteq H^c(e)$.

Thus $(F, A)^c\tilde{\cap}_R(G, B)^c\tilde{\subseteq}\left((F, A)\tilde{\cap}_R(G, B)\right)^c$.

Proposition 3.18

For two fuzzy soft multi sets *(F, A)* and *(G, B)* over U, then we have

1. $\left((F, A)\tilde{\cap}_R(G, B)\right)^c\subseteq(F, A)^c\tilde{\cup}_E(G, B)^c$
2. $(F, A)^c\tilde{\cap}_R(G, B)^c\subseteq\left((F, A)\tilde{\cup}_E(G, B)\right)^c$

Proof.

1. Let $(F, A)\tilde{\cap}_R(G, B)=(H,C)$, where $C=A\cap B$ and $\forall e\in C$,

$$H(e)=\bigcap\left(F(e),G(e)\right)=\left(\left\{\frac{u}{\min\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Thus $\left((F,\,A)\tilde{\cap}_R\,(G,\,B)\right)^c=(H,C)^c=(H^c,C),$ where $C=A\cap B$ and $\forall e\in C,$

$$H^c(e)=\left(\left\{\frac{u}{1-\min\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

$$=\left(\left\{\frac{u}{\max\left\{1-\mu_{F(e)}(u),1-\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Again, $(F,\,A)^c\,\tilde{\cup}_E\,(G,\,B)^c=(F^c,\,A)\tilde{\cup}_E\,(G^c,\,B)=(K,D)$. Where D=$A\cup B$ and $\forall e\in D,$

$$K(e)=\begin{cases}F^c(e), & \text{if } e\in A-B\\ G^c(e), & \text{if } e\in B-A\\ \bigcup\left(F^c(e),G^c(e)\right), & \text{if } e\in A\cap B,\end{cases}$$

$$\text{where, } \bigcup\left(F^c(e),G^c(e)\right)=\left(\left\{\frac{u}{\max\left\{1-\mu_{F(e)}(u),1-\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

We see that $C\subseteq D$ and $\forall e\in C,\ H^c(e)=K(e)$.

Thus $\left((F,\,A)\tilde{\cap}_R\,(G,\,B)\right)^c\subseteq(F,\,A)^c\,\tilde{\cup}_E\,(G,\,B)^c$.

2. Let $(F,\,A)^c\,\tilde{\cap}_R\,(G,\,B)^c=(F^c,\,A)\tilde{\cap}_R\,(G^c,\,B)=(H,C),$ where $C=A\cap B$ and $\forall e\in C,$

$$H(e)=\bigcap\left(F^c(e),G^c(e)\right)=\left(\left\{\frac{u}{\min\left\{1-\mu_{F(e)}(u),1-\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

$$=\left(\left\{\frac{u}{1-\max\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Again, let $(F,\,A)\tilde{\cup}_E\,(G,\,B)=(K,D)$. Where D=$A\cup B$ and $\forall e\in D,$

$$K(e)=\begin{cases}F(e), & \text{if } e\in A-B\\ G(e), & \text{if } e\in B-A\\ \bigcup\left(F(e),G(e)\right), & \text{if } e\in A\cap B,\end{cases}$$

$$\text{where, } \bigcup\left(F(e),G(e)\right)=\left(\left\{\frac{u}{\max\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Thus $\left((F,\,A)\tilde{\cup}_E\,(G,\,B)\right)^c=\left(K,D\right)^c=(K^c,D),$ where D=$A\cup B$ and $\forall e\in D,$

$$K^c(e)=\begin{cases}F^c(e), & \text{if } e\in A-B\\ G^c(e), & \text{if } e\in B-A\\ \left(\bigcup\left(F(e),G(e)\right)\right)^c, & \text{if } e\in A\cap B,\end{cases}$$

$$\text{where, } \left(\bigcup\left(F(e),G(e)\right)\right)^c=\left(\left\{\frac{u}{1-\max\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

We see that $C\subseteq D$ and $\forall e\in C,\ H(e)=K^c(e)$.

Thus $(F,\,A)^c\,\tilde{\cap}_R\,(G,\,B)^c\subseteq\left((F,\,A)\tilde{\cup}_E\,(G,\,B)\right)^c$.

Proposition 3.19(De Morgan Laws)

For two fuzzy soft multi sets (F, A) and (G, B) over U, then we have

[1]. $\left((F, A)\tilde{\cup}_R (G, B)\right)^c = (F, A)^c \tilde{\cap}_R (G, B)^c$

[2]. $\left((F, A)\tilde{\cap}_R (G, B)\right)^c = (F, A)^c \tilde{\cup}_R (G, B)^c$

Proof.

[1]. Let $(F, A)\tilde{\cup}_R (G, B) = (H, C),$ where $C = A \cap B$ and $\forall e \in C,$

$$H(e) = \bigcup\left(F(e), G(e)\right) = \left(\left\{\frac{u}{\max\left\{\mu_{F(e)}(u), \mu_{G(e)}(u)\right\}} : u \in U_i\right\} : i \in I\right).$$

Thus $\left((F, A)\tilde{\cup}_R (G, B)\right)^c = (H, C)^c = (H^c, C),$ where $C = A \cap B$ and $\forall e \in C,$

$$H^c(e) = \left(\left\{\frac{u}{1-\max\left\{\mu_{F(e)}(u), \mu_{G(e)}(u)\right\}} : u \in U_i\right\} : i \in I\right)$$
$$= \left(\left\{\frac{u}{\min\left\{1-\mu_{F(e)}(u), 1-\mu_{G(e)}(u)\right\}} : u \in U_i\right\} : i \in I\right)$$

Again, $(F, A)^c \tilde{\cap}_R (G, B)^c = (F^c, A)\tilde{\cap}_R (G^c, B) = (K, D),$ where $D = A \cap B$ and $\forall e \in D,$

$$K(e) = \bigcap\left(F^c(e), G^c(e)\right) = \left(\left\{\frac{u}{\min\left\{1-\mu_{F(e)}(u), 1-\mu_{G(e)}(u)\right\}} : u \in U_i\right\} : i \in I\right).$$

We see that $C=D$ and $\forall e \in C,\ H^c(e) = K(e)$. Hence proved.

[2]. Let $(F, A)\tilde{\cap}_R (G, B) = (H, C),$ where $C = A \cap B$ and $\forall e \in C,$

$$H(e) = \bigcap\left(F(e), G(e)\right) = \left(\left\{\frac{u}{\min\left\{\mu_{F(e)}(u), \mu_{G(e)}(u)\right\}} : u \in U_i\right\} : i \in I\right).$$

Thus $\left((F, A)\tilde{\cap}_R (G, B)\right)^c = (H, C)^c = (H^c, C),$ where $C = A \cap B$ and $\forall e \in C,$

$$H^c(e) = \left(\left\{\frac{u}{1-\min\left\{\mu_{F(e)}(u), \mu_{G(e)}(u)\right\}} : u \in U_i\right\} : i \in I\right)$$
$$= \left(\left\{\frac{u}{\max\left\{1-\mu_{F(e)}(u), 1-\mu_{G(e)}(u)\right\}} : u \in U_i\right\} : i \in I\right)$$

Again, $(F, A)^c \tilde{\cup}_R (G, B)^c = (F^c, A)\tilde{\cup}_R (G^c, B) = (K, D),$ where $D = A \cap B$ and $\forall e \in D,$

$$K(e) = \bigcup\left(F^c(e), G^c(e)\right) = \left(\left\{\frac{u}{\max\left\{1-\mu_{F(e)}(u), 1-\mu_{G(e)}(u)\right\}} : u \in U_i\right\} : i \in I\right).$$

We see that $C=D$ and $\forall e \in C,\ H^c(e) = K(e)$. Hence proved.

Proposition 3.20(De Morgan Laws)

For two fuzzy soft multi sets *(F, A) and (G, B)* over U, then we have

1. $\left((F, A)\tilde{\cup}_E (G, B)\right)^c = (F, A)^c \tilde{\cap}_E (G, B)^c$
2. $\left((F, A)\tilde{\cap}_E (G, B)\right)^c = (F, A)^c \tilde{\cup}_E (G, B)^c$

Proof.

1. Let $(F,\ A)\tilde{\cup}_E(G,\ B)=(H,C)$, where $C=A\cup B$ and $\forall e\in C$,

$$H(e)=\begin{cases} F(e), & if\ \ e\in A-B \\ G(e), & if\ \ e\in B-A \\ \bigcup\left(F(e),G(e)\right), & if\ \ e\in A\cap B, \end{cases}$$

$$where\ \bigcup\left(F(e),G(e)\right)=\left(\left\{\frac{u}{\max\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Thus $\left((F,\ A)\tilde{\cup}_E(G,\ B)\right)^c=(H,C)^c=(H^c,C)$, where $C=A\cup B$ and $\forall e\in C$,

$$H^c(e)=\begin{cases} F^c(e), & if\ \ e\in A-B \\ G^c(e), & if\ \ e\in B-A \\ \left(\bigcup\left(F(e),G(e)\right)\right)^c, & if\ \ e\in A\cap B, \end{cases}$$

$$\begin{aligned} where\ \left(\bigcup\left(F(e),G(e)\right)\right)^c &=\left(\left\{\frac{u}{1-\max\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right) \\ &=\left(\left\{\frac{u}{\min\left\{1-\mu_{F(e)}(u),1-\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right) \end{aligned}$$

Again, $(F,\ A)^c\tilde{\cap}_E(G,\ B)^c=(F^c,\ A)\tilde{\cap}_E(G^c,\ B)=(K,D)$, where $D{=}A{\cup}B$ and $\forall e\in D$,

$$K(e)=\begin{cases} F^c(e), & if\ \ e\in A-B \\ G^c(e), & if\ \ e\in B-A \\ \bigcap\left(F^c(e),G^c(e)\right), & if\ \ e\in A\cap B, \end{cases}$$

$$where\ \bigcap\left(F^c(e),G^c(e)\right)=\left(\left\{\frac{u}{\min\left\{1-\mu_{F(e)}(u),1-\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

We see that $C{=}D$ and $\forall e\in C,\ H^c(e)=K(e)$. Thus the result.

2. Let $(F,\ A)\tilde{\cap}_E(G,\ B)=(H,C)$, where $C=A\cup B$ and $\forall e\in C$,

$$H(e)=\begin{cases} F(e), & if\ \ e\in A-B \\ G(e), & if\ \ e\in B-A \\ \bigcap\left(F(e),G(e)\right), & if\ \ e\in A\cap B, \end{cases}$$

$$where\ \bigcap\left(F(e),G(e)\right)=\left(\left\{\frac{u}{\min\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Thus $\left((F,\ A)\tilde{\cap}_E(G,\ B)\right)^c=(H,C)^c=(H^c,C)$, where $C=A\cup B$ and $\forall e\in C$,

$$H^c(e)=\begin{cases} F^c(e), & if\ \ e\in A-B \\ G^c(e), & if\ \ e\in B-A \\ \left(\bigcap\left(F(e),G(e)\right)\right)^c, & if\ \ e\in A\cap B, \end{cases}$$

$$where\ \left(\cap\left(F(e),G(e)\right)\right)^c = \left(\left\{\frac{u}{1-\min\left\{\mu_{F(e)}(u),\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$
$$= \left(\left\{\frac{u}{\max\left\{1-\mu_{F(e)}(u),1-\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

Again, $(F,\ A)^c\ \tilde{\cup}_E\ (G,\ B)^c = (F^c,\ A)\ \tilde{\cup}_E\ (G^c,\ B) = (K,D)$, where $D=A\cup B$ and $\forall e\in D$,

$$K(e)=\begin{cases} F^c(e), & if\ e\in A-B \\ G^c(e), & if\ e\in B-A \\ \cup\left(F^c(e),G^c(e)\right), & if\ e\in A\cap B, \end{cases}$$

$$where\ \cup\left(F^c(e),G^c(e)\right) = \left(\left\{\frac{u}{\max\left\{1-\mu_{F(e)}(u),1-\mu_{G(e)}(u)\right\}}:u\in U_i\right\}:i\in I\right)$$

We see that $C=D$ and $\forall e\in C,\ H^c(e)=K(e)$. Hence proved.

Definition 3.21

If (F, A) and (G, B) be two fuzzy soft multi sets over U, then "(F,A) AND (G,B)" is a fuzzy soft multi set denoted by $(F,A)\wedge(G,B)$ and is defined by $(F,A)\wedge(G,B)=(H,A\times B)$, where H is mapping given by H: A×B→U and

$$\forall(a,b)\in A\times B,\ H(a,b)=\left(\left\{\frac{u}{\min\left\{\mu_{F(a)}(u),\mu_{G(b)}(u)\right\}}:u\in U_i\right\}:i\in I\right).$$

Example 3.22

If we consider two fuzzy soft multi sets (F, A) and (G, B) as in example 3.15, then we have

$$\begin{aligned}(F,A)\wedge(G,B)=\Bigg\{&\left((e_1,e_1),\left(\left\{\frac{h_1}{0.2},\frac{h_2}{0.3},\frac{h_3}{0.7}\right\},\left\{\frac{c_1}{0.7},\frac{c_2}{0.5}\right\}\right)\right),\left((e_1,e_2),\left(\left\{\frac{h_1}{0.2},\frac{h_2}{0.4},\frac{h_3}{0.8}\right\},\left\{\frac{c_1}{0.4},\frac{c_2}{0.5}\right\}\right)\right),\\
&\left((e_1,e_4),\left(\left\{\frac{h_1}{0.2},\frac{h_2}{0.4},\frac{h_3}{0.8}\right\},\left\{\frac{c_1}{0.5},\frac{c_2}{0.2}\right\}\right)\right),\left((e_2,e_1),\left(\left\{\frac{h_1}{0.3},\frac{h_2}{0.3},\frac{h_3}{0.7}\right\},\left\{\frac{c_1}{0.7},\frac{c_2}{0.6}\right\}\right)\right),\\
&\left((e_2,e_2),\left(\left\{\frac{h_1}{0.5},\frac{h_2}{0.7},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.4},\frac{c_2}{0.6}\right\}\right)\right),\left((e_2,e_4),\left(\left\{\frac{h_1}{0.7},\frac{h_2}{0.7},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.5},\frac{c_2}{0.2}\right\}\right)\right),\\
&\left((e_3,e_1),\left(\left\{\frac{h_1}{0.3},\frac{h_2}{0.3},\frac{h_3}{0.7}\right\},\left\{\frac{c_1}{0.3},\frac{c_2}{0.5}\right\}\right)\right),\left((e_3,e_2),\left(\left\{\frac{h_1}{0.5},\frac{h_2}{0.7},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.3},\frac{c_2}{0.5}\right\}\right)\right),\\
&\left((e_3,e_4),\left(\left\{\frac{h_1}{0.8},\frac{h_2}{0.7},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.3},\frac{c_2}{0.2}\right\}\right)\right)\Bigg\}\end{aligned}$$

Definition 3.23

If $(F,\ A)$ and $(G,\ B)$ be two fuzzy soft multi sets over U, then "(F,A) OR (G,B)" is a fuzzy soft multi set denoted by $(F,A)\vee(G,B)$ and is defined by $(F,A)\vee(G,B)=(K,A\times B)$, where K is mapping given by K: A×B→U and

$$\forall (a,b)\in A\times B,\ K(a,b)=\left(\left\{\frac{u}{\max\{\mu_{F(a)}(u),\mu_{G(b)}(u)\}}:u\in U_i\right\}:i\in I\right)$$

Example 3.24

If we consider two fuzzy soft multi sets (*F, A*) and (*G, B*) as in example 3.15, then we have

$$\begin{aligned}(F,A)\vee(G,B)=\Bigg\{&\left((e_1,e_1),\left(\left\{\frac{h_1}{0.3},\frac{h_2}{0.4},\frac{h_3}{0.8}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.6}\right\}\right)\right),\left((e_1,e_2),\left(\left\{\frac{h_1}{0.5},\frac{h_2}{0.8},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.6}\right\}\right)\right),\\ &\left((e_1,e_4),\left(\left\{\frac{h_1}{0.8},\frac{h_2}{0.9},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.5}\right\}\right)\right),\left((e_2,e_1),\left(\left\{\frac{h_1}{0.7},\frac{h_2}{0.7},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.6}\right\}\right)\right),\\ &\left((e_2,e_2),\left(\left\{\frac{h_1}{0.7},\frac{h_2}{0.8},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.6}\right\}\right)\right),\left((e_2,e_4),\left(\left\{\frac{h_1}{0.8},\frac{h_2}{0.9},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.6}\right\}\right)\right),\\ &\left((e_3,e_1),\left(\left\{\frac{h_1}{0.9},\frac{h_2}{0.7},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.7},\frac{c_2}{0.6}\right\}\right)\right),\left((e_3,e_2),\left(\left\{\frac{h_1}{0.9},\frac{h_2}{0.8},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.4},\frac{c_2}{0.6}\right\}\right)\right),\\ &\left((e_3,e_4),\left(\left\{\frac{h_1}{0.9},\frac{h_2}{0.9},\frac{h_3}{1}\right\},\left\{\frac{c_1}{0.5},\frac{c_2}{0.5}\right\}\right)\right)\Bigg\}\end{aligned}$$

Proposition 3.25

For two fuzzy soft multi sets (*F, A*) and (*G, B*) over U, we have the following

1. $((F,\ A)\wedge(G,\ B))^c=(F,\ A)^c\vee(G,\ B)^c$
2. $((F,\ A)\vee(G,\ B))^c=(F,\ A)^c\wedge(G,\ B)^c$

Proof.

1. Let $(F,\ A)\wedge(G,\ B)=(H,A\times B),$ where $\forall a\in A\ and\ \ \forall b\in B,$

$$H(a,b)=\bigcap\left(F(a),G(b)\right)=\left(\left\{\frac{u}{\min\{\mu_{F(a)}(u),\mu_{G(b)}(u)\}}:u\in U_i\right\}:i\in I\right)$$

Thus $((F,\ A)\wedge(G,\ B))^c=(H,A\times B)^c=(H^c,A\times B),$ where $\forall (a,b)\in A\times B,$

$$\begin{aligned}H^c(a,b)&=\left(\left\{\frac{u}{1-\min\{\mu_{F(a)}(u),\mu_{G(b)}(u)\}}:u\in U_i\right\}:i\in I\right)\\ &=\left(\left\{\frac{u}{\max\{1-\mu_{F(a)}(u),1-\mu_{G(b)}(u)\}}:u\in U_i\right\}:i\in I\right)\end{aligned}$$

Again, let $(F,\ A)^c\vee(G,\ B)^c=(F^c,\ A)\vee(G^c,\ B)=(K,A\times B)$, where $\forall (a,b)\in A\times B,$

$$K(a,b)=\bigcup\left(F^c(a),G^c(b)\right)=\left(\left\{\frac{u}{\max\{1-\mu_{F(a)}(u),1-\mu_{G(b)}(u)\}}:u\in U_i\right\}:i\in I\right).$$

Thus it follows that $((F,\ A)\wedge(G,\ B))^c=(F,\ A)^c\vee(G,\ B)^c$.

2. Let $(F,\ A)\vee(G,\ B)=(H,A\times B),$ where $\forall a\in A\ and\ \ \forall b\in B,$

$$H(a,b)=\bigcup\left(F(a),G(b)\right)=\left(\left\{\frac{u}{\max\{\mu_{F(a)}(u),\mu_{G(b)}(u)\}}:u\in U_i\right\}:i\in I\right)$$

Thus $((F, A)\vee(G, B))^c = (H, A\times B)^c = (H^c, A\times B),$ where $\forall (a,b)\in A\times B,$

$$H^c(a,b) = \left(\left\{\frac{u}{1-\max\{\mu_{F(a)}(u),\mu_{G(b)}(u)\}} : u\in U_i\right\} : i\in I\right)$$
$$= \left(\left\{\frac{u}{\min\{1-\mu_{F(a)}(u),1-\mu_{G(b)}(u)\}} : u\in U_i\right\} : i\in I\right)$$

Again, let $(F, A)^c \wedge (G, B)^c = (F^c, A)\wedge(G^c, B) = (K, A\times B)$, where $\forall (a,b)\in A\times B,$

$$K(a,b) = \bigcap\left(F^c(a), G^c(b)\right) = \left(\left\{\frac{u}{\min\{1-\mu_{F(a)}(u),1-\mu_{G(b)}(u)\}} : u\in U_i\right\} : i\in I\right).$$

Thus it follows that $((F, A)\vee(G, B))^c = (F, A)^c \wedge (G, B)^c$.

4. CONCLUSION

In this paper, we have made an investigation on existing basic notions and results on fuzzy soft multi sets. Some new results have been stated in our work. Here we shall define some new operations in fuzzy soft multi set theory and show that the De Morgan's type of results holds in fuzzy soft multi set theory with respect to these newly defined operations in our way. These properties may be used in real life problems, like decision making problem, inventory control problem, etc.

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