ANTI-SYNCHRONIZATION OF HYPERCHAOTIC BAO AND HYPERCHAOTIC XU SYSTEMS VIA ACTIVE CONTROL

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Abstract

This paper investigates the anti-synchronization of identical hyperchaotic Bao systems (Bao and Liu, 2008), identical hyperchaotic Xu systems (Xu, Cai and Zheng, 2009) and non-identical hyperchaotic Bao and hyperchaotic Xu systems. Active nonlinear control has been deployed for the anti- synchronization of the hyperchaotic systems addressed in this paper and the main results have been established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the active nonlinear control method is very effective and convenient to achieve anti-synchronization of identical and non-identical hyperchaotic Bao and hyperchaotic Xu systems. Numerical simulations have been provided to validate and demonstrate the effectiveness of the anti-synchronization results for the hyperchaotic Cao and hyperchaotic Xu systems.

KEYWORDS

Active Control, Anti-Synchronization, Hyperchaos, Hyperchaotic Bao System, Hyperchaotic Xu System.

1. INTRODUCTION

Chaos is a complex nonlinear phenomenon. The first chaotic attractor was discovered by Lorenz ([1], 1963), when he was studying the atmospheric convection in the weather models. Since then, chaos has been received extensive investigations and chaos phenomenon has been observed in a variety of fields including physical systems [2], chemical systems [3], ecological systems [4], secure communications [5-6], etc.

The seminal work on chaos synchronization problem was published by Pecora and Carroll ([7], 1990). From then on, chaos synchronization has been extensively and intensively studied in the last three decades [8-30].

In most of the anti-synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of anti-synchronization is to use the output of the master system to control the slave system so that the states of the slave system have the same amplitude but opposite signs as the states of the master system asymptotically. Thus, when anti-synchronization is achieved, the sum of the states of the master and slave systems tends to zero asymptotically with time.

In the recent years, various schemes have been deployed for chaos synchronization such as PC method [2], OGY method [8], active control [9-12], adaptive control [13-15], backstepping design [16], sampled-data feedback [17], sliding mode control [18-20], etc. Recently, active control method has been applied to achieve anti-synchronization of two identical chaotic systems [21-22].

Hyperchaotic system is usually defined as a chaotic system with at least two positive Lyapunov exponents, implying that its dynamics are expanded in several different directions simultaneously. Thus, the hyperchaotic systems have more complex dynamical behaviour which can be used to improve the security of a chaotic communication system. Hence, the theoretical design and circuit realization of various hyperchaotic signals have become important research topics [23-27].

In this paper, we use active control to derive new results for the anti-synchronization of identical hyperchaotic Bao systems ([28], 2008), identical hyperchaotic Xu systems ([29], 2009) and non-identical hyperchaotic Bao and hyperchaotic Xu systems.

This paper is organized as follows. In Section 2, we describe the problem statement and our methodology using Lyapunov stability theory. In Section 3, we provide a description of the hyperchaotic systems addressed in this paper. In Section 4, we discuss the anti-synchronization of identical hyperchaotic Bao systems (2008) using active nonlinear control. In Section 5, we discuss the anti-synchronization of identical hyperchaotic Xu systems (2009) using active nonlinear control. In Section 6, we discuss the anti-synchronization of hyperchaotic Bao and hyperchaotic Xu systems using active nonlinear control. In Section 7, we summarize the main results obtained in this paper.

2. PROBLEM STATEMENT AND OUR METHODOLOGY

As the *master* or *drive* system, we consider the chaotic system described by

$$\dot{x} = Ax + f(x), \tag{1}$$

where $x \in R^n$ is the state vector, A is the $n \times n$ matrix of system parameters and $f : R^n \to R^n$ is the nonlinear part of the system.

As the *slave* or *response* system, we consider the following chaotic system described by

$$\dot{y} = By + g(y) + u, \tag{2}$$

where $y \in R^n$ is the state of the slave system, B is the $n \times n$ matrix of system parameters, $g : R^n \to R^n$ is the nonlinear part of the system and u is the active controller to be designed. If $A = B$ and $f = g$, then x and y are the states of two *identical* chaotic systems. If $A \neq B$ or $f \neq g$, then x and y are the states of two *different* chaotic systems.

For the anti-synchronization of the chaotic systems (1) and (2) using active control, we design a state feedback controller u, which anti-synchronizes the states of the master system (1) and the slave system (2) for all initial conditions $x(0), y(0) \in R^n$.

Thus, we define the *anti-synchronization error* as

$$e = y + x, \tag{3}$$

Hence, the error dynamics is obtained as

$$\dot{e} = By + Ax + g(y) + f(x) + u \tag{4}$$

Thus, the anti-synchronization problem is essentially to find a feedback controller (active controller) u so as to stabilize the error dynamics (4) for all initial conditions, *i.e.*

$$\lim_{t \to \infty} \| e(t) \| = 0, \quad \text{for all } e(0) \in R^n \tag{5}$$

We use the Lyapunov stability theory as our methodology.

We take as a candidate Lyapunov function

$$V(e) = e^T P e, \tag{6}$$

where P is a positive definite matrix. Note that $V : R^n \to R$ is a positive definite function by construction.

If we find a feedback controller u so that

$$\dot{V}(e) = -e^T Q e, \tag{7}$$

where Q is a positive definite matrix, then $\dot{V} : R^n \to R$ is a negative definite function.

Thus, by Lyapunov stability theory [25], the error dynamics (4) is globally exponentially stable.

Hence, the states of the master system (1) and the slave system (2) will be globally and exponentially anti-synchronized for all values of the initial conditions $x(0), y(0) \in R^n$.

3. SYSTEMS DESCRIPTION

The hyperchaotic Bao system ([28], 2008) is described by the 4D dynamics

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_4 \\
\dot{x}_2 &= cx_2 - x_1 x_3 \\
\dot{x}_3 &= x_1 x_2 - bx_3 \\
\dot{x}_4 &= \varepsilon x_1 + dx_2 x_3
\end{aligned} \tag{8}$$

where x_1, x_2, x_3, x_4 are the state variables and a, b, c, d, ε are positive constants.

The four-dimensional Bao system (8) is *hyperchaotic* when

$$a = 36, \quad b = 3, \quad c = 20, \quad d = 0.1 \quad \text{and} \quad \varepsilon = 21.$$

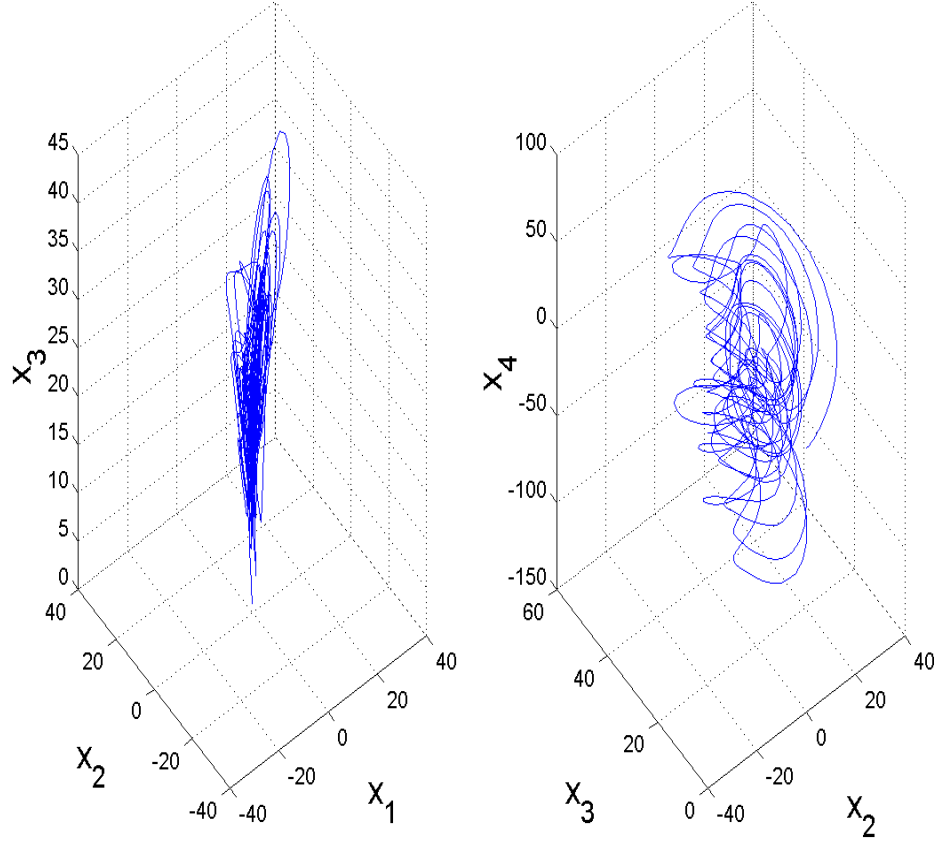The phase portrait of the hyperchaotic Bao system is shown in Figure 1.

Figure 1. Phase Portrait of the Hyperchaotic Bao System

The hyperchaotic Xu system ([29], 2009) is described by the 4D dynamics

$$
\begin{aligned}
\dot{x}_1 &= \alpha(x_2 - x_1) + x_4 \\
\dot{x}_2 &= \beta x_1 + r x_1 x_3 \\
\dot{x}_3 &= -\gamma x_3 - l x_1 x_2 \\
\dot{x}_4 &= x_1 x_3 - m x_2
\end{aligned}
\tag{9}
$$

where x_1, x_2, x_3, x_4 are the state variables and $\alpha, \beta, \gamma, l, m$ are positive constants.

The four-dimensional Xu system (9) is *hyperchaotic* when

$$\alpha = 10, \ \ \beta = 40, \ \ \gamma = 2.5, \ \ r = 16, \ \ l = 1 \ \text{and} \ \ m = 2.$$

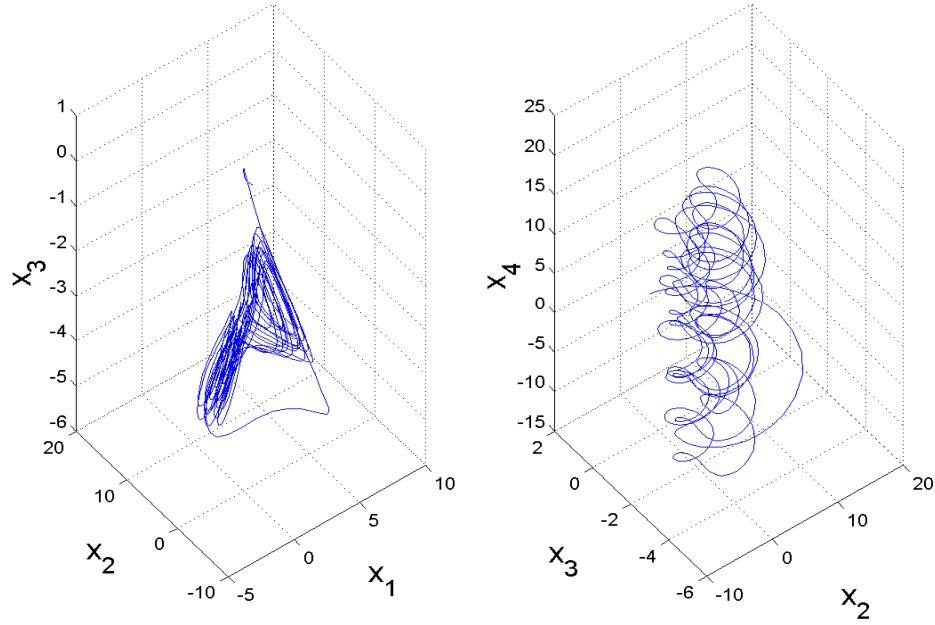The phase portrait of the hyperchaotic Xu system is shown in Figure 2.

Figure 2. Phase Portrait of the Hyperchaotic Xu System

3. ANTI-SYNCHRONIZATION OF HYPERCHAOTIC BAO SYSTEMS

In this section, we discuss the anti-synchronization of identical hyperchaotic Bao systems ([28], 2008).

As the master system, we consider the hyperchaotic Bao system described by

$$
\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_4 \\
\dot{x}_2 &= cx_2 - x_1 x_3 \\
\dot{x}_3 &= x_1 x_2 - bx_3 \\
\dot{x}_4 &= \varepsilon x_1 + dx_2 x_3
\end{aligned}
\tag{10}
$$

where x_1, x_2, x_3, x_4 are the state variables and a, b, c, d, ε are positive constants.

As the slave system, we consider the controlled hyperchaotic Bao dynamics described by

$$
\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + y_4 + u_1 \\
\dot{y}_2 &= cy_2 - y_1 y_3 + u_2 \\
\dot{y}_3 &= y_1 y_2 - by_3 + u_3 \\
\dot{y}_4 &= \varepsilon y_1 + dy_2 y_3 + u_4
\end{aligned}
\tag{11}
$$

where y_1, y_2, y_3, y_4 are the state variables and u_1, u_2, u_3, u_4 are the active controls.

The anti-synchronization error is defined as

$$
\begin{aligned}
e_1 &= y_1 + x_1 \\
e_2 &= y_2 + x_2 \\
e_3 &= y_3 + x_3 \\
e_4 &= y_4 + x_4
\end{aligned}
\tag{12}
$$

A simple calculation gives the error dynamics

$$
\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) + e_4 + u_1 \\
\dot{e}_2 &= ce_2 - y_1 y_3 - x_1 x_3 + u_2 \\
\dot{e}_3 &= -be_3 + y_1 y_2 + x_1 x_2 + u_3 \\
\dot{e}_4 &= \varepsilon e_1 + d(y_2 y_3 + x_2 x_3) + u_4
\end{aligned}
\tag{13}
$$

We consider the active nonlinear controller defined by

$$
\begin{aligned}
u_1 &= -a(e_2 - e_1) - e_4 - k_1 e_1 \\
u_2 &= -ce_2 + y_1 y_3 + x_1 x_3 - k_2 e_2 \\
u_3 &= be_3 - y_1 y_2 - x_1 x_2 - k_3 e_3 \\
u_4 &= -\varepsilon e_1 - d(y_2 y_3 + x_2 x_3) - k_4 e_4
\end{aligned}
\tag{14}
$$

where k_1, k_2, k_3, k_4 are positive constants.

Substitution of (14) into (13) yields the linear error dynamics

$$
\dot{e}_1 = -k_1 e_1, \quad \dot{e}_2 = -k_2 e_2, \quad \dot{e}_3 = -k_3 e_3, \quad \dot{e}_4 = -k_4 e_4
\tag{15}
$$

We consider the quadratic Lyapunov function defined by

$$
V(e) = \frac{1}{2} e^T e = \frac{1}{2} \left(e_1^2 + e_2^2 + e_3^2 + e_4^2 \right),
\tag{16}
$$

which is a positive definite function on R^4.

Next, we establish the main result of this section using Lyapunov stability theory.

Theorem 1. *The identical hyperchaotic Bao systems (10) and (11) are globally and exponentially anti-synchronized with the active nonlinear controller (13).*

Proof. Differentiating (16) along the trajectories of the error system (15), we get

$$
\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2,
\tag{17}
$$

which is a negative definite function on R^4

Thus, by Lyapunov stability theory [30], the error dynamics (15) is globally exponentially stable. This completes the proof. ∎

Numerical Simulations:

For the numerical simulations, the fourth order Runge-Kutta method with initial time-step $h = 10^{-8}$ is used to solve the two systems of differential equations (10) and (11) with the active nonlinear controller (14). We take the gains as $k_i = 5$ for $i = 1, 2, 3, 4$.

The parameters of the identical hyperchaotic Bao systems (8) and (9) are selected as

$$a = 36, \quad b = 3, \quad c = 20, \quad d = 0.1 \quad \text{and} \quad \varepsilon = 21.$$

The initial values for the master system (8) are taken as

$$x_1(0) = 5, \quad x_2(0) = -2, \quad x_3(0) = 14, \quad x_4(0) = 20$$

and the initial values for the slave system (9) are taken as

$$y_1(0) = 24, \quad y_2(0) = 17, \quad y_3(0) = -18, \quad y_4(0) = -22$$

Figure 4 depicts the anti-synchronization of the identical hyperchaotic Bao systems (10) and (11).

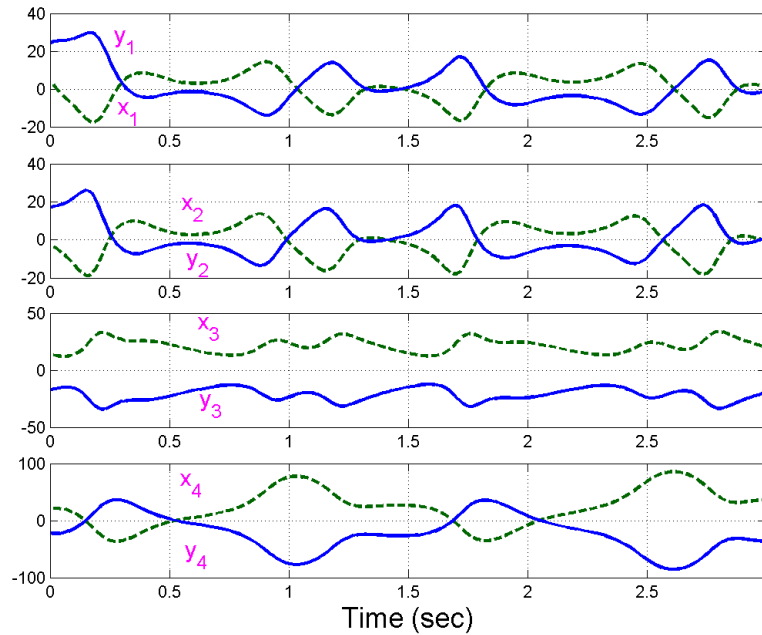Figure 4 depicts the time-history of the anti-synchronization error.

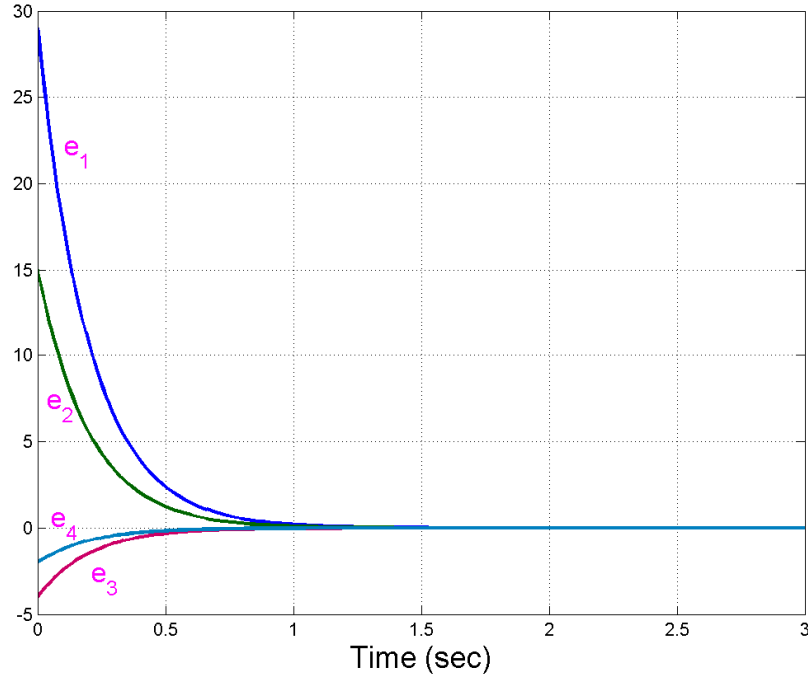Figure 3. Anti-Synchronization of Identical Hyperchaotic Bao Systems

Figure 4. Time-History of Anti-Synchronization Error e_1, e_2, e_3, e_4

4. ANTI-SYNCHRONIZATION OF HYPERCHAOTIC XU SYSTEMS

In this section, we discuss the anti-synchronization of identical hyperchaotic Xu systems ([29], 2009).

As the master system, we consider the hyperchaotic Xu system described by

$$
\begin{aligned}
\dot{x}_1 &= \alpha(x_2 - x_1) + x_4 \\
\dot{x}_2 &= \beta x_1 + r x_1 x_3 \\
\dot{x}_3 &= -\gamma x_3 - l x_1 x_2 \\
\dot{x}_4 &= x_1 x_3 - m x_2
\end{aligned}
\tag{18}
$$

where x_1, x_2, x_3, x_4 are the state variables and $\alpha, \beta, \gamma, r, l, m$ are positive constants.

As the slave system, we consider the controlled hyperchaotic Xu dynamics described by

$$
\begin{aligned}
y_1 &= \alpha(y_2 - y_1) + y_4 + u_1 \\
y_2 &= \beta y_1 + r y_1 y_3 + u_2 \\
y_3 &= -\gamma y_3 - l y_1 y_2 + u_3 \\
y_4 &= y_1 y_3 - m y_2 + u_4
\end{aligned}
\tag{19}
$$

where y_1, y_2, y_3, y_4 are the state variables and u_1, u_2, u_3, u_4 are the active controls.

The anti-synchronization error is defined as

$$
\begin{aligned}
e_1 &= y_1 + x_1 \\
e_2 &= y_2 + x_2 \\
e_3 &= y_3 + x_3 \\
e_4 &= y_4 + x_4
\end{aligned}
\tag{20}
$$

A simple calculation gives the error dynamics

$$
\begin{aligned}
\dot{e}_1 &= \alpha(e_2 - e_1) + e_4 + u_1 \\
\dot{e}_2 &= \beta e_1 + r(y_1 y_3 + x_1 x_3) + u_2 \\
\dot{e}_3 &= -\gamma e_3 - l(y_1 y_2 + x_1 x_2) + u_3 \\
\dot{e}_4 &= -m e_2 + y_1 y_3 + x_1 x_3 + u_4
\end{aligned}
\tag{21}
$$

We consider the active nonlinear controller defined by

$$
\begin{aligned}
u_1 &= -\alpha(e_2 - e_1) - e_4 - k_1 e_1 \\
u_2 &= -\beta e_1 - r(y_1 y_3 + x_1 x_3) - k_2 e_2 \\
u_3 &= \gamma e_3 + l(y_1 y_2 + x_1 x_2) - k_3 e_3 \\
u_4 &= m e_2 - y_1 y_3 - x_1 x_3 - k_4 e_4
\end{aligned}
\tag{22}
$$

where k_1, k_2, k_3, k_4 are positive constants.

Substitution of (22) into (21) yields the linear error dynamics

$$
\dot{e}_1 = -k_1 e_1, \quad \dot{e}_2 = -k_2 e_2, \quad \dot{e}_3 = -k_3 e_3, \quad \dot{e}_4 = -k_4 e_4
\tag{23}
$$

We consider the quadratic Lyapunov function defined by

$$
V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right),
\tag{24}
$$

which is a positive definite function on R^4.

Next, we establish the main result of this section using Lyapunov stability theory.

Theorem 2. *The identical hyperchaotic Xu systems (18) and (19) are globally and exponentially anti-synchronized with the active nonlinear controller (22).*

Proof. Differentiating (16) along the trajectories of the error system (15), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2, \tag{25}$$

which is a negative definite function on R^4

Thus, by Lyapunov stability theory [30], the error dynamics (23) is globally exponentially stable.

Hence, the states of the identical hyperchaotic Xu systems (18) and (19) are globally and exponentially anti-synchronized for all initial conditions $x(0), y(0) \in R^n$.

This completes the proof. ■

Numerical Simulations:

For the numerical simulations, the fourth order Runge-Kutta method with initial time-step $h = 10^{-6}$ is used to solve the two systems of differential equations (18) and (19) with the active nonlinear controller (22).

We take the feedback gains as

$$k_1 = 5, \ \ k_2 = 5, \ \ k_3 = 5, \ \ k_4 = 5$$

The parameters of the identical hyperchaotic Xu systems (18) and (19) are selected as

$$\alpha = 10, \ \ \beta = 40, \ \ \gamma = 2.5, \ \ r = 16, \ \ l = 1 \ \text{ and } \ m = 2.$$

The initial values for the master system (18) are taken as

$$x_1(0) = 12, \ \ x_2(0) = -14, \ \ x_3(0) = -7, \ \ x_4(0) = 21$$

and the initial values for the slave system (19) are taken as

$$y_1(0) = 26, \ \ y_2(0) = 10, \ \ y_3(0) = 18, \ \ y_4(0) = -4$$

Figure 5 depicts the anti-synchronization of the identical hyperchaotic Xu systems (18) and (19).

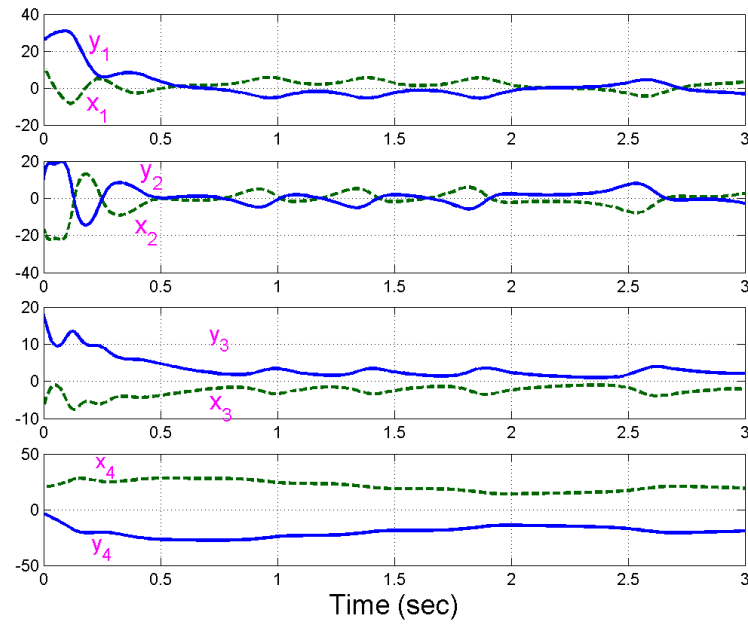Figure 6 depicts the time-history of the anti-synchronisation error e_1, e_2, e_3, e_4.

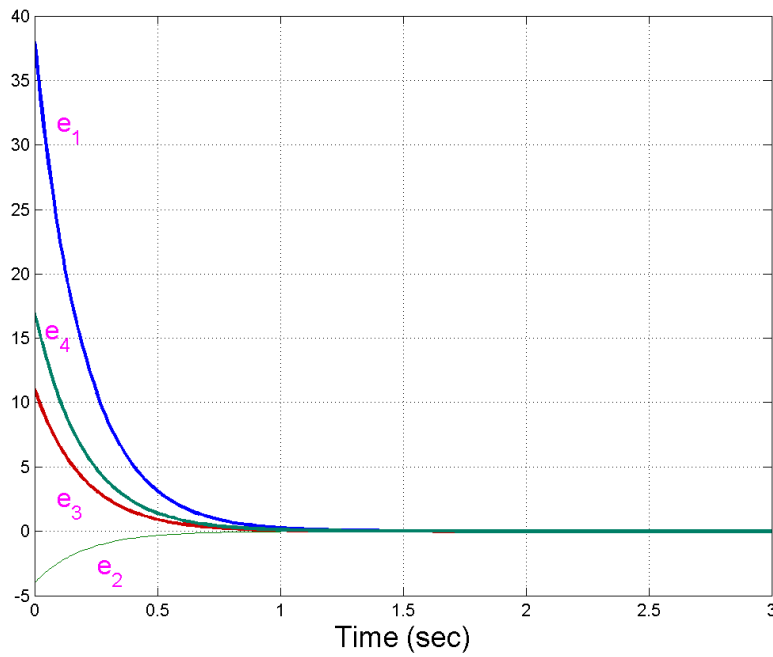Figure 5. Anti-Synchronization of Identical Hyperchaotic Xu Systems

Figure 6. Time-History of the Anti-Synchronization Error e_1, e_2, e_3, e_4

5. ANTI-SYNCHRONIZATION OF HYPERCHAOTIC BAO AND HYPERCHAOTIC XU SYSTEMS

In this section, we consider the anti-synchronization of non-identical hyperchaotic Bao system ([28], 2008) and hyperchaotic Xu system ([29], 2009).

As the master system, we consider the hyperchaotic Bao system described by

$$
\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_4 \\
\dot{x}_2 &= cx_2 - x_1 x_3 \\
\dot{x}_3 &= x_1 x_2 - bx_3 \\
\dot{x}_4 &= \varepsilon x_1 + dx_2 x_3
\end{aligned}
\tag{26}
$$

where x_1, x_2, x_3, x_4 are the state variables and a, b, c, d, ε are positive constants.

As the slave system, we consider the controlled hyperchaotic Xu dynamics described by

$$
\begin{aligned}
y_1 &= \alpha(y_2 - y_1) + y_4 + u_1 \\
y_2 &= \beta y_1 + ry_1 y_3 + u_2 \\
y_3 &= -\gamma y_3 - ly_1 y_2 + u_3 \\
y_4 &= y_1 y_3 - my_2 + u_4
\end{aligned}
\tag{27}
$$

where y_1, y_2, y_3, y_4 are the state variables, $\alpha, \beta, \gamma, r, l, m$ are positive constants and u_1, u_2, u_3, u_4 are the active controls.

The anti-synchronization error is defined as

$$
\begin{aligned}
e_1 &= y_1 + x_1 \\
e_2 &= y_2 + x_2 \\
e_3 &= y_3 + x_3 \\
e_4 &= y_4 + x_4
\end{aligned}
\tag{28}
$$

A simple calculation gives the error dynamics

$$
\begin{aligned}
\dot{e}_1 &= \alpha(e_2 - e_1) + e_4 - (\alpha - a)(x_2 - x_1) + u_1 \\
\dot{e}_2 &= \beta e_1 - \beta x_1 + cx_2 + ry_1 y_3 - x_1 x_3 + u_2 \\
\dot{e}_3 &= -\gamma e_3 - (b - \gamma)x_3 - ly_1 y_2 + x_1 x_2 + u_3 \\
\dot{e}_4 &= -me_2 + \varepsilon x_1 + mx_2 + y_1 y_3 + dx_2 x_3 + u_4
\end{aligned}
\tag{29}
$$

We consider the active nonlinear controller defined by

$$
\begin{aligned}
u_1 &= -\alpha(e_2 - e_1) - e_4 + (\alpha - a)(x_2 - x_1) - k_1 e_1 \\
u_2 &= -\beta e_1 + \beta x_1 - c x_2 - r y_1 y_3 + x_1 x_3 - k_2 e_2 \\
u_3 &= \gamma e_3 + (b - \gamma) x_3 + l y_1 y_2 - x_1 x_2 - k_3 e_3 \\
u_4 &= m e_2 - \varepsilon x_1 - m x_2 - y_1 y_3 - d x_2 x_3 - k_4 e_4
\end{aligned}
\tag{30}
$$

Substitution of (30) into (29) yields the linear error dynamics

$$
\begin{aligned}
\dot{e}_1 &= -k_1 e_1 \\
\dot{e}_2 &= -k_2 e_2 \\
\dot{e}_3 &= -k_3 e_3 \\
\dot{e}_4 &= -k_4 e_4
\end{aligned}
\tag{31}
$$

We consider the quadratic Lyapunov function defined by

$$
V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right),
\tag{32}
$$

which is a positive definite function on R^4.

Next, we establish the main result of this section using Lyapunov stability theory.

Theorem 3. *The non-identical hyperchaotic Bao system (26) and hyperchaotic Xu system (27) are globally and exponentially anti-synchronized with the active nonlinear controller (29).*

Proof. Differentiating (32) along the trajectories of the error system (31), we get

$$
\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2,
\tag{33}
$$

which is a negative definite function on R^4 since α and γ are positive constants.

Thus, by Lyapunov stability theory [30], the error dynamics (31) is globally exponentially stable.

Hence, it follows that the states of the hyperchaotic Bao system (26) and hyperchaotic Xu system (27) are globally and exponentially anti-synchronized for all initial conditions $x(0), y(0) \in R^n$.

This completes the proof.∎

Numerical Simulations

For the numerical simulations, the fourth order Runge-Kutta method with initial time-step $h = 10^{-6}$ is used to solve the two systems of differential equations (26) and (27) with the active nonlinear controller (30).

We take the feedback gains as

$$k_1 = 5, \quad k_2 = 5, \quad k_3 = 5 \text{ and } k_4 = 5.$$

The parameters of the identical hyperchaotic Bao system (24) are selected as

$$a = 36, \quad b = 3, \quad c = 20, \quad d = 0.1 \text{ and } \varepsilon = 21.$$

The parameters of the identical hyperchaotic Xu system (25) are selected as

$$\alpha = 10, \quad \beta = 40, \quad \gamma = 2.5, \quad r = 16, \quad l = 1 \text{ and } m = 2.$$

The initial values for the master system (26) are taken as

$$x_1(0) = 25, \quad x_2(0) = -6, \quad x_3(0) = 9, \quad x_4(0) = -14$$

and the initial values for the slave system (27) are taken as

$$y_1(0) = 12, \quad y_2(0) = 15, \quad y_3(0) = -20, \quad y_4(0) = -8$$

Figure 7 depicts the anti-synchronization of the non-identical hyperchaotic systems, *viz.* hyperchaotic Bao system (26) and hyperchaotic Xu system (28).

Figure 8 depicts the time-history of the anti-synchronization error.

6. CONCLUSIONS

In this paper, using the active control method, new results have been derived for the anti-synchronization for the following pairs of hyperchaotic systems:

    (A) Identical hyperchaotic Bao systems (2008)

    (B) Identical hyperchaotic Xu systems (2009)

    (C) Non-identical hyperchaotic Bao and hyperchaotic Xu systems.

The anti-synchronization results derived in this paper have been proved using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the active control method is very convenient and efficient for the anti- synchronization of identical and non-identical hyperchaotic Bao and hyperchaotic Xu systems. Numerical simulation results have been presented to illustrate the anti-synchronization results derived in this paper.
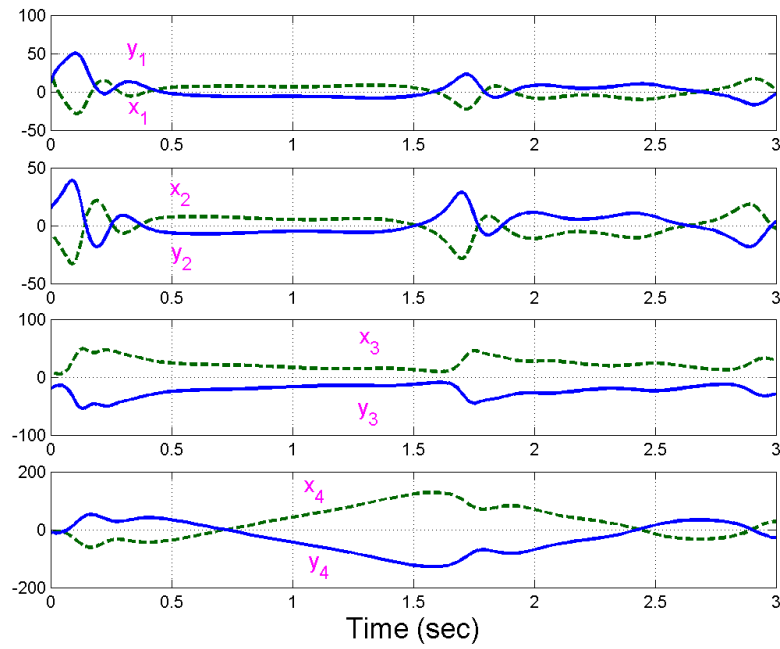
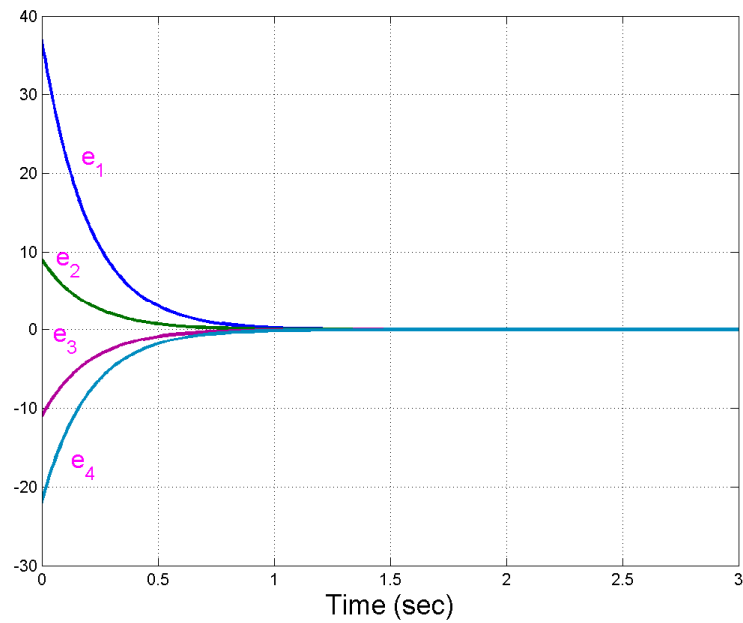Figure 7. Anti-Synchronization of Hyperchaotic Bao and Hyperchaotic Xu Systems

Figure 8. Time-History of the Anti-Synchronization Error e_1, e_2, e_3, e_4

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