PERFORMANCE OF ITERATIVE LDPC-BASED
SPACE-TIME TRELLIS CODED MIMO-OFDM
SYSTEM OVER AWGN AND FADING CHANNELS

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ABSTRACT
This paper presents the bit error rate (BER) performance of the low density parity check (LDPC) based
space-time trellis coded 2×2 multiple-input multiple-output orthogonal frequency-division multiplexing
(STTC-MIMO-OFDM) system on text message transmission. The system under investigation incorporates
1/2-rated LDPC encoding scheme under various digital modulations (BPSK, QPSK and QAM) over an
additive white gaussian noise (AWGN) and other fading (Rayleigh and Rician) channels for two transmit
and two receive antennas. At the receiving section of the simulated system, Minimum Mean-Square-Error
(MMSE) channel equalization technique has been implemented to extract transmitted symbols without
enhancing noise power level. The effectiveness of the proposed system is analyzed in terms of BER with
signal-to-noise ratio (SNR). It is observable from the Matlab based simulation study that the proposed
system outperforms with BPSK as compared to other digital modulation schemes at relatively low SNRs
under AWGN, Rayleigh and Rician fading channels. The transmitted text message is found to have
retrieved effectively at the receiver under implementation of iterative sum-product LDPC decoding
algorithm. It has also been anticipated that the performance of the LDPC-based STTC-MIMO-OFDM
system degrades with the increase of noise power.

KEYWORDS
Multiple-Input Multiple-Output (MIMO), Orthogonal Frequency Division Multiplexing (OFDM),
Space-time trellis coding (STTC), Low density parity check (LDPC) Codes, Minimum Mean-Square-
Error (MMSE) Channel Equalization.

1. INTRODUCTION
Orthogonal frequency division multiplexing (OFDM) and space-time coding have been receiving
increased attention due to their potential to provide increased capacity for next generation
wireless systems. The OFDM technique is currently widely used to combat intersymbol
interference (ISI) by transforming a frequency selective fading channel into a set of parallel
correlated fading channels. The capacity and performance of a wireless link can be greatly
improved by using multiple antennas at both the transmitter and receiver was shown in [1,2]. The
combination of OFDM and MIMO seems to be very promising when aiming at the design of very
high-rate wireless mobile systems and space-time codes (STCs) are the codes designed for the use
in MIMO systems. Recently space-time trellis coding (STTC) technique has been proposed to achieve both the diversity and coding gain in multi-input multi-output (MIMO) fading channels [3]. Space-time trellis codes operate on one input symbol at a time and produce a sequence of spatial vector outputs. Various STTCs in OFDM systems, referred to as STTC-OFDMs, in frequency fading channels have been investigated [4],[5],[6],[7]. The combination of space-time coding and OFDM results in an enhanced system performance in wideband wireless channels [8],[9].

For the sake of further improving the performance of space-time trellis coded MIMO-OFDM system, forward error correction (FEC) coding schemes may be invoked for protecting the subcarriers against frequency selective fading in an OFDM environment. Many error-correcting codes have been applied to OFDM such as convolutional codes, Reed-Solomon codes, Turbo codes [10], and so on. Recently low-density parity-check (LDPC) codes have attracted much attention particularly in the field of coding theory. LDPC codes were proposed by Gallager in 1962 [11],[12] and are now recognized as good error-correcting codes achieving near Shannon limit performance[13]. In order to provide channel coding gain, the family of LDPC codes has emerged as an attractive alternative to turbo coding [9]. The performance of LDPC codes has been also evaluated on a block fading channel and it has been shown that the LDPC codes achieve a large gain with respect to convolutional codes for large packet length [14]. It has also been shown in [15] that LDPC based space-time coded OFDM systems are capable of efficiently exploiting the achievable spatial diversity in wireless channel.

Throughout this paper, we only consider a 2×2 LDPC-based STTC-OFDM system with two transmit and two receive antennas. At the receiver, Minimum Mean-Square-Error (MMSE) equalization technique is employed to separate transmitted symbols without enhancing the noise [16],[17],[18]. In this contribution we evaluate the bit error rate (BER) of the proposed LDPC aided STTC-MIMO-OFDM system under text message transmission and compare the BER performance of the system working under each of the three types of digital modulation (BPSK, QPSK and QAM) on the AWGN, Rayleigh and Rician fading channels.

2. RELATED WORKS

A brief survey of literature in the area relevant to this paper is as follows. To enhance system performance M.Y. Alias et al. [17] proposed concatenated LDPC and Turbo coding assisted space-time block coded wireless OFDM system. J. Wu and H-N Lee [18] have shown that channel capacity can be significantly increased by using LDPC coded modulation in MIMO multiple-access systems. The impact of concatenated block codes and interleaver with STTC on the performance of a MIMO-OFDM wireless communication system on color image transmission has been investigated by D. Haque et al. [19]. In [20], authors presented the performance of STTCs over Nakagami fading and Rayleigh fading channels and shown that the STTCs designed for Rayleigh fading and Rician fading channels are also suitable for Nakagami fading channels. Ben Lu et al. [21] shown that compared with the conventional STTC, LDPC-based STTC can significantly improve the system performance by efficiently exploiting both the spatial diversity and selective-fading diversity in wireless channels. They also shown that compared with the recently proposed turbo-code-based STC scheme, LDPC-based STC exhibits lower receiver complexity and more flexible scalability. In a recent study, M. Mesbahul Alam et al. [22] investigated the impact of concatenation of an interleaved LDPC code with convolutional channel coding on the performance of the space time block coded (STBC) OFDM wireless communication system on text message transmission. They showed that the BER performance of the concatenated channel encoded OFDM system with 1/2-rated channel coding is better as compared to 3/4-rated channel coding.
3. **LDPC Code**

Low-density parity-check (LDPC) codes are a class of linear block codes which have better block error performance than turbo codes, because the minimum distance of an LDPC code increases proportional to the code length with a high probability. Such a property is desirable for the high-bit-rate transmission that requires very low frame error probability. Among the advantages, LDPC codes have over turbo codes are: they allow a parallelizable decoder, they are more amenable to high code rates, they generally possess a lower error-rate floor (for the same length and rate), they possess superior performance in bursts (due to interference, fading, and so on), they require no interleavers in the encoder and decoder, and a single LDPC code can be universally good over a collection of channels. Among the disadvantages are: most LDPC codes have somewhat complex encoders, the connectivity among the decoder component processors can be large and unwieldy, and turbo codes can often perform better when the code length is short.

Basically there are two different possibilities to represent LDPC codes, matrix and graphical representations. An LDPC code is defined by $m \times n$ parity-check matrix $H$ as $(n, k)$ LDPC, where $k = n - m$ and the code rate is $r = k / n$. For a matrix to be called low-density the two conditions $w_c \ll n$ and $w_r \ll m$ must be satisfied where $w_c$ and $w_r$ defining the number of 1’s in each row and the number of 1’s in each column respectively. Tanner introduced an effective graphical representation for LDPC codes. The two types of nodes in a Tanner graph are called variable nodes ($v$-nodes) and check nodes ($c$-nodes). Figure 1 is an example for such a Tanner graph of a parity check matrix $H$ with dimension $m \times n$ for a $(8, 4)$ code.

Low-density parity-check codes are classified into two groups, regular and irregular LDPC codes. A LDPC code is called regular if $w_c$ is constant for every column and $w_r = w_c$. $(n/m)$ is also constant for every row. If $H$ is low density but the numbers of 1’s in each row or column aren’t constant the code is called an irregular LDPC code. Irregular LDPC codes have better performance than regular codes.

The generator matrix $G$ is found from $H$ by Gaussian elimination. If $H$ is put in the form $H = [P^T : I]$, the generator matrix will become $G = [I : P]$. The general encoding and decoding of systematic linear block codes can be expressed respectively as

$$c = xG = [x : xP]$$

and

$$cH^T = 0$$

if $c$ is a valid codeword and $x$ is a message word. There are several algorithms used to decode LDPC codes defined to date and the most common ones are message passing algorithm, belief propagation algorithm and sum-product algorithm [23]. In this paper, we have employed sum-product decoding algorithm as presented in [24].

![Tanner graph](image-url)
4. STTC Encoder

The encoder structure for a 4-PSK scheme with two transmits antennas and memory order \( v \) of its STTC coding section is shown in Fig.2 [19]. The binary information bits are spatially demultiplexed into two sub streams, \( X_1^t \) and \( X_2^t \) and are fed into the upper and lower branches of the STTC encoder with \( X_1^t \) being the most significant bit. The memory orders of the upper and lower branches are \( v_1 \) and \( v_2 \), respectively, where \( v = v_1 + v_2 \) and

\[
v_i = \left\lfloor \frac{v + i - 1}{2} \right\rfloor, \quad i = 1, 2,
\]

where \([x]\) denotes the largest integer smaller than \( x \). The two streams of input bits are delayed and multiplied by coefficient pairs \( \left\{ a_p^1, a_p^2 \right\} \) and \( \left\{ b_q^1, b_q^2 \right\} \) respectively, where \( a_p^1, b_q^1 \in \{0, 1, 2, 3\}, i = 1, 2, p = 0, 1, \ldots, v_1 \), \( v_1, q = 0, 1, \ldots, v_2 \). The encoder outputs are computed as

\[
C_k^k = \sum_{p=0}^{v_1} X_{t-p}^1 a_p^k + \sum_{q=0}^{v_2} X_{t-q}^2 b_q^k \mod 4, \quad k=1,2
\]

and are fed into two OFDM modulating channels.

![Figure 2. STTCM encoder for 4-PSK scheme.](image)

5. MMSE Equalization Algorithm

Let us assume that in the memory less MIMO channel, \( H \) is random and spatially white and its entries (path gains), \( h_{ij} \) are the independent and identically distributed zero-mean Gaussian
random variables for multi-path Rayleigh fading. In case of Rician fading, the channel has a fixed line-of-sight (LOS) component and the received signal equals the superposition of a complex Gaussian component and a LOS component. The Gaussian random variables are uncorrelated and do not have zero mean. The path gains are constant during one OFDM block symbol and vary independently from one OFDM block to another. At any time slot \( t \), the complex path gain \( h_{i,j} \) from a transmitting antenna \( i \) to a receiving antenna \( j \), the transmitted complex symbol, \( c_t^i \) and the received complex symbol, \( r_t^j \) are related as [19],

\[
r_t^j = \sum_{i=1}^{2} h_{i,j} c_t^i + n_t^j
\]

where, \( n_t^j \) is the AWGN sample for receiving antenna \( j \) in time \( t \). The noise samples are independent samples of a zero mean complex Gaussian random variable with spectral density \( N_0/2 \) per dimension [25],[26]. In matrix notation, the received signal (Eq. 5) can be written as

\[
R = Hc + n
\]

where \( n \) is the noise vector and \( H \) is the fading (Rayleigh/Rician) channel matrix. The eigen decomposition of the channel matrix \( H \) is given by

\[
H = U \Sigma V^H
\]

where \( \Sigma \) is a 2x2 matrix with the singular values of \( H, \{\sqrt{\lambda_1}, \sqrt{\lambda_2}\} \) as its main diagonal elements, \( U \) and \( V \) are the 2x2 unitary matrices respectively. The solution of the linear MMSE is given by

\[
\hat{C} = (\frac{1}{SNR} I_{N_R} + H^H H)^{-1} H^H \times R
\]

where the superscript ‘\( H \)’ denotes the complex conjugate transpose, \( I_{N_R} \) is the identity matrix \( \left[UU^T\right] \) and SNR is the signal to noise ratio. The MMSE receiver can minimize the overall error caused by noise and mutual interference between the co-channels signals which is also at the cost of reduced signal separation quality [27],[28].

### 6. Simulation Model

Figure 3 shows a simulation model for the LDPC channel encoded MIMO OFDM system with the implementation of MMSE channel equalization scheme for text message transmission. The system utilizes space-time-trellis coding for the antenna diversity. In such a communication system, two transmit antennas (Tx\(_1\) & Tx\(_2\)) and two receive antennas (Rx\(_1\) & Rx\(_2\)) are used. At the transmitter, a text messages “Information Technology is a very important sector in modern world. It is a computer based education.” is taken as input. The text message is converted into binary data of length 696 bits. The binary bits are encoded at the LDPC encoder. Here the 1/2-rated irregular LDPC matrix of size 696x1392 is used. The output of the LDPC encoder is 1392 bits which are then passed through interleaver to minimize bursts errors and modulated at the modulator. The complex data streams from the modulator output are then passed through STTC encoder to encode or divide the bit streams into two parts, which are converted from serial into parallel in each path independently and fed to the input of multi-carrier modulator IFFT. Inverse Fast Fourier Transform (IFFT) does modulation and multiplexing in one step. To mitigate the effects of inter-symbol Interference (ISI) caused by channel time spread, each block of IFFT coefficients is typically preceded by a cycle prefix of length (0.1x8), where 8 is the length of MIMO-OFDM block. After cycle prefixing these data are converted in serial form using parallel-to-serial converter and transmitted over the channel. Here AWGN (Additive white Gaussian
noise) channel and two fading channels (Rayleigh, Rician) are used. For fading channel the bit rate and Doppler shift are considered as 100000 and 100 Hz, respectively.

At the receiving section, the multiple versions of the signal created by different diversity schemes are combined to improve the performance. After combining the signal, it is converted into parallel form. Then the cyclic prefix is removed and the data streams are then passed through multi-carrier demodulator FFT (Fast Fourier Transform) which performs demodulation and de-multiplexing in one step. Outputs of the multi-carrier demodulator are then converted into serial form. These serial data streams are then separated and decoded using Minimum Mean-Square Error (MMSE) channel equalization algorithm and subsequently passed through digital demodulator, de-interleaver and LDPC decoder. LDPC decoder used iterative sum-product algorithm. Finally the LDPC decoded binary bit stream is converted into text message.

7. Simulation Results and Discussions

The computer simulation has been performed using Matlab 7.5 programming language to evaluate the BER performance of the LDPC based space-time trellis coded MIMO-OFDM system under different modulation schemes on text message transmission. The simulation
parameters are listed in Table 1. Figures 4 through 6 show the BER performance of the proposed MIMO-OFDM system under three types of digital modulations (BPSK, QPSK and QAM) on both AWGN and fading channels.

Table 1. Simulation Parameters

<table>
<thead>
<tr>
<th>Modulation</th>
<th>BPSK, QPSK, QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFDM Block Size</td>
<td>8</td>
</tr>
<tr>
<td>Cyclic Prefix Length</td>
<td>1</td>
</tr>
<tr>
<td>Channel</td>
<td>AWGN, Rayleigh, Rician</td>
</tr>
<tr>
<td>Bit Rate</td>
<td>100000</td>
</tr>
<tr>
<td>Doppler Shift</td>
<td>100 Hz</td>
</tr>
<tr>
<td>Size of the parity check matrix in LDPC code</td>
<td>$696 \times 1392$</td>
</tr>
<tr>
<td>Total number of bits in a codeword</td>
<td>1392</td>
</tr>
<tr>
<td>Number of information bits in a codeword</td>
<td>696</td>
</tr>
<tr>
<td>Number of parity bits in a codeword</td>
<td>696</td>
</tr>
</tbody>
</table>

As seen from Figure 4 that the 1/2-rated LDPC channel encoded STTC-MIMO-OFDM system outperforms at BPSK modulation and shows worst performance in the case of QPSK on AWGN channel. For a typical $E_b/N_0$ value of 0.75dB, the bit error rate values for BPSK and QPSK are 0.0656 and 0.1058 respectively which implies that the system performance with BPSK modulation is improved by 2.0758 dB than that of the system with QPSK modulation for a typical $E_b/N_0$ value of 0.75dB.

The BER performance of the 1/2-rated LDPC encoded STTC-MIMO-OFDM system on Rician fading channel is shown in Figure 5. The system provides satisfactory performance with BPSK modulation. Due to fading effect, the system performance undergoes significant degradation in QPSK modulation. In comparison of the BPSK with QPSK modulation, it is found that the system performance is improved by 2.797 dB in the case of BPSK for a typical $E_b/N_0$ value of 0.75 dB (where the BER values for BPSK and QPSK are 0.0669 and 0.1274, respectively).

Figure 6 demonstrates that the BER performance of the 1/2-rated LDPC channel encoded STTC-MIMO-OFDM system under different digital modulations on a Raleigh fading channel degrades due to fading channel effect. The system outperforms at BPSK modulation and shows worst performance at QPSK modulation; the system with QPSK modulation is more influenced by the Doppler frequency shift and its performance degrades. At $E_b/N_0$ value of 0.75 dB (where the BER values for BPSK and QPSK are 0.0687 and 0.1285, respectively), the system performance is improved by 2.7195 dB in the case of BPSK modulation as compared with QPSK.
Figure 4. BER of the $1/2$-rated LDPC encoded STTC-MIMO-OFDM system under different modulation schemes and AWGN channel.

Figure 5. BER of the $1/2$-rated LDPC encoded STTC-MIMO-OFDM system under different modulation schemes and Rician fading channel.
Figure 6. BER of the $\frac{1}{2}$-rated LDPC encoded STTC-MIMO-OFDM system under different modulation schemes and Rayleigh fading channel.

Figure 7. Impact of number of iterations on the BER simulation of LDPC encoded MIMO-OFDM system under BPSK modulation over AWGN channel.
Comparing the system performance on both the Rician and Rayleigh fading channels in Figures 5 and 6, respectively, it is observable that the BER performance under Rician fading channel is better than that in Rayleigh fading channel. For a typical $E_b/N_0$ value of 1.0 dB, where the BER values for BPSK under Rayleigh and Rician fading channels are 0.0428 and 0.0389 respectively which implies that the system performance is improved by 0.42 dB.

Figure 7 shows the impact of number of iterations on the BER simulation of LDPC encoded STTC-MIMO-OFDM system under BPSK modulation over AWGN channel. As iterative sum-product LDPC decoding algorithm has been used in retrieving transmitted text messages, the number of iterations has impact on the bit error rate. From Fig. 7 it is observable that the system performance is well appreciated at higher number of iterations.

Under Rayleigh fading channel environment, the transmitted and retrieved text messages are shown in Figures 8(a) through 8(e) for $E_b/N_0$ values of 0.5 dB, 1.0 dB, 1.5 dB and 2.0 dB, respectively for the LDPC encoded STTC-MIMO-OFDM system. The erroneous characters in the retrieved text messages are shown in bold faces (Figure 8).

8. CONCLUSIONS

In this paper, we evaluated the BER performance of LDPC-based space-time trellis coded MIMO-OFDM systems with the implementation of MMSE channel equalizer at the receiver under different digital modulations over AWGN, Raleigh and Rician fading channels. On the basis of the results obtained in the present simulation study, it can be concluded that the deployment of LDPC coding scheme in space-time trellis coded 2x2 MIMO-OFDM wireless communication system under BPSK modulation is very much effective in proper identification and retrieval of transmitted text message in noisy and fading environments.
REFERENCES


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