ANALYSIS AND SLIDING CONTROLLER DESIGN FOR HYBRID SYNCHRONIZATION OF HYPERCHAOTIC YUJUN SYSTEMS

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ABSTRACT

Hybrid synchronization of chaotic systems is a research problem with a goal to synchronize the states of master and slave chaotic systems in a hybrid manner, namely, their even states are completely synchronized (CS) and odd states are anti-synchronized. This paper deals with the research problem of hybrid synchronization of chaotic systems. First, a detailed analysis is made on the qualitative properties of hyperchaotic Yujun system (2010). Then sliding controller has been derived for the hybrid synchronization of identical hyperchaotic Yujun systems, which is based on a general hybrid result derived in this paper. MATLAB simulations have been shown in detail to illustrate the new results derived for the hybrid synchronization of hyperchaotic Yujun systems. The results are proved using Lyapunov stability theory.

KEYWORDS

Hyperchaotic Yujun System, Hyperchaos, Sliding Control, Synchronization, Lyapunov Stability Theory.

1. INTRODUCTION

By definition, a *hyperchaotic system* is a chaotic system with more than one positive Lyapunov exponent which implies that the dynamics can be expanded in several directions.

In 1979, Rössler [1] discovered the hyperchaotic system. Hyperchaotic systems have found many research applications in neural networks [2], secure communications [3], etc.

In the chaos literature, there has been active interest in developing new methodology for synchronization of chaotic and hyperchaotic systems.

Some important methods developed for chaos synchronization are OGY method [4], PC method [5], sampled-data feedback method [6], time-delay feedback method [7], active control method [8-10], adaptive control method [11-13], backstepping design method [14-15] and sliding mode control method [16-18].

In this paper, a qualitative analysis has been presented for the 4-D hyperchaotic Yujun system ([19], 2010). Next, a sliding mode controller has been derived for the hybrid synchronization of identical hyperchaotic Yujun systems.

The organization of this paper can be detailed as follows. Section 2 contains general sliding control results for the hybrid synchronization of master and slave systems. Section 3 describes the hyperchaotic Yujun system. Section 4 describes the sliding controller design for the hybrid synchronization of hyperchaotic Yujun systems and MATLAB simulations.

As the *drive* system, we take the chaotic system

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in R^n$ is the system state, A is an $n \times n$ constant matrix and $f : R^n \to R^n$ is the nonlinear part.

As the *response* system, we take the identical controlled chaotic system

$$\dot{y} = Ay + f(y) + u \tag{2}$$

where $y \in R^n$ is the system state and $u \in R^m$ is the sliding controller to be constructed.

A formal definition of the *hybrid synchronization error* between the drive system (1) and the response system (2) can be provided as

$$e_i = \begin{cases} y_i - x_i, & \text{when} \quad i \text{ is an odd integer} \\ y_i + x_i, & \text{when} \quad i \text{ is an even integer} \end{cases} \tag{3}$$

Differentiating (3) and simplifying, we obtain the error dynamics as

$$\dot{e} = Ae + \eta(x, y) + u, \tag{4}$$

The design goal is to determine $u(t)$ so as to drive the error to zero, *i.e.*

$$\lim_{t \to \infty} \left\| e(t) \right\| = 0 \quad \text{for each } e(0) \in R^n. \tag{5}$$

As methodology, we use sliding mode control (SMC) to solve the above design problem.

First, we define the controller u as

$$u = -\eta(x, y) + Bv \tag{6}$$

where B is chosen carefully such that (A, B) is completely controllable.

If we substitute (5) into (4), then the error dynamics takes the simple form

$$\dot{e} = Ae + Bv \tag{7}$$

Eq. (7) represents a single-input linear time-invariant control system.

In SMC control design, we take the sliding variable as

$$s(e) = Ce = c_1 e_1 + c_2 e_2 + \cdots + c_n e_n \tag{8}$$

where $C = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}$ is a constant vector to be determined.

In SMC design, the error system (7) is confined to the sliding manifold defined by

$$S = \left\{ x \in R^n \mid s(e) = 0 \right\}$$

The sliding control theory demands that the manifold S be invariant under the flow of (7).

Thus, the error system (7) must satisfy the following two conditions, *viz.*

$$s(e) = 0 \tag{9}$$

and

$$\dot{s}(e) = 0 \tag{10}$$

By making use of Eqns. (7) and (8), Eq. (10) can be simplified in an equivalent form as

$$\dot{s}(e) = C\left[Ae + Bv \right] = 0 \tag{11}$$

Next, we find the solution v of the equation (11). Thus, we get the equivalent control law

$$v_{\text{eq}}(t) = -(CB)^{-1} CA\, e(t) \tag{12}$$

In Eq. (12), C is selected such that CB is a non-zero scalar.

Plugging in the equivalent control law (12) into (7) yields the simplified dynamics

$$\dot{e} = \left[I - B(CB)^{-1} C \right] Ae \tag{13}$$

We choose C so that the system matrix $\left[I - B(CB)^{-1} C \right] A$ is stable.

It is noted that the resulting system (13) is GAS (globally asymptotically stable).

In our sliding control methodology, to obtain the SMC control for (7), we make use of the constant plus proportional rate law. This is given by the equation

$$\dot{s} = -q\,\text{sgn}(s) - k\, s \tag{14}$$

where the gains $q > 0, \;\; k > 0$ are constants.

We shall find the gains so that the following hold:

(1) The sliding condition is satisfied.

(2) The sliding motion will occur.

By the use of equations (11) and (14), we get the sliding control $v(t)$ as

$$v(t) = -(CB)^{-1}\left[C(kI + A)e + q\,\mathrm{sgn}(s)\right] \tag{15}$$

Theorem 1. *The drive system (1) and the response system (2), which are identical chaotic systems, are globally and asymptotically hybrid synchronized for all initial conditions* $x(0), y(0) \in R^n$ *by the sliding control law*

$$u(t) = -\eta(x, y) + Bv(t) \tag{16}$$

where $v(t)$ *is given by (15) and* B *is chosen such that* (A, B) *is controllable.*

Proof. We prove this theorem by applying Lyapunov stability theory [20].

First, we make a substitution of (16) and (15) into the error system (4).

After this substitution, a simple calculation shows that the error dynamics reduces to

$$\dot{e} = Ae - B(CB)^{-1}\left[C(kI + A)e + q\,\mathrm{sgn}(s)\right] \tag{17}$$

As we prove using Lyapunov stability theory, we have to construct a Lyapunov function for the error dynamics (17). We consider the following function as a candidate, *viz.*

$$V(e) = \frac{1}{2}s^2(e) \tag{18}$$

We note that V is a positive definite function on R^n.

Next, we find the time derivative of V along the trajectories of (16) or the equivalent dynamics (14). Thus, we obtain

$$\dot{V}(e) = s(e)\dot{s}(e) = -ks^2 - q\,\mathrm{sgn}(s)s \tag{19}$$

We note that $\dot{V}$ is a negative definite function on R^n.

Hence, the proof is complete by Lyapunov stability theory [20].

3. Analysis of Hyperchaotic Yujun System

The hyperchaotic Yujun system ([19], 2010) is a four-dimensional system given by

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_2x_3 \\
\dot{x}_2 &= cx_1 - x_2 + x_4 - x_1x_3 \\
\dot{x}_3 &= -bx_3 + x_1x_2 \\
\dot{x}_4 &= rx_4 - x_1x_3
\end{aligned} \tag{20}$$

where a, b, c, r are positive parameters and x_1, x_2, x_3, x_4 are the states of the system.

The system (20) is *hyperchaotic* when the parameter values are chosen as

$$a = 35, \quad b = 8/3, \quad c = 55 \text{ and } r = 1.3 \tag{21}$$

When the parameters are given by (21), the system Lyapunov exponents are obtained as

$$L_1 = 1.4164, \quad L_2 = 0.5318, \quad L_3 = 0, \quad L_4 = -39.1015. \tag{22}$$

Since there are two positive Lyapunov exponents, the above calculation confirms that the system (20) is hyperchaotic when the parameter values are given by (21).

The projections of the hyperchaotic Yujun attractor are shown in Figure 1.

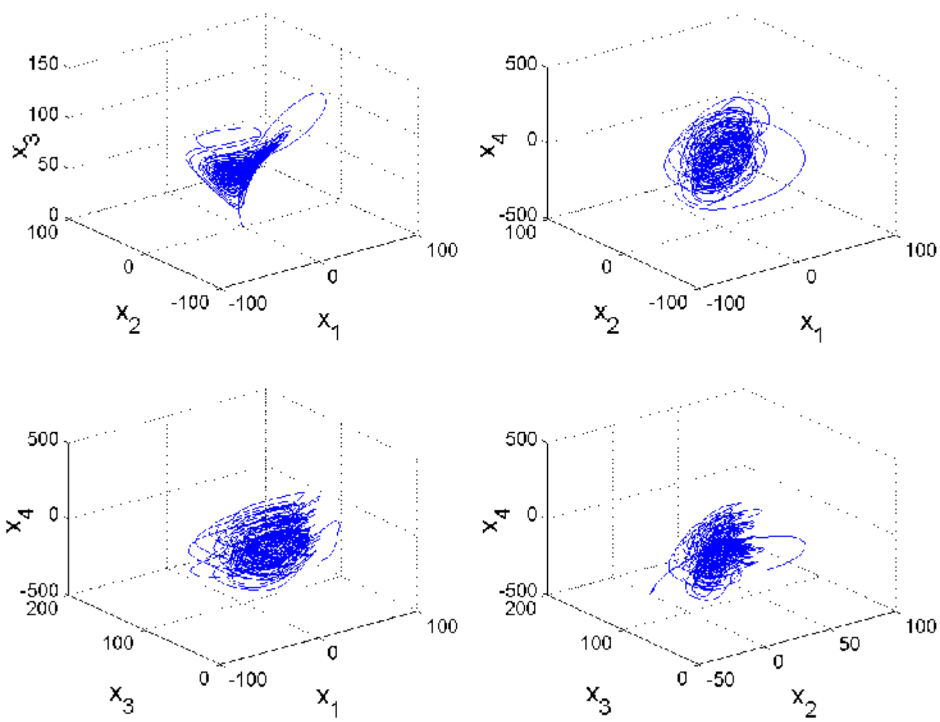

Figure 1. The Projections of the Hyperchaotic Yujun Attractor

4. SMC Design for Hybrid Synchronization of Hyperchaotic Yujun Systems

In this section, we take the drive system as the hyperchaotic Yujun system given by

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\
\dot{x}_2 &= c x_1 - x_2 + x_4 - x_1 x_3 \\
\dot{x}_3 &= -b x_3 + x_1 x_2 \\
\dot{x}_4 &= r x_4 - x_1 x_3
\end{aligned} \tag{23}$$

where a, b, c, r are positive, constant parameters of the system.

The response system is described by the controlled hyperchaotic Yujun system as

$$\begin{aligned}\dot{y}_1 &= a(y_2 - y_1) + y_2 y_3 + u_1 \\ \dot{y}_2 &= c y_1 - y_2 + y_4 - y_1 y_3 + u_2 \\ \dot{y}_3 &= -b y_3 + y_1 y_2 + u_3 \\ \dot{y}_4 &= r y_4 - y_1 y_3 + u_4\end{aligned} \tag{24}$$

where u_1, u_2, u_3, u_4 are the sliding mode controllers to be designed using the systematic procedure outlined in Section 2.

The hybrid synchronization error is given by the definition

$$\begin{aligned}e_1 &= y_1 - x_1 \\ e_2 &= y_2 + x_2 \\ e_3 &= y_3 - x_3 \\ e_4 &= y_4 + x_4\end{aligned} \tag{25}$$

A simple workout yields the dynamics of (25) as

$$\begin{aligned}\dot{e}_1 &= a(e_2 - e_1) - 2ae_2 + u_1 \\ \dot{e}_2 &= 4e_1 + ce_2 + 4e_4 + 8x_1 - 10(y_1 y_3 + x_1 x_3) + u_2 \\ \dot{e}_3 &= -be_3 + y_2^2 - x_2^2 + u_3 \\ \dot{e}_4 &= -de_1 - 2dx_1 + u_4\end{aligned} \tag{26}$$

We write the error dynamics (26) in a matrix form.

Thus, we get the system

$$\dot{e} = Ae + \eta(x, y) + u \tag{27}$$

In Eq. (27), the associated matrix and vectors are

$$A = \begin{bmatrix} -a & a & 0 & 0 \\ 4 & c & 0 & 4 \\ 0 & 0 & -b & 0 \\ -d & 0 & 0 & 0 \end{bmatrix}, \quad \eta(x, y) = \begin{bmatrix} -2ax_2 \\ 8x_1 - 10(y_1 y_3 + x_1 x_3) \\ y_2^2 - x_2^2 \\ -2dx_1 \end{bmatrix} \text{ and } u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{28}$$

Next, we carry out the sliding controller design for the hybrid synchronization of the hyperchaotic systems considered in this paper.

For this paper, we first define u as

$$u = -\eta(x, y) + Bv \tag{29}$$

In (29), the matrix B is chosen such that (A, B) is controllable.

A simple selection for B is

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{30}$$

We take the parameter values of the hyperchaotic Yujun system as

$$a = 35, \quad b = 3, \quad c = 21 \quad \text{and} \quad d = 2 \tag{31}$$

We also take the sliding mode variable as

$$s = Ce = \begin{bmatrix} -1 & -2 & 0 & 1 \end{bmatrix} e = -e_1 - 2e_2 + e_4 \tag{32}$$

The choice of C renders the system globally asymptotically stable in the sliding mode.

We take the sliding mode gains k and q as

$$k = 6 \ \text{ and } \ q = 0.2$$

By making use of Eq. (15), we derive the sliding control $v(t)$ as

$$v(t) = 9.5e_1 - 44.5e_2 - e_4 + 0.1\,\text{sgn}(s) \tag{33}$$

Finally, for the hybrid synchronization of hyperchaotic Yujun systems, we get the sliding control law given by the equation

$$u = -\eta(x, y) + Bv \tag{34}$$

Theorem 2. *The identical hyperchaotic Yujun systems (23) and (24) are asymptotically hybrid synchronized for all initial conditions by implementing the sliding mode controller* u *defined by (34).*

3.2 Numerical Results

The initial values of the master system (23) are taken as

$$x_1(0) = 20, \ x_2(0) = -6, \ x_3(0) = 15, \ x_4(0) = 26$$

and the initial values of the slave system (24) are taken as

$$y_1(0) = -4, \ y_2(0) = 26, \ y_3(0) = -11, \ y_4(0) = 28$$

Figure 2 depicts the hybrid synchronization of the identical hyperchaotic Yujun systems.

Figure 3 depicts the time history of the synchronization error states $e_1(t), e_2(t), e_3(t), e_4(t)$.

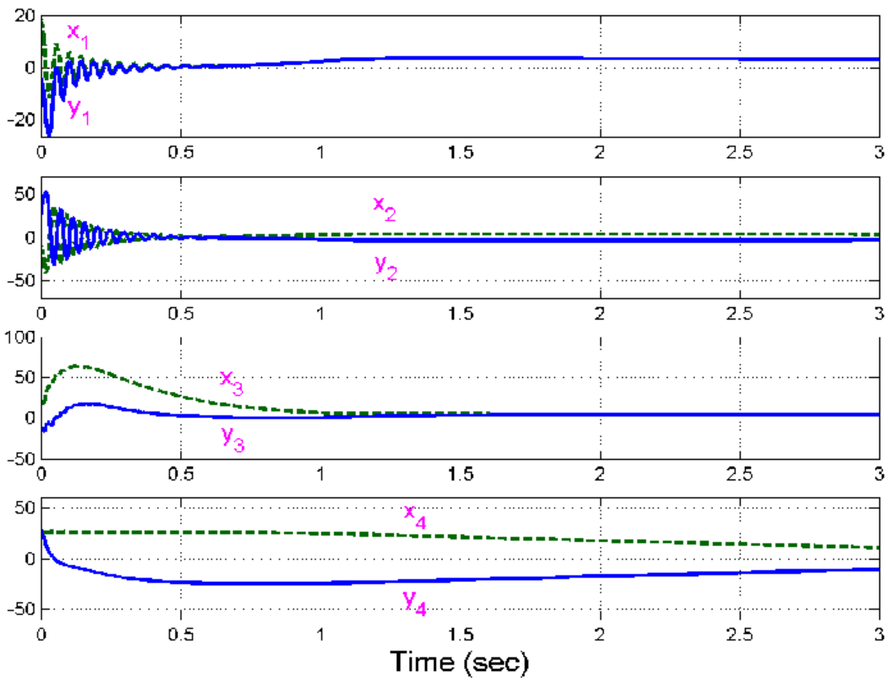

Figure 2. Hybrid Synchronization of Identical Hyperchaotic Yujun Systems

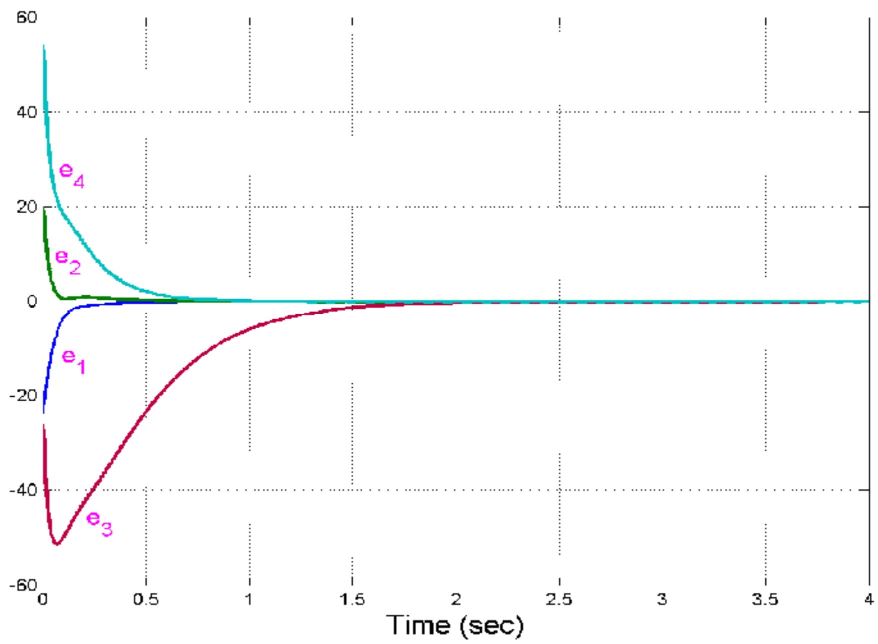

Figure 3. Time History of Error States $e_1(t),\ e_2(t),\ e_3(t),\ e_4(t)$

4. CONCLUSIONS

This paper investigated the hybrid synchronization problem for the hyperchaotic systems using sliding mode control (SMC) and derived some general results. Lyapunov stability theory was the methodology used for proving these results. Using this general result for hybrid synchronization, we derived a new sliding controller for achieving hybrid synchronization of the identical hyperchaotic Yujun systems (2010). MATLAB simulation results were provided to illustrate the results derived in this paper.

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