NEW BER ANALYSIS OF OFDM SYSTEM OVER NAKAGAMI-n (RICE) FADING CHANNEL

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ABSTRACT

Modern wireless communication systems support high speed multimedia services. These services require high data rates with acceptable error rates. Orthogonal Frequency Division Multiplexing (OFDM) is a capable candidate to solve this problem. In this paper, a new expression for the BER of OFDM system has been derived over Nakagami–n (Rice) fading channels using characteristics function (CHF) approach. The exact probability density function of first order of Nakagami-n (Rice) random vector is used to derive the expression for the error rates of OFDM system. The BER derivation of Rician fading channel is slightly more complex compared to the Nakagami–m distribution because the PDF of the Rician RV contains an explicit term of a modified Bessel function of first kind. Earlier, this problem was solved by replacing the Bessel function with its infinite series and exponential integral representation. Here we propose an integral expression to remove the complexity of the expression.

KEYWORD

Error Rate Analysis, Fading Channel, Orthogonal Frequency Division Multiplexing (OFDM),Nakagami-n (Rice) distribution.

1. INTRODUCTION

Today the demand of wireless communication is increasing exponentially and next generation of wireless broadband multimedia communication system will require high-speed, high-quality digital mobile portable reception and transmission. A receiver has to cope with a signal that is often weaker than desirable and that contains many echoes. Simple digital systems do not work well in the multipath environment. When solving this problem, first question is how to put this large bit stream on wireless channel with sufficient Quality of services (QoS) guaranties, i.e. which modulation can compromise all contradicting requirements in the best way. Using different equalization techniques at the receiver side could be the solution, but there are many practical difficulties to implement this equalization in real-time at several Mbps with compact, low-cost hardware. A capable candidate that eliminates a need for the complex equalizers at receiver side is the OFDM, a multiple carrier modulation technique so that it has been employed in many wireless standards [1].

In an ideal radio channel, the received signal would consist of only a single direct path signal, which would be a perfect reconstruction of the transmitted signal [2]. However, in a real channel, the signal is modified during transmission in the channel. The received signal consists of a combination of attenuated, reflected, refracted, and diffracted replicas of the transmitted signal [24]. Understanding of these effects on the signal is important because the performance of a radio system is dependent on the radio channel characteristics. The rapid fluctuations of the instantaneous received power due to multipath effects are usually described with Rayleigh, Rician, Nakagami-m or Nakagami-q model [3, 4]. Among them Nakagami-n (Rice) and Nakagami-q (Hoyt) fading channels, has not yet received as much attention as the Rayleigh and Nakagami-m fading channels, mainly due to the complex form of their PDF, despite the fact that these models exhibit an excellent fit to experimental fading channel measurements for land, mobile, terrestrial, and satellite telecommunications[5,6]. In [5] by using an alternative moments-based approach performance analysis of equal-gain combining (EGC) receivers over independent, not necessarily identically distributed Rice and Hoyt-fading channels has been derived. Moreover, using Padé rational approximation to the moment-generating function of the output SNR, the average symbol error probability and the outage probability are evaluated. In [6] complexity of the PDF has been solved by replacing the Bessel function with its infinite series representation and [7] used exponential integral representation for Bessel function. Further [20] the combination of OFDM and diversity has been becoming popular in wireless communications.

In digital communication system the bit error rate has been very extensively used as a performance measurement. This measurement is one of the prime considerations in determining signal quality and quantifies the reliability of the entire radio system. It is the one that is most revealing about the nature of the system behavior and the one most often illustrated in documents containing system performance evaluations; thus, it is of primary interest to have a method for its evaluation that reduces the degree of difficulty as much as possible[4,8]. Generally researchers perform the averaging using the probability density function method or the moment generating function (MGF). The third technique is CHF approach which is more efficient to evaluate the system performance and overcomes all the limitations of the MGF method [9].

So our motivation behind this paper is to study the performance of OFDM system over Nakagami-n (Rice) fading channel using CHF method to remove the complexity of the system, and within a single common framework, developed a general method for calculating the average error rates of single channel and multiple channel reception.

The main objective of paper is to derive the analytical expression for error rates of OFDM systems over Nakagami-n (Rice) fading channel. To accomplish this, in first step we have derived the PDF of Nakagami-n (Rice) random phase vector in an integral form by using CHF method. Further this PDF is used to calculate the error rate performance of OFDM system over frequency selective Nakagami-n fading channels. The error rate derivation of Nakagami-n (Rice) fading channel is slightly more complex compared to the Nakagami–m fading case because the PDF of the Rician RVs contains an explicit term of a modified Bessel function of first kind. To remove this complexity we are using an integral expression, to express CHF in a closed form expression, which has resulted into less complexity and easy numerical computation of the expression. Here the average error rate is expressed in terms of exponential and imaginary error function. Finally it is observed through numerical results, that error rate performance of an OFDM signal over multi propagation Nakagami-n channels does not improve with increasing Nakagami-n fading parameter n.

The rest of the paper organized as follows. The section 2 deals with the OFDM system model. In section 3 multipath fading channels is explained. In section 4 we derive an integral expression for the PDF of Nakagami-n (Rice) random phase vectors. In section 5 the error rate expressions for OFDM system has been derived. The section 6 discusses about the numerical results and finally section 7 concludes the work.

2. OFDM SYSTEM MODEL

Let an OFDM system with N sub-carriers and X (k) is the k[th] OFDM data block to be transmitted with narrowband N subcarriers. These data are used to modulate N orthogonal sub carriers. Then IDFT is used to modulate the input signal. After modulation signal can be represented as [10]:

$$x(n) = {1}/{N}\sum_{k=0}^{N-1} X(k)\exp\left(\frac{j2\pi kn}{N}\right), n = 0,1...N-1 \tag{1}$$

Cyclic prefix is inserted after IDFT modulation, which is removed before demodulation at the OFDM receiver. The received signal after removal of cyclic prefix can be demodulated using DFT. Output of DFT can be represented as [19]:

$$R(k) = \sum_{n=0}^{N-1} r(n)\exp(-j2\pi k\, n/N) = X(k)H(k) + W_k,\ k=0,\ 1....N-1 \tag{2}$$

Where W_k is additive complex Gaussian noise with zero mean and unit variance and H (k) is frequency domain channel impulse response.

3. MULTIPATH FADING CHANNEL MODEL

Due to multipath phenomenon many reflected signals arrive at receiver side with different times. The reason behind these delayed signals is reflection, diffraction, scattering etc. from the hills, buildings, trees, vehicles. To understand this whole effect on the signal, we express the multipath channel with its impulse response. The channel impulse response of a multipath fading channel is modeled as a finite impulse response filter with taps h_l (n).We can express channel impulse response as [11]:

$$H_l(k) = \sum_{n=0}^{M-1} h_l(n)\exp(-j2\pi k\, n/N), k = 0,1,...M-1 \tag{3}$$

Where n identify each path number, M is the number of paths.

4. PDF OF NAKAGAMI RV & BER PERFORMANCE ANALYSIS

In this section, an integral expression for the first order PDF is evaluated based on CHF approach. Assume an auxiliary function Z=R cos (θ), where R is the fading amplitude and θ is the random phase distributed uniformly over [0, 2π]. The Nakagami-n (Rice) distribution is given by [12]:

$$f(r) = \frac{2(1+K)}{\Omega} r e^{\left(-K-{(1+K)r^2}/{\Omega}\right)} I_0\left(2r\sqrt{\frac{K(1+K)}{\Omega}}\right), r \ge 0 \tag{4}$$

The standard derivation for the CHF, as derived in [13, 17, 21]

$$\Psi_{X n} = \int_0^\infty f(r) J_0(\eta r) dr \tag{5}$$

Using the eq. (4), eq. (5) can be written as:

$$\Psi_{X n} = \int_0^\infty \frac{2(1+K)}{\Omega} r e^{-K} e^{\left(-(1+K)r^2/\Omega\right)} I_0\left(2r\sqrt{\frac{K(1+K)}{\Omega}}\right) J_0(\eta r) dr \tag{6}$$

To simplify the eq. (6) we use integral identity [14, eq. 6.633.4] and eq. (6) can be expressed as:

$$\Psi_{X n}(\eta) = \exp\left(\frac{-\Omega_n}{4(1+K_n)}\eta^2\right) J_0\left(\eta\sqrt{\frac{K_n\Omega_n}{(1+K_n)}}\right) \tag{7}$$

Because channel tap coefficients are independent so CHF can be written as [11]:

$$\Psi_{X n}(\eta) = \prod_0^{M-1} \exp\left(\frac{-\Omega_n}{4(1+K_n)}\eta^2\right) J_0\left(\eta\sqrt{\frac{K_n\Omega_n}{(1+K_n)}}\right) \cong \Psi(\eta) \tag{8}$$

To obtain the PDF for X the inverse FT is used to get the equation as given in [13, 15]:

$$f_X(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-j\eta x}\Psi(\eta) d\eta \tag{9}$$

After some simplification eq. (9) can be written as [13]:

$$f_X(x) = \frac{1}{\pi}\int_0^{\infty} \cos(\eta x)\Psi(\eta) d\eta \tag{10}$$

Using eq. (7) the integral PDF is given by

$$f_X(x) = \frac{1}{\pi}\int_0^{\infty} \cos(\eta x) \exp\left(\frac{-\Omega_n}{4(1+K_n)}\eta^2\right) J_0\left(\eta\sqrt{\frac{K_n\Omega_n}{(1+K_n)}}\right) d\eta \tag{11}$$

Equation (11) is integral expression for PDF ,which is furher used to derive the BER expression for OFDM system.

5. BER PERFORMANCE ANALYSIS

The conditional BER of a particular modulation is given by Q (Sr), where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{\left(\frac{-t^2}{2}\right)} dt$ Using the alternative representation of above equation as given in [16].

$$Q(Sx) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{\left(\frac{-S^2x^2}{2\sin^2(\phi)}\right)} d\phi \tag{12}$$

Where S is a factor related to signal to noise ratio (SNR), as given in [8] $S = \sqrt{2gE_b/N_0}$, where g=1 for coherent binary phase shift keying (BPSK). The parameter *x* is the fading amplitude. Averaging the CEP over derived PDF the error rate denoted by P(S) can be expressed as [11]:

$$P(S)=\int_0^{\infty} Q(Sx) f_X(x)dx \tag{13}$$

$$P(S)=\int_0^{\pi/2}\frac{1}{\pi}\exp\left(-\frac{S^2x^2}{2\sin^2(\phi)}\right)d\phi\times\left(\frac{1}{\pi}\int_0^{\infty}\cos(\eta x)\Psi(\eta)d\eta\right)dx \tag{14}$$

With an exchange of order of integration eq. (14) can be given as:

$$P(S)=\frac{1}{\pi^2}\int_0^{\infty}\Psi(\eta)d\eta\left(\int_0^{\pi/2}\int_0^{\infty}\cos(\eta x)\exp\left(-\frac{S^2x^2}{2\sin^2(\phi)}\right)dx\right)d\phi \tag{15}$$

Using the identity[14,eq. 3.896.4],eq.(15) can be written as:

$$P(S)=\frac{1}{\pi^2}\int_0^{\infty}\Psi(\eta)d\eta\left(\int_0^{\pi/2}\sqrt{\frac{2\pi\sin^2(\phi)}{S^2}}\exp\left(-\frac{\eta^2\sin^2(\phi)}{2S^2}\right)d\phi\right) \tag{16}$$

After integration and some simplification eq.(16) can be expressed as :

$$P(S)=\frac{1}{2\pi}\int_0^{\infty}\prod\nolimits_0^{M-1}\exp\left(\frac{-\Omega_n}{4(1+K_n)}\eta^2\right)J_0\left(\eta\sqrt{\frac{K_n\Omega_n}{(1+K_n)}}\right)e^{\frac{-\eta^2}{2S^2}}\frac{erfi\left(\frac{\eta}{\sqrt{2}S}\right)}{\eta}d\eta \tag{17}$$

Where *erfi* is imaginary error function, eq. (17) can be used for numerical computation of BER for OFDM signals transmitted over Nakagami-n (Rice) channels. Further eq. (17) is used to express the BER expression for the single tap and two tap multipath channels. For single tap BER expression can be given as:

$$P(S)=\frac{1}{2\pi}\int_0^{\infty}\exp\left(\frac{-\Omega_0}{4(1+K_0)}\eta^2\right)J_0\left(\eta\sqrt{\frac{K_0\Omega_0}{(1+K_0)}}\right)e^{\frac{-\eta^2}{2S^2}}\frac{erfi\left(\frac{\eta}{\sqrt{2}S}\right)}{\eta}d\eta \tag{18}$$

In the similar way for the two tap multipath channel we have expression as:

$$P(S)=\frac{1}{2\pi}\int_0^{\infty}\exp\left(\frac{-\Omega_0\eta^2}{4(1+K_0)}\right)J_0\left(\eta\sqrt{\frac{K_0\Omega_0}{(1+K_0)}}\right)\exp\left(\frac{-\Omega_1\eta^2}{4(1+K_1)}\right)J_0\left(\eta\sqrt{\frac{K_1\Omega_1}{(1+K_1)}}\right)e^{\frac{-\eta^2}{2S^2}}\frac{erfi\left(\frac{\eta}{\sqrt{2}S}\right)}{\eta}d\eta \tag{19}$$

When we have condition $\Omega_0=\Omega_1=\Omega$ and $K_0=K_1=K$, eq. (19) can be expressed as

$$P(S)=\frac{1}{2\pi}\int_{0}^{\infty}\exp\left(\frac{-\Omega}{2(1+K)}\eta^{2}\right)\left(J_{0}\left(\eta\sqrt{\frac{K\Omega}{(1+K)}}\right)\right)^{2}e^{\frac{-\eta^{2}}{2S^{2}}}\frac{erfi\left(\frac{\eta}{\sqrt{2S}}\right)}{\eta}d\eta \tag{20}$$

Above derived eq. (18) and (20) is used for numerical evaluation of OFDM-BPSK and OFDM-BFSK system.

6. Results & Discussions

In this section the BER performance of an OFDM-BPSK system over Nakagami-n (Rice) fading channels are analytically evaluated. Fig.1 illustrates the average BER performance versus the SNR in a BPSK modulated OFDM system for single tap channel. It is well known, that at n=0 Nakagami-n fading corresponds to Rayleigh fading. So, results for the same have been achieved through numerical evaluation. Further when value of n is increased, the BER starts reducing at different values of n.We have further analyzed OFDM-BFSK system as shown in Fig.2.

Results obtained from numerical evaluation for OFDM-BFSK system is similar in nature to that of OFDM-BPSK system. BER starts decreasing with increasing value of n. Fig.3 shows the error rate curves for a two-tap OFDM BPSK system over frequency-selective Nakagami-n channel with $n_0=n_1=n$ and $\Omega_0=\Omega_1=\Omega=1$.

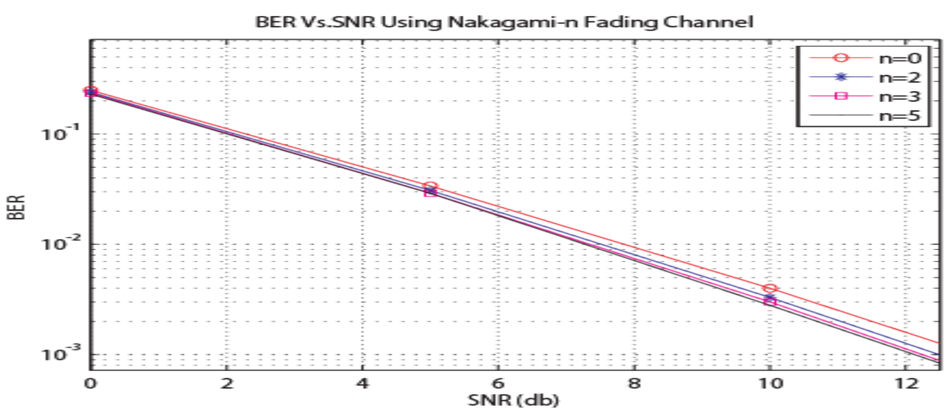

Fig.1 BER Vs SNR for OFDM-BPSK system with single tap

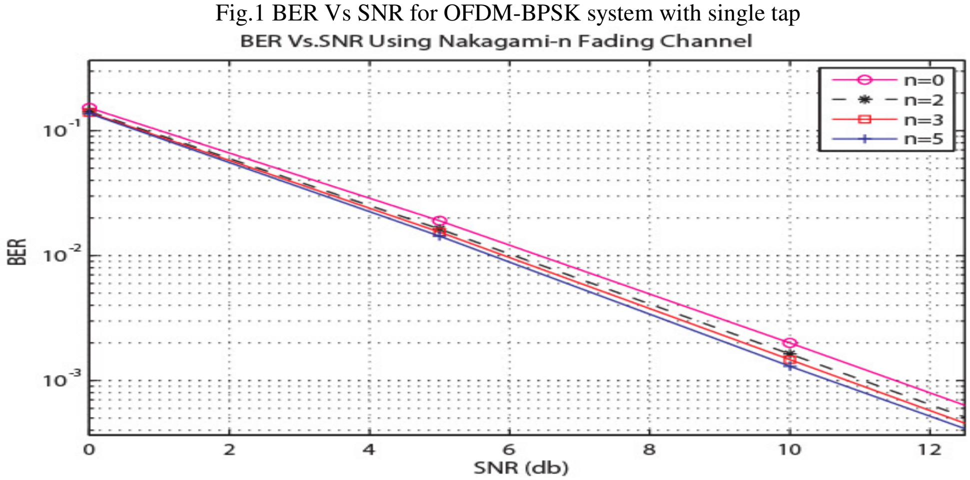

Fig.2 BER Vs SNR for OFDM-BFSK system with single tap

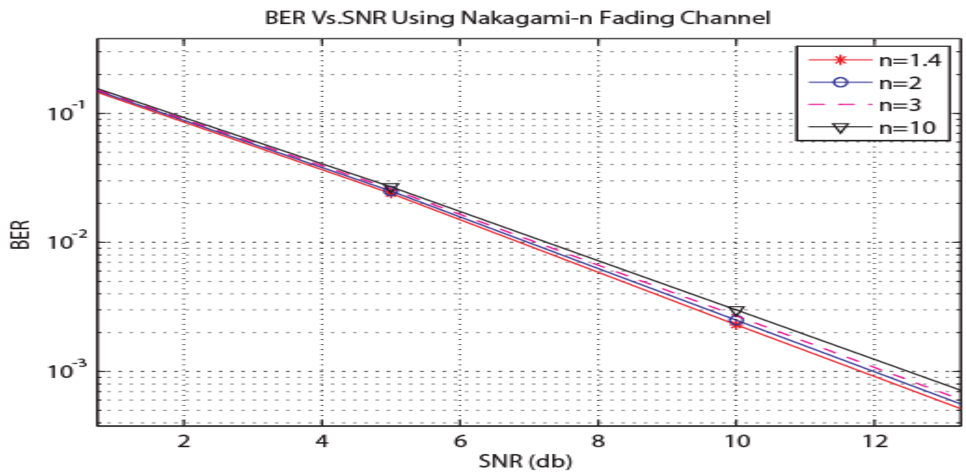

Fig.3 BER Vs SNR for OFDM-BPSK system with two tap

If fading parameter n increases, slopes of the error rate performance increases. But further, if we increase n, no reduction in BER has been reported rather it starts increasing. So this put a limit to increase the value of n beyond the certain value. This interesting fact is expected because according to [4, 6, 12] we can approximate the Rician distribution by a Nakagami-m distribution that means if for the Nakagami-m (Two Tap) case error rate performance degrades with increasing values of m as reported in [11, 16, 19] then same nature of the BER performance should be shown by the Nakagami-n (Rice) fading distribution, and same results, we are achieved in our numerical analysis. Here the threshold value for Nakagami-n is achieved to be 1.4 through analytical results.

We have further analyzed OFDM-BFSK system as shown in Fig.4. Results obtained from numerical evaluation for OFDM-BFSK system is similar in nature to that of OFDM-BPSK system. BER starts decreasing with increasing value of n.

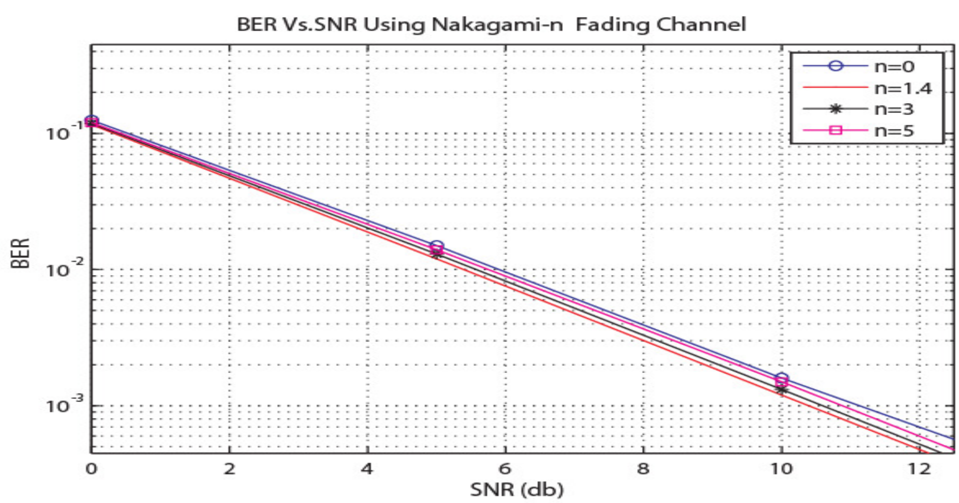

Fig.4 BER Vs SNR for OFDM-BFSK system with two tap

7. CONCLUSION

In this paper, Nakagami-n (Rice) PDF has been derived in an integral form using CHF approach and further utilized for evaluating the average error rate expression for OFDM system over Nakagami-n (Rice) fading channel with the fading severity index n. The derived PDF remove the complexity of the SER expression efficiently. The evaluated analytical error rate expression over Nakagami-n (Rice) fading is useful where model propagation path consisting of one strong direct LOS component and many random weaker components. Further as per the previous papers, we can approximate the Nakagami-m fading channels with the Nakagami-n (Rice) channel, i.e. the nature of the error rate performance should be same. This fact is proved by our numerical result. Finally it has been concluded that, depending on the number of channel taps, larger Nakagami-n (Rice) fading parameters do not necessarily give smaller error rates.

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