HYBRID CHAOS SYNCHRONIZATION OF HYPERCHAOTIC LIU AND HYPERCHAOTIC CHEN SYSTEMS BY ACTIVE NONLINEAR CONTROL

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ABSTRACT

This paper investigates the hybrid chaos synchronization of identical 4-D hyperchaotic Liu systems (2006), 4-D identical hyperchaotic Chen systems (2005) and hybrid synchronization of 4-D hyperchaotic Liu and hyperchaotic Chen systems. The hyperchaotic Liu system (Wang and Liu, 2005) and hyperchaotic Chen system (Li, Tang and Chen, 2006) are important models of new hyperchaotic systems. Hybrid synchronization of the 4-dimensional hyperchaotic systems addressed in this paper is achieved through complete synchronization of two pairs of states and anti-synchronization of the other two pairs of states of the underlying systems. Active nonlinear control is the method used for the hybrid synchronization of identical and different hyperchaotic Liu and hyperchaotic Chen and the stability results have been established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the proposed nonlinear control method is effective and convenient to achieve hybrid synchronization of the hyperchaotic Liu and hyperchaotic Chen systems. Numerical simulations are shown to demonstrate the effectiveness of the proposed chaos synchronization schemes.

KEYWORDS

Active Nonlinear Control, Hybrid Synchronization, Hyperchaos, Hyperchaotic Liu System, Hyperchaotic Chen System.

1. INTRODUCTION

Chaotic systems are dynamical systems that are highly sensitive to initial conditions. The sensitive nature of chaotic systems is commonly called as the *butterfly effect* [1].

Synchronization of chaotic systems is a phenomenon that may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect, which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem in the chaos literature [1-23].

In 1990, Pecora and Carroll [2] introduced a method to synchronize two identical chaotic systems and showed that it was possible for some chaotic systems to be completely synchronized. From then on, chaos synchronization has been widely explored in a variety of fields including physical systems [3], chemical systems [4], ecological systems [5], secure communications [6-7], etc.

In most of the chaos synchronization approaches, the *master-slave* or *drive-response* formalism has been used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

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Since the seminal work by Pecora and Carroll [2], a variety of impressive approaches have been proposed for the synchronization of chaotic systems such as the sampled-data feedback synchronization method [8], OGY method [9], time-delay feedback method [10], backstepping method [11], adaptive design method [12], sliding mode control method [13], etc.

So far, many types of synchronization phenomenon have been presented such as complete synchronization [2], phase synchronization [5, 14], generalized synchronization [7, 15], anti-synchronization [16, 17], projective synchronization [18], generalized projective synchronization [19, 20], etc.

Complete synchronization (CS) is characterized by the equality of state variables evolving in time, while anti-synchronization (AS) is characterized by the disappearance of the sum of relevant variables evolving in time. Projective synchronization (PS) is characterized by the fact that the master and slave systems could be synchronized up to a scaling factor, whereas in generalized projective synchronization (GPS), the responses of the synchronized dynamical states synchronize up to a constant scaling matrix α. It is easy to see that the complete synchronization (CS) and anti-synchronization (AS) are special cases of the generalized projective synchronization (GPS) where the scaling matrix $\alpha = I$ and $\alpha = -I$, respectively.

In hybrid synchronization of chaotic systems [20], one part of the system is synchronized and the other part is anti-synchronized so that the complete synchronization (CS) and anti-synchronization (AS) coexist in the system. The coexistence of CS and AS is highly useful in secure communication and chaotic encryptation schemes.

This paper is organized as follows. In Section 2, we derive results for the hybrid synchronization of identical hyperchaotic Liu systems ([22], 2006). In Section 3, we derive results for the hybrid synchronization of identical hyperchaotic Chen systems ([23], 2005). In Section 4, we derive results for the hybrid synchronization of non-identical hyperchaotic Liu and hyperchaotic Chen systems. The nonlinear controllers are derived using Lyapunov stability theory for the hybrid synchronization of the two hyperchaotic systems. In Section 5, we summarize the main results obtained in this paper.

2. Hybrid Synchronization of Identical Hyperchaotic Liu Systems

2.1 Theoretical Results

In this section, we discuss the hybrid synchronization of identical hyperchaotic Liu systems (Wang and Liu, [22], 2006). The hyperchaotic Liu system [22] is one of the important models of recently discovered hyperchaotic systems.

Thus, we consider the master system as the hyperchaotic Liu dynamics described by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= bx_1 - kx_1x_3 + x_4 \\ \dot{x}_3 &= -cx_3 + hx_1^2 \\ \dot{x}_4 &= -dx_1 \end{aligned} \tag{1}$$

where $x_i \ (i = 1, 2, 3, 4)$ are the state variables and a, b, c, d, h, k are positive constants.

The system (1) is hyperchaotic when the parameter values are taken as

$$a = 10, \quad b = 40, \quad c = 2.5, \quad d = 10.6, \quad h = 4 \text{ and } k = 1.$$

The hyperchaotic portrait of the system (1) is illustrated in Figure 1.

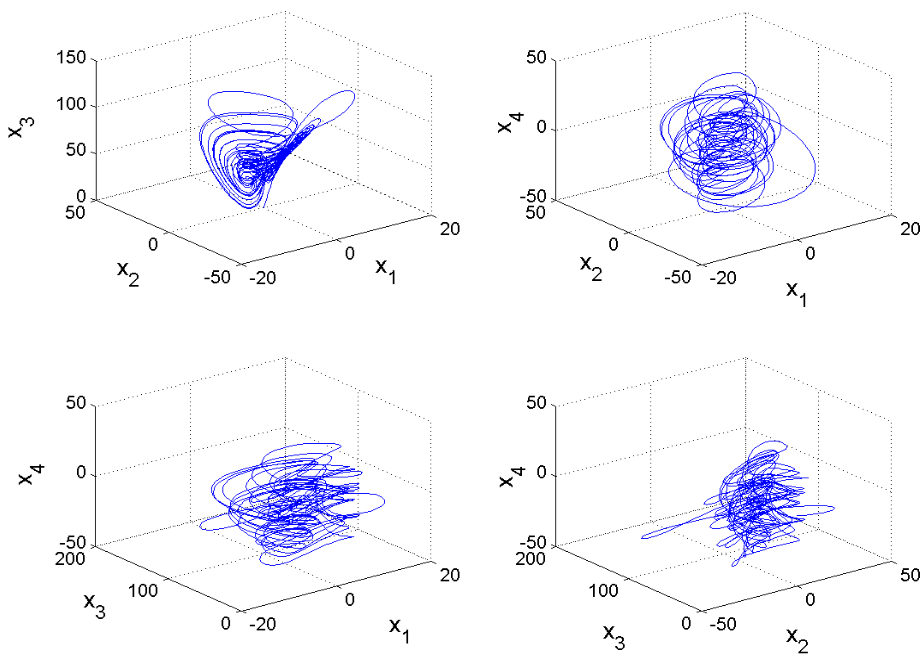

Figure 1. State Orbits of the Hyperchaotic Liu System

We consider the hyperchaotic Liu dynamics also as the slave system, which is described by

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + u_1 \\
\dot{y}_2 &= by_1 - ky_1 y_3 + y_4 + u_2 \\
\dot{y}_3 &= -cy_3 + hy_1^2 + u_3 \\
\dot{y}_4 &= -dy_1 + u_4
\end{aligned} \tag{2}$$

where $y_i \ (i = 1, 2, 3, 4)$ are the state variables and $u_i \ (i = 1, 2, 3, 4)$ are the active controls.

For the hybrid synchronization of the identical hyperchaotic Liu systems (1) and (2), the synchronization errors are defined as

$$\begin{aligned}
e_1 &= y_1 - x_1 \\
e_2 &= y_2 + x_2 \\
e_3 &= y_3 - x_3 \\
e_4 &= y_4 + x_4
\end{aligned} \tag{3}$$

From the error equations (3), it is clear that one part of the two hyperchaotic systems is completely synchronized (first and third states), while the other part is completely anti-synchronized (second and fourth states) so that complete synchronization (CS) and anti-synchronization (AS) coexist in the synchronization process of the two hyperchaotic systems (1) and (2).

A simple calculation yields the error dynamics as

$$\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) - 2ax_2 + u_1 \\
\dot{e}_2 &= be_1 + e_4 + 2bx_1 - k(y_1 y_3 + x_1 x_3) + u_2 \\
\dot{e}_3 &= -ce_3 + h(y_1^2 - x_1^2) + u_3 \\
\dot{e}_4 &= -de_1 - 2dx_1 + u_4
\end{aligned} \tag{4}$$

We consider the nonlinear controller defined by

$$\begin{aligned}
u_1 &= -ae_2 + 2ax_2 \\
u_2 &= -be_1 - e_2 - e_4 - 2bx_1 + k(y_1 y_3 + x_1 x_3) \\
u_3 &= -h(y_1^2 - x_1^2) \\
u_4 &= de_1 - e_4 + 2dx_1
\end{aligned} \tag{5}$$

Substitution of (5) into (4) yields the linear error dynamics

$$\begin{aligned}
\dot{e}_1 &= -ae_1 \\
\dot{e}_2 &= -e_2 \\
\dot{e}_3 &= -ce_3 \\
\dot{e}_4 &= -e_4
\end{aligned} \tag{6}$$

We consider the candidate Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right) \tag{7}$$

which is a positive definite function on R^4.

Differentiating (7) along the trajectories of the system (6), we get

$$\dot{V}(e) = -ae_1^2 - e_2^2 - ce_3^2 - e_4^2,$$

which is a negative definite function on $\boldsymbol{R}^4$ since a and c are positive constants.

Thus, by Lyapunov stability theory [24], the error dynamics (6) is globally exponentially stable. Hence, we obtain the following result.

Theorem 1. The identical hyperchaotic Liu systems (1) and (2) are globally and exponentially hybrid synchronized for all initial conditions $x(0), y(0) \in R^4$ with the active nonlinear controller (5). ■

2.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the two systems of differential equations (1) and (2) with the nonlinear controller (5).

The parameters of the identical hyperchaotic Liu systems (1) and (2) are selected as

$$a = 10, \quad b = 40, \quad c = 2.5, \quad d = 10.6, \quad h = 4, \quad k = 1$$

so that the systems (1) and (2) exhibit hyperchaotic behaviour.

The initial values of the master system (1) are taken as

$$x_1(0) = 20, \quad x_2(0) = 12, \quad x_3(0) = 6, \quad x_4(0) = 32$$

The initial values of the slave system (2) are taken as

$$y_1(0) = 40, \quad y_2(0) = 15, \quad y_3(0) = 20, \quad y_4(0) = 18$$

Figure 2 exhibits the hybrid synchronization of the hyperchaotic systems (1) and (2).

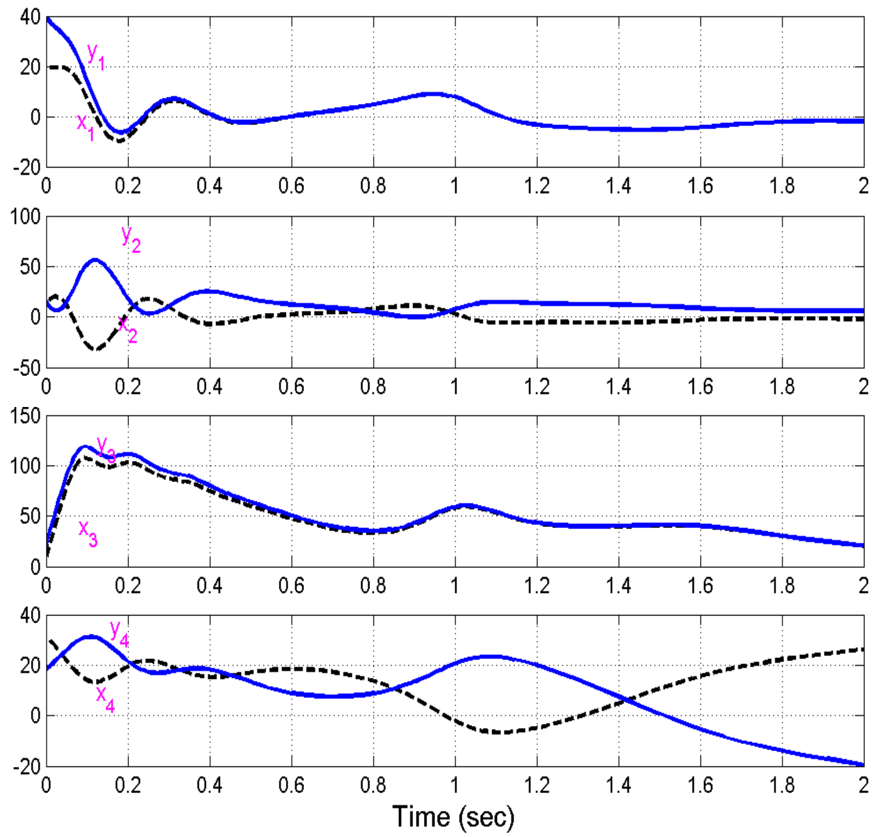

Figure 2. Hybrid Synchronization of the Identical Hyperchaotic Liu Systems

3. HYBRID SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC CHEN SYSTEMS

3.1 Theoretical Results

In this section, we discuss the hybrid synchronization of identical hyperchaotic Chen systems (Li, Tang and Chen, [23], 2005). The hyperchaotic Chen system [23] is one of the important models of recently discovered hyperchaotic systems.

Thus, we consider the master system as the hyperchaotic Chen dynamics described by

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) + x_4 \\ \dot{x}_2 &= \delta x_1 - x_1 x_3 + \gamma x_2 \\ \dot{x}_3 &= x_1 x_2 - \beta x_3 \\ \dot{x}_4 &= x_2 x_3 + r x_4 \end{aligned} \tag{8}$$

where $x_i \ (i = 1, 2, 3, 4)$ are the state variables and $\alpha, \beta, \gamma, \delta, r$ are positive constants.

The system (8) is hyperchaotic when the parameter values are taken as

$\alpha = 35, \ \ \beta = 3, \ \ \gamma = 12, \ \ \delta = 7$ and $r = 0.5$

The hyperchaotic portrait of the system (8) is illustrated in Figure 3.

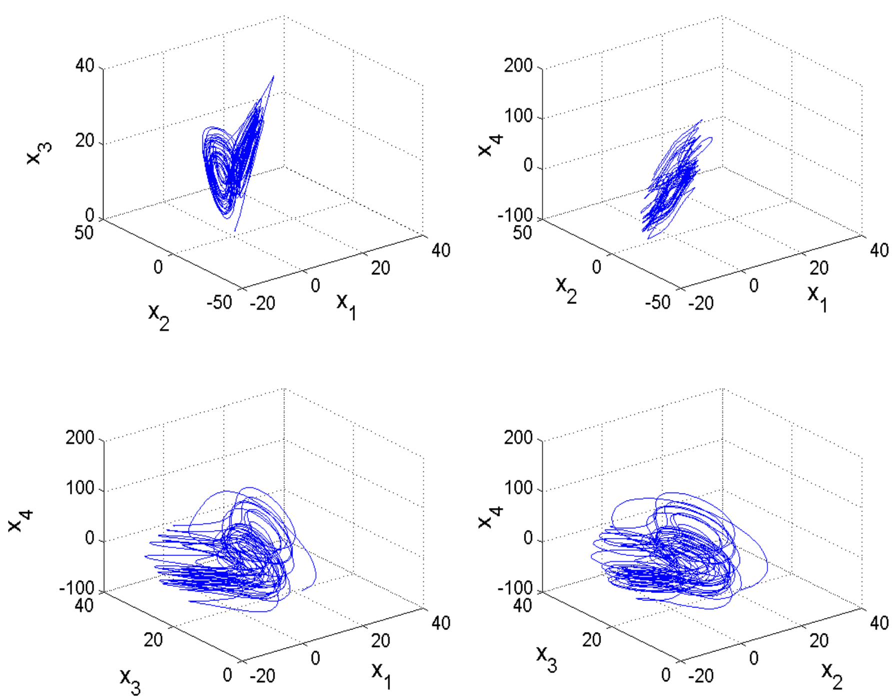

Figure 3. State Orbits of the Hyperchaotic Chen System

We consider the hyperchaotic Chen dynamics also as the slave system, which is described by

$$\begin{aligned}
\dot{y}_1 &= \alpha(y_2 - y_1) + y_4 + u_1 \\
\dot{y}_2 &= \delta y_1 - y_1 y_3 + \gamma y_2 + u_2 \\
\dot{y}_3 &= y_1 y_2 - \beta y_3 + u_3 \\
\dot{y}_4 &= y_2 y_3 + r y_4 + u_4
\end{aligned} \tag{9}$$

where $y_i \ (i = 1, 2, 3, 4)$ are the state variables and $u_i \ (i = 1, 2, 3, 4)$ are the active controls.

For the hybrid synchronization of the identical hyperchaotic Chen systems (8) and (9), the synchronization errors are defined as

$$\begin{aligned}
e_1 &= y_1 - x_1 \\
e_2 &= y_2 + x_2 \\
e_3 &= y_3 - x_3 \\
e_4 &= y_4 + x_4
\end{aligned} \tag{10}$$

From the error equations (10), it is clear that one part of the two hyperchaotic systems is completely synchronized (first and third states), while the other part is completely anti-synchronized (second and fourth states) so that complete synchronization (CS) and anti-synchronization (AS) coexist in the synchronization process of the two hyperchaotic systems (8) and (9).

A simple calculation yields the error dynamics as

$$\begin{aligned}
\dot{e}_1 &= \alpha(e_2 - e_1) + e_4 - 2\alpha x_2 - 2x_4 + u_1 \\
\dot{e}_2 &= \delta e_1 + \gamma e_2 + 2\delta x_1 - y_1 y_3 - x_1 x_3 + u_2 \\
\dot{e}_3 &= -\beta e_3 + y_1 y_2 - x_1 x_2 + u_3 \\
\dot{e}_4 &= r e_4 + y_2 y_3 + x_2 x_3 + u_4
\end{aligned} \tag{11}$$

We consider the nonlinear controller defined by

$$\begin{aligned}
u_1 &= -\alpha e_2 - e_4 + 2\alpha x_2 + 2x_4 \\
u_2 &= -\delta e_1 - (\gamma + 1) e_2 - 2\delta x_1 + y_1 y_3 + x_1 x_3 \\
u_3 &= -y_1 y_2 + x_1 x_2 \\
u_4 &= -(r + 1) e_4 - y_2 y_3 - x_2 x_3
\end{aligned} \tag{12}$$

Substitution of (12) into (11) yields the linear error dynamics

$$\begin{aligned}
\dot{e}_1 &= -\alpha e_1 \\
\dot{e}_2 &= -e_2 \\
\dot{e}_3 &= -\beta e_3 \\
\dot{e}_4 &= -e_4
\end{aligned} \tag{13}$$

We consider the candidate Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right) \tag{14}$$

which is a positive definite function on R^4.

Differentiating (14) along the trajectories of the system (13), we get

$$\dot{V}(e) = -\alpha e_1^2 - e_2^2 - \beta e_3^2 - e_4^2,$$

which is a negative definite function on $\boldsymbol{R}^4$ since α and β are positive constants.

Thus, by Lyapunov stability theory [24], the error dynamics (13) is globally exponentially stable. Hence, we obtain the following result.

Theorem 2. The identical hyperchaotic Chen systems (8) and (9) are globally and exponentially hybrid synchronized for all initial conditions $x(0), y(0) \in R^4$ with the active nonlinear controller (12). ■

3.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the two systems of differential equations (8) and (9) with the nonlinear controller (12).

The parameters of the identical hyperchaotic Chen systems (8) and (8) are selected as

$$\alpha = 35, \quad \beta = 3, \quad \gamma = 12, \quad \delta = 7, \quad r = 0.5$$

so that the systems (8) and (9) exhibit hyperchaotic behaviour.

The initial values of the master system (8) are taken as

$$x_1(0) = 5, \quad x_2(0) = 18, \quad x_3(0) = 26, \quad x_4(0) = 14$$

The initial values of the slave system (9) are taken as

$$y_1(0) = 20, \quad y_2(0) = 35, \quad y_3(0) = 10, \quad y_4(0) = 22$$

Figure 4 exhibits the hybrid synchronization of the hyperchaotic Chen systems (8) and (9).

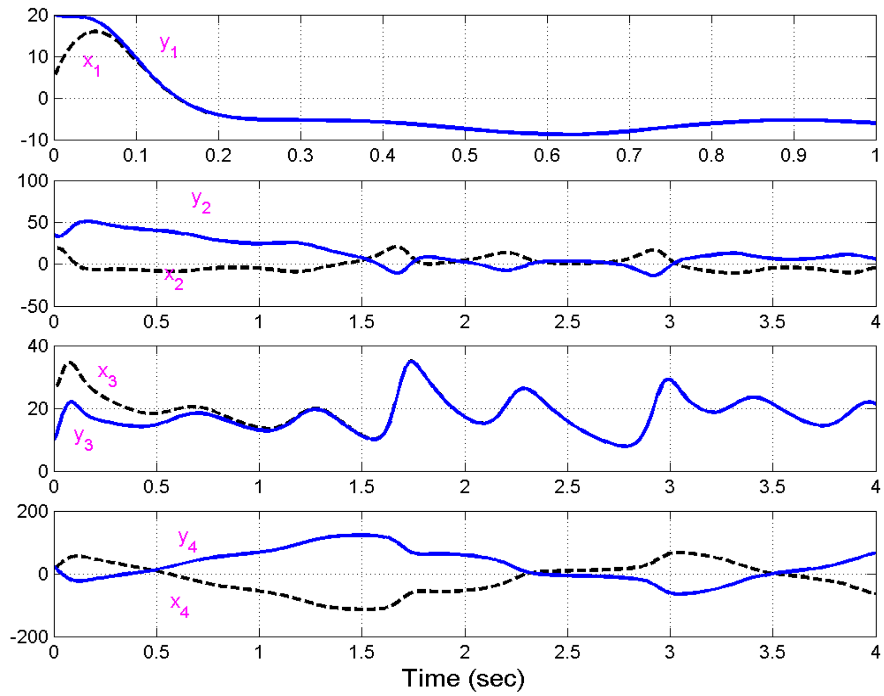

Figure 4 Hybrid Synchronization of the Identical Hyperchaotic Chen Systems

4. Hybrid Chaos Synchronization of Hyperchaotic Liu and Hyperchaotic Chen Systems

4.1 Theoretical Results

In this section, we discuss the hybrid chaos synchronization of non-identical hyperchaotic Liu system (Wang and Liu, [22], 2006) and hyperchaotic Chen system (Li, Tang and Chen, [23], 2005). Hyperchaotic Liu and hyperchaotic Chen systems are important models of recently discovered hyperchaotic systems.

Here, we consider the master system as the hyperchaotic Liu dynamics described by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= bx_1 - kx_1x_3 + x_4 \\ \dot{x}_3 &= -cx_3 + hx_1^2 \\ \dot{x}_4 &= -dx_1 \end{aligned} \tag{15}$$

where $x_i \ (i = 1,2,3,4)$ are the state variables and a,b,c,d,h,k are positive constants.

The system (15) is hyperchaotic when the parameter values are taken as

$a = 10, \ \ b = 40, \ \ c = 2.5, \ \ d = 10.6, \ \ h = 4$ and $k = 1.$

We consider the hyperchaotic Chen dynamics as the slave system, which is described by

$$\begin{aligned}\dot{y}_1 &= \alpha(y_2 - y_1) + y_4 + u_1 \\ \dot{y}_2 &= \delta y_1 - y_1 y_3 + \gamma y_2 + u_2 \\ \dot{y}_3 &= y_1 y_2 - \beta y_3 + u_3 \\ \dot{y}_4 &= y_2 y_3 + r y_4 + u_4\end{aligned} \tag{16}$$

where $y_i \ (i = 1,2,3,4)$ are the state variables, $\alpha, \beta, \gamma, \delta, r$ are positive constants and $u_i \ (i = 1,2,3,4)$ are the active controls.

The system (16) is hyperchaotic when the parameter values are taken as

$$\alpha = 35, \ \ \beta = 3, \ \ \gamma = 12, \ \ \delta = 7 \ \text{ and } \ r = 0.5$$

For the hybrid synchronization of the non-identical hyperchaotic systems (15) and (16), the synchronization errors are defined as

$$\begin{aligned}e_1 &= y_1 - x_1 \\ e_2 &= y_2 + x_2 \\ e_3 &= y_3 - x_3 \\ e_4 &= y_4 + x_4\end{aligned} \tag{17}$$

From the error equations (17), it is clear that one part of the two hyperchaotic systems is completely synchronized (first and third states), while the other part is completely anti-synchronized (second and fourth states) so that complete synchronization (CS) and anti-synchronization (AS) coexist in the synchronization process of the two hyperchaotic systems (15) and (16).

A simple calculation yields the error dynamics as

$$\begin{aligned}\dot{e}_1 &= \alpha(e_2 - e_1) + e_4 + (a - \alpha)x_1 - (a + \alpha)x_2 - x_4 + u_1 \\ \dot{e}_2 &= \delta e_1 + \gamma e_2 + (b + \delta)x_1 - \gamma x_2 + x_4 - y_1 y_3 - k x_1 x_3 + u_2 \\ \dot{e}_3 &= -\beta e_3 + (c - \beta)x_3 + y_1 y_2 - h x_1^2 + u_3 \\ \dot{e}_4 &= r e_4 - d x_1 - r x_4 + y_2 y_3 + u_4\end{aligned} \tag{18}$$

We consider the nonlinear controller defined by

$$\begin{aligned}u_1 &= -\alpha e_2 - e_4 - (a - \alpha)x_1 + (a + \alpha)x_2 + x_4 \\ u_2 &= -\delta e_1 - (\gamma + 1)e_2 - (b + \delta)x_1 + \gamma x_2 - x_4 + y_1 y_3 + k x_1 x_3 \\ u_3 &= -(c - \beta)x_3 - y_1 y_2 + h x_1^2 \\ u_4 &= -(r + 1)e_4 + d x_1 + r x_4 - y_2 y_3\end{aligned} \tag{19}$$

Substitution of (19) into (18) yields the linear error dynamics

$$\begin{aligned} \dot{e}_1 &= -\alpha e_1 \\ \dot{e}_2 &= -e_2 \\ \dot{e}_3 &= -\beta e_3 \\ \dot{e}_4 &= -e_4 \end{aligned} \tag{20}$$

We consider the candidate Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right) \tag{21}$$

which is a positive definite function on R^4.

Differentiating (21) along the trajectories of the system (20), we get

$$\dot{V}(e) = -\alpha e_1^2 - e_2^2 - \beta e_3^2 - e_4^2,$$

which is a negative definite function on $\boldsymbol{R}^4$ since α and β are positive constants.

Thus, by Lyapunov stability theory [24], the error dynamics (20) is globally exponentially stable. Hence, we obtain the following result.

Theorem 3. The non-identical hyperchaotic Liu system (15) and hyperchaotic Chen system (16) are globally and exponentially hybrid synchronized for all initial conditions $x(0), y(0) \in R^4$ with the active nonlinear controller (19). ■

4.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the two systems of differential equations (15) and (16) with the nonlinear controller (19).

The parameters of the hyperchaotic Liu system (15) are selected as

$$a = 10, \quad b = 40, \quad c = 2.5, \quad d = 10.6, \quad h = 4, \quad k = 1$$

The parameters of the hyperchaotic Chen system (16) are selected as

$$\alpha = 35, \quad \beta = 3, \quad \gamma = 12, \quad \delta = 7, \quad r = 0.5$$

The initial values of the master system (15) are taken as

$$x_1(0) = 14, \quad x_2(0) = 36, \quad x_3(0) = 15, \quad x_4(0) = 12$$

The initial values of the slave system (16) are taken as

$$y_1(0) = 30, \quad y_2(0) = 18, \quad y_3(0) = 22, \quad y_4(0) = 10$$

Figure 5 exhibits the hybrid synchronization of the hyperchaotic systems (15) and (16).

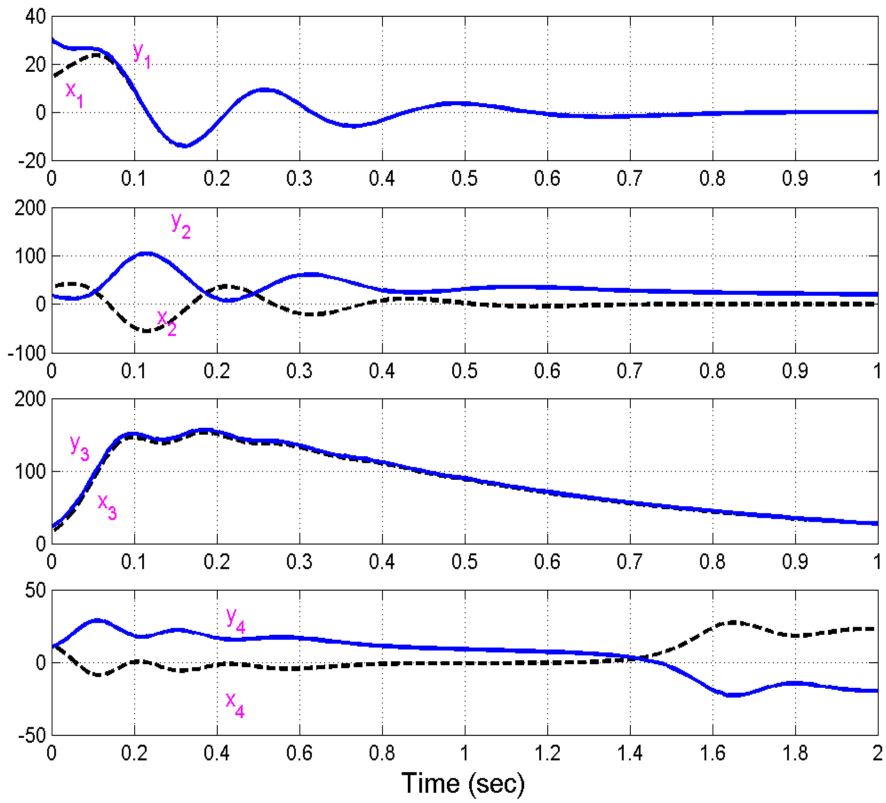

Figure 5. Hybrid Synchronization of Hyperchaotic Liu and Hyperchaotic Chen Systems

5. Conclusions

In this paper, active nonlinear control method has been deployed to achieve global chaos hybrid synchronization of the following hyperchaotic systems:

(A) Identical hyperchaotic Liu systems (2006)

(B) Identical hyperchaotic Chen systems (2005)

(C) Non-identical hyperchaotic Liu and hyperchaotic Chen systems.

The stability results for the hybrid chaos synchronization of the above hyperchaotic systems were established using Lyapunov stability theory. Since Lyapunov exponents are not required for these calculations, the nonlinear control method is effective and convenient to achieve hybrid synchronization of the identical and non-identical hyperchaotic Liu and hyperchaotic Chen systems. Numerical simulations have been shown to demonstrate the effectiveness of the hybrid synchronization schemes derived in this paper.

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