ANTI-SYNCHRONIZING SLIDING CONTROLLER DESIGN FOR IDENTICAL PAN SYSTEMS

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ABSTRACT

The anti-synchronization of identical Pan systems (Pan, Xu and Zhou, 2010) has been investigated in this paper and new results have been derived by sliding mode control. In this paper, we first derive a general result for the anti-synchronization of identical chaotic systems. As an application, we derive anti-synchronizing sliding controller for identical Pan systems. A numerical example using MATLAB has been presented so as to demonstrate the anti-synchronizing sliding controller for the Pan systems.

KEYWORDS

Pan System, Anti-Synchronization, Sliding Control, Chaos.

1. INTRODUCTION

Chaos theory is an active research area that deals with nonlinear dynamical systems which are sensitive to initial conditions [1]. This area has diverse applications like Economics [2], Physics [3], Biology [4], Chemistry [5], Image Processing [6] and Secure Communications [7].

The chaos synchronization problem, which deals with the synchronization of a pair of chaotic systems called the master and slave systems, has been studied rigorously in the literature. Some miscellaneous methods of synchronization can be listed as PC method [8], OGY method [9], active control method [10-11], adaptive control method [12], backstepping method [13], sampled-data feedback method [14], etc.

This paper deals with the design of sliding controller design for the anti-synchronization of identical Pan systems ([15], 2010). First, a general result is derived for anti-synchronization of chaotic systems. Next, as an application, new results are derived for the anti-synchronization of identical Pan systems. Numerical simulations using MATLAB are shown to illustrate the main results of this paper.

2. PROBLEM STATEMENT AND OUR METHODOLOGY USING SMC

We consider the chaotic system described by

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in R^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : R^n \to R^n$ is the nonlinear part of the system.

We take the system (1) as the *master* system.

Next, we consider the following chaotic system as the *slave system* described by

$$\dot{y} = Ay + f(y) + u \tag{2}$$

where $y \in R^n$ is the state of the system and $u \in R^m$ is the controller to be designed.

If we define the *anti-synchronization error* as

$$e = y + x, \tag{3}$$

then the error dynamics is obtained as

$$\dot{e} = Ae + \eta(x, y) + u, \tag{4}$$

where

$$\eta(x, y) = f(y) + f(x) \tag{5}$$

The design goal is to find a controller u such that

$$\lim_{t \to \infty} \|e(t)\| = 0$$

for all $e(0) \in R^n$.

Our methodology is detailed next. We define the control u as

$$u = -\eta(x, y) + Bv \tag{6}$$

In Eq. (6), B is an $n \times 1$ matrix carefully selected such that (A, B) is controllable.

Next, we substitute the definition of u from (6) into (4).

Then the error dynamics becomes

$$\dot{e} = Ae + Bv \tag{7}$$

It is noted that Eq. (7) is a linear single-input, time-invariant control system.

In SMC, we define the sliding variable as

$$s(e) = Ce = c_1 e_1 + c_2 e_2 + \cdots + c_n e_n \tag{8}$$

In SMC, we constrain the motion of the system (7) to the sliding manifold defined by

$$S = \left\{ e \in R^n : s(e) = 0 \right\}$$

such that the motion is invariant under the flow of the error dynamics (7).

When in sliding manifold S, the system (7) satisfies the following conditions:

$$s(e) = 0 \tag{9}$$

and

$$\dot{s}(e) = 0 \tag{10}$$

With the help of equations (7) and (8), the equation (10) can be rearranged as

$$\dot{s}(e) = C\left[Ae + Bv\right] = 0 \tag{11}$$

Solving (11) for v, we obtain the equivalent control law

$$v_{\text{eq}}(t) = -(CB)^{-1}CA\, e(t) \tag{12}$$

where C is chosen such that $CB \neq 0$.

Next, we substitute (12) into the error dynamics (7).

Then we obtain the closed-loop error dynamics as

$$\dot{e} = \left[I - B(CB)^{-1}C\right]Ae \tag{13}$$

The row vector C is chosen so that the system linearization matrix of the controlled dynamics $\left[I - B(CB)^{-1}C\right]A$ is Hurwitz, *i.e.* it has all eigenvalues with negative real parts. Then, by Lyapunov stability theory, the controlled system (13) is globally asymptotically stable.

A sliding mode controller for (7) is designed as follows.

Our strategy is to apply the constant plus proportional rate reaching law given by

$$\dot{s} = -q\,\text{sgn}(s) - k\, s \tag{14}$$

where the gains q and k are positive.

We obtain the control $v(t)$ from equations (11) and (14) as follows:

$$v(t) = -(CB)^{-1}\left[C(kI + A)e + q\,\text{sgn}(s)\right] \tag{15}$$

which yields

$$v(t) = \begin{cases} -(CB)^{-1}\left[C(kI + A)e + q\right], & \text{if } s(e) > 0 \\ -(CB)^{-1}\left[C(kI + A)e - q\right], & \text{if } s(e) < 0 \end{cases} \tag{16}$$

Theorem 2.1. *Consider the master system (1) and the slave system (2). These two chaotic systems are globally and asymptotically anti-synchronized for all initial conditions* $x(0), y(0) \in R^n$ *by the feedback control law*

$$u(t) = -\eta(x, y) + Bv(t) \tag{17}$$

where $v(t)$ *is defined by (15) and* B *is a column vector such that* (A, B) *is controllable. Also, the sliding mode gains* k, q *are positive.*

Proof. To start with, we substitute (17) and (15) into the error dynamics (4).

Then we obtain the closed-loop error dynamics as

$$\dot{e} = Ae - B(CB)^{-1}\left[C(kI + A)e + q\,\text{sgn}(s)\right] \tag{18}$$

We wish to prove that the error dynamics (18) is globally asymptotically stable.

For this purpose, we consider the candidate Lyapunov function defined by

$$V(e) = \frac{1}{2}s^2(e) \tag{19}$$

which is a positive definite function on R^n.

We differentiate V along the trajectories of (18) or the equivalent dynamics (14).

This yields

$$\dot{V}(e) = s(e)\dot{s}(e) = -ks^2 - q\,\text{sgn}(s)s \tag{20}$$

which is a negative definite function on R^n.

Thus, the dynamics (18) is globally asymptotically stable by Lyapunov stability theory [16].

This completes the proof. ■

3. ANTI-SYNCHRONIZATION OF IDENTICAL PAN SYSTEMS USING SLIDING MODE CONTROL

3.1 Theoretical Results

We take the Pan system as the master system given by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= cx_1 - x_1x_3 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \end{aligned} \tag{21}$$

where the constants a, b, c are positive.

We take the controlled Pan system as the slave system given by

$$\begin{aligned}\dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= cy_1 - y_1 y_3 + u_2 \\ \dot{y}_3 &= -by_3 + y_1 y_2 + u_3\end{aligned} \tag{22}$$

where u_1, u_2, u_3 are the controllers to be designed.

The Pan system is chaotic when

$$a = 10, \quad b = 8/3 \quad \text{and} \quad c = 16.$$

Figure 1 illustrates the strange attractor of the chaotic Pan system (21).

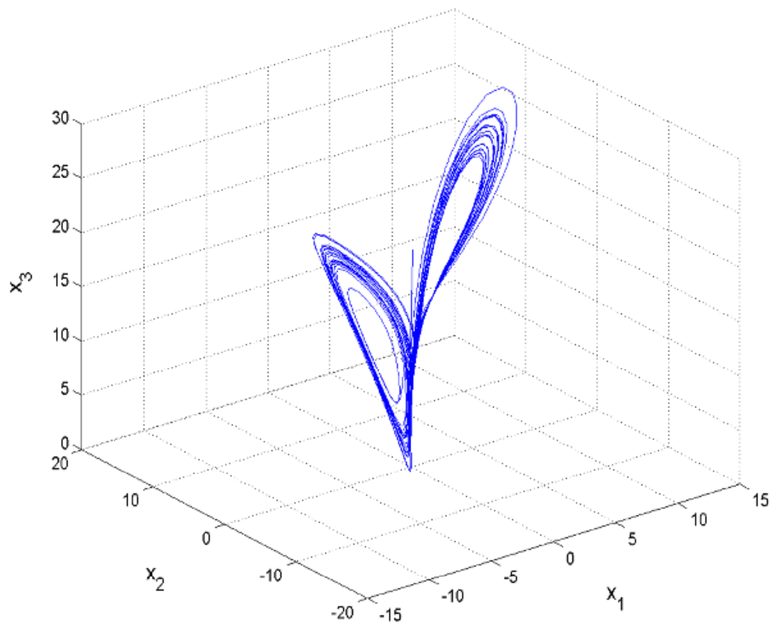

Figure 1. Strange Attractor of the Pan System

The chaos *anti-synchronization error* is defined by

$$e_i = y_i + x_i, \quad (i = 1, 2, 3) \tag{23}$$

The error dynamics is easily obtained as

$$\begin{aligned}\dot{e}_1 &= a(e_2 - e_1) + u_1 \\ \dot{e}_2 &= ce_1 - y_1 y_3 - x_1 x_3 + u_2 \\ \dot{e}_3 &= -be_3 + y_1 y_2 + x_1 x_2 + u_3\end{aligned} \tag{24}$$

The error dynamics (24) can be compactly written in matrix form as

$$\dot{e} = Ae + \eta(x, y) + u \tag{25}$$

where

$$A = \begin{bmatrix} -a & a & 0 \\ c & 0 & 0 \\ 0 & 0 & -b \end{bmatrix}, \quad \eta(x, y) = \begin{bmatrix} 0 \\ -y_1 y_3 - x_1 x_3 \\ y_1 y_2 + x_1 x_2 \end{bmatrix} \text{ and } u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}. \tag{26}$$

Using the results outlined in Section 2, we design the anti-synchronizing sliding mode controller for identical Pan systems as follows.

First, we set u as

$$u = -\eta(x, y) + Bv \tag{27}$$

where B is taken so that (A, B) is controllable.

We choose B as

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{28}$$

In the chaotic case, the parameter values are taken as

$$a = 10, \quad b = 8/3 \quad \text{and} \quad c = 16.$$

The sliding mode variable is selected as

$$s = Ce = \begin{bmatrix} 6 & 6 & 1 \end{bmatrix} e = 6e_1 + 6e_2 + e_3 \tag{29}$$

which renders the sliding dynamics asymptotically stable.

We choose the sliding mode gains as

$$k = 6 \text{ and } q = 0.1.$$

From Eq. (15), we can obtain $v(t)$ as

$$v(t) = -5.5385\, e_1 - 7.3846 e_2 - 0.2564\, e_3 - 0.0077\ \operatorname{sgn}(s) \tag{30}$$

Thus, we have derived the required anti-synchronizing controller as

$$u = -\eta(x, y) + Bv \tag{31}$$

where $\eta(x, y), B$ and $v(t)$ are given by the equations (26), (28) and (30).

By Theorem 2.1, we arrive at the following result.

Theorem 3.1. *The identical Pan systems (21) and (22) are globally and asymptotically anti-synchronized for all initial conditions with the sliding mode controller* u *defined by (31).* ■

3.2 Numerical Results

In this section, we show the numerical simulations obtained via the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ using MATLAB for solving the Pan systems (21) and (22). Note that the sliding mode controller u has been given by the formula (31).

The parameter values of the Pan system are taken as in the chaotic case, *viz.* $a = 10,\ b = 8/3$ and $c = 16$. The sliding mode gains are chosen as $k = 6$ and $q = 0.1$.

The initial values of the Pan system (21) are taken as

$$x_1(0) = 30,\ x_2(0) = -5,\ x_3(0) = 14$$

The initial values of the controlled Pan system (22) are taken as

$$y_1(0) = 14,\ y_2(0) = 10,\ y_3(0) = -26$$

Figure 2 illustrates the anti-synchronization of the identical Pan systems (21) and (22).

Figure 3 illustrates the time-history of the anti-synchronization errors e_1, e_2, e_3.

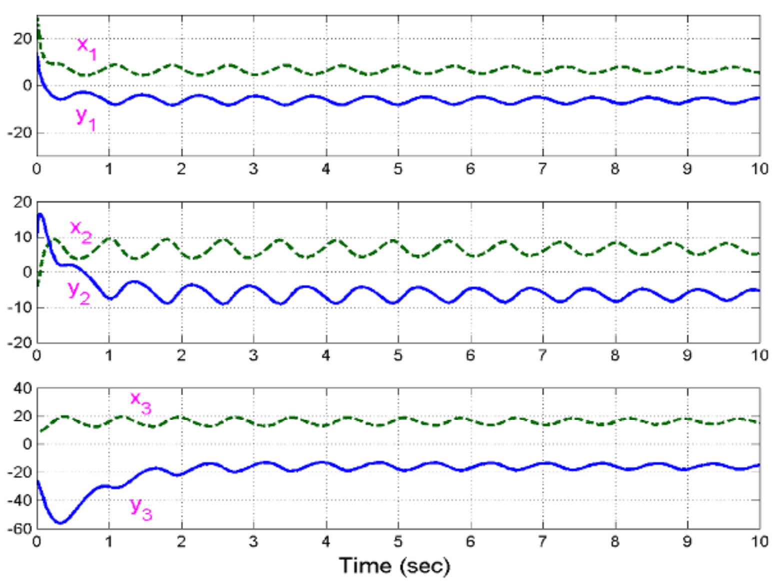

Figure 2. Anti-Synchronization of Identical Pan Systems

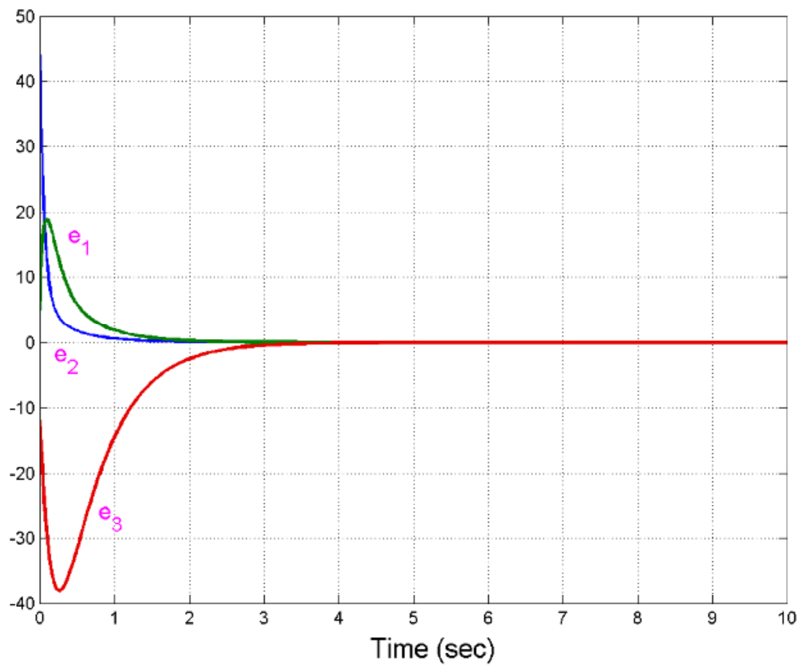

Figure 3. Time-History of the Anti-Synchronization Error

4. CONCLUSIONS

This paper derived new results for the anti-synchronization of chaotic systems via sliding mode control (SMC). As an application, new results were derived using sliding control to achieve anti-synchronization for the identical Pan systems (2010). Our anti-synchronization results for the identical Pan systems have been proved using Lyapunov stability theory.

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