GLOBAL SYNCHRONIZATION OF FOUR-WING CHAOTIC SYSTEMS BY SLIDING MODE CONTROL

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ABSTRACT

This paper investigates the global chaos synchronization of identical Qi four-wing chaotic systems (Qi et al., 2008), identical Liu four-wing chaotic systems (Liu, 2009) and identical Wang chaotic systems (Wang et al., 2009). The stability results derived in this paper for the complete synchronization of the three pairs of identical four-wing chaotic systems are established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the sliding mode control method is very effective and convenient to achieve global chaos synchronization of the four-wing chaotic systems. Numerical simulations are shown to illustrate and validate the synchronization schemes derived in this paper for the identical four-wing chaotic systems.

KEYWORDS

Sliding Mode Control, Chaos Synchronization, Chaotic Systems, Four-Wing Systems.

1. INTRODUCTION

Chaotic systems are dynamical systems that are highly sensitive to initial conditions. The sensitive nature of chaotic systems is commonly called as the *butterfly effect* [1]. Synchronization of chaotic systems is a phenomenon which may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem.

In most of the chaos synchronization approaches, the *master-slave* or *drive-response* formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

Since the pioneering work by Pecora and Carroll ([2], 1990), chaos synchronization problem has been studied extensively and intensively in the literature [2-29]. Chaos theory has been applied to a variety of fields such as physical systems [3], chemical systems [4], ecological systems [5], secure communications [6-8], etc.

In the last two decades, various schemes have been successfully applied for chaos synchronization such as PC method [2], OGY method [9], active control method [10-14], adaptive control method [15-18], time-delay feedback method [19], backstepping design method [20-22], sampled-data feedback method [23], etc.

In this paper, we derive new results based on the sliding mode control [24-26] for the global chaos synchronization of identical Qi four-wing chaotic systems ([27], 2008), identical Liu four-wing chaotic systems ([28], 2009) and identical Wang four-wing chaotic systems ([29], 2009). In robust control systems, the sliding mode control method is often adopted due to its inherent advantages of easy realization, fast response and good transient performance as well as its insensitivity to parameter uncertainties and external disturbances.

This paper has been organized as follows. In Section 2, we describe the problem statement and our methodology using sliding mode control (SMC). In Section 3, we discuss the global chaos synchronization of identical Qi four-wing chaotic systems (2008) using sliding mode control. In Section 4, we discuss the global chaos synchronization of identical Liu four-wing chaotic systems (2009) using sliding mode control. In Section 5, we discuss the global chaos synchronization of identical Wang four-wing chaotic systems (2009) using sliding mode control. In Section 6, we summarize the main results obtained in this paper.

2. Problem Statement and Our Methodology using SMC

In this section, we describe the problem statement for the global chaos synchronization for identical chaotic systems and our methodology using sliding mode control (SMC).

Consider the chaotic system described by

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in R^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : R^n \to R^n$ is the nonlinear part of the system. We consider the system (1) as the *master* or *drive* system.

As the *slave* or *response* system, we consider the following chaotic system described by the dynamics

$$\dot{y} = Ay + f(y) + u \tag{2}$$

where $y \in R^n$ is the state of the system and $u \in R^m$ is the controller to be designed.

If we define the *synchronization error* as

$$e = y - x, \tag{3}$$

then the error dynamics is obtained as

$$\dot{e} = Ae + \eta(x, y) + u, \tag{4}$$

where

$$\eta(x, y) = f(y) - f(x) \tag{5}$$

The objective of the global chaos synchronization problem is to find a controller u such that

$$\lim_{t\to\infty} \|e(t)\| = 0 \quad \text{for all } e(0) \in R^n.$$

To solve this problem, we first define the control u as

$$u = -\eta(x, y) + Bv \tag{6}$$

where B is a constant gain vector selected such that (A, B) is controllable.

Substituting (5) into (4), the error dynamics simplifies to

$$\dot{e} = Ae + Bv \tag{7}$$

which is a linear time-invariant control system with single input v.

Thus, the original global chaos synchronization problem can be replaced by an equivalent problem of stabilizing the zero solution $e = 0$ of the system (7) by a suitable choice of the sliding mode control. In the sliding mode control, we define the variable

$$s(e) = Ce = c_1e_1 + c_2e_2 + \cdots + c_ne_n \tag{8}$$

where $C = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}$ is a constant vector to be determined.

In the sliding mode control, we constrain the motion of the system (7) to the sliding manifold defined by

$$S = \left\{ x \in R^n \mid s(e) = 0 \right\}$$

which is required to be invariant under the flow of the error dynamics (7).

When in sliding manifold S, the system (7) satisfies the following conditions:

$$s(e) = 0 \tag{9}$$

which is the defining equation for the manifold S and

$$\dot{s}(e) = 0 \tag{10}$$

which is the necessary condition for the state trajectory $e(t)$ of (7) to stay on the sliding manifold S.

Using (7) and (8), the equation (10) can be rewritten as

$$\dot{s}(e) = C\left[Ae + Bv\right] = 0 \tag{11}$$

Solving (11) for v, we obtain the equivalent control law

$$v_{\text{eq}}(t) = -(CB)^{-1}CA\ e(t) \tag{12}$$

where C is chosen such that $CB \neq 0$.

Substituting (12) into the error dynamics (7), we obtain the closed-loop dynamics as

$$\dot{e} = \left[I - B(CB)^{-1}C \right] Ae \tag{13}$$

The row vector C is selected such that the system matrix of the controlled dynamics $\left[I - B(CB)^{-1}C \right] A$ is Hurwitz, *i.e.* it has all eigenvalues with negative real parts. Then the controlled system (13) is globally asymptotically stable.

To design the sliding mode controller for (7), we apply the constant plus proportional rate reaching law

$$\dot{s} = -q\,\text{sgn}(s) - k\ s \tag{14}$$

where $\text{sgn}(\cdot)$ denotes the sign function and the gains $q > 0,\ \ k > 0$ are determined such that the sliding condition is satisfied and sliding motion will occur.

From equations (11) and (14), we can obtain the control $v(t)$ as

$$v(t) = -(CB)^{-1}\left[C(kI + A)e + q\,\text{sgn}(s) \right] \tag{15}$$

which yields

$$v(t) = \begin{cases} -(CB)^{-1}\left[C(kI + A)e + q \right], & \text{if } s(e) > 0 \\ -(CB)^{-1}\left[C(kI + A)e - q \right], & \text{if } s(e) < 0 \end{cases} \tag{16}$$

Theorem 1. *The master system (1) and the slave system (2) are globally and asymptotically synchronized for all initial conditions* $x(0), y(0) \in R^n$ *by the feedback control law*

$$u(t) = -\eta(x, y) + Bv(t) \tag{17}$$

where $v(t)$ *is defined by (15) and* B *is a column vector such that* (A, B) *is controllable. Also, the sliding mode gains* k, q *are positive.*

Proof. First, we note that substituting (17) and (15) into the error dynamics (4), we obtain the closed-loop error dynamics as

$$\dot{e} = Ae - B(CB)^{-1}\left[C(kI + A)e + q\,\text{sgn}(s) \right] \tag{18}$$

To prove that the error dynamics (18) is globally asymptotically stable, we consider the candidate Lyapunov function defined by the equation

$$V(e) = \frac{1}{2}s^2(e) \tag{19}$$

which is a positive definite function on R^n.

Differentiating V along the trajectories of (18) or the equivalent dynamics (14), we get

$$\dot{V}(e) = s(e)\dot{s}(e) = -ks^2 - q\,\mathrm{sgn}(s)s \tag{20}$$

which is a negative definite function on R^n.

This calculation shows that V is a globally defined, positive definite, Lyapunov function for the error dynamics (18), which has a globally defined, negative definite time derivative $\dot{V}$.

Thus, by Lyapunov stability theory [30], it is immediate that the error dynamics (18) is globally asymptotically stable for all initial conditions $e(0) \in R^n$.

Hence, it follows that the master system (1) and the slave system (2) are globally and asymptotically synchronized for all initial conditions $x(0), y(0) \in R^n$.

This completes the proof. ■

3. Global chaos Synchronization of Identical Qi Four-Wing Systems using Sliding Mode Control

3.1 Theoretical Results

In this section, we apply the sliding mode control results derived in Section 2 for the global chaos synchronization of identical Qi four-wing chaotic systems ([27], Qi *et al*., 2008).

Thus, the master system is described by the Qi dynamics

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + \varepsilon x_2 x_3 \\
\dot{x}_2 &= cx_1 + dx_2 - x_1 x_3 \\
\dot{x}_3 &= -bx_3 + x_1 x_2
\end{aligned} \tag{21}$$

where x_1, x_2, x_3 are state variables, a, b, d are all real positive constant parameters and c, ε are real constant parameters of the system.

The slave system is described by the controlled Qi dynamics

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + \varepsilon y_2 y_3 + u_1 \\
\dot{y}_2 &= cy_1 + dy_2 - y_1 y_3 + u_2 \\
\dot{y}_3 &= -by_3 + y_1 y_2 + u_3
\end{aligned} \tag{22}$$

where y_1, y_2, y_3 are state variables and u_1, u_2, u_3 are the controllers to be designed.

The Qi systems (21) and (22) are chaotic when

$$a = 14,\ b = 43,\ c = -1,\ d = 16 \ \text{ and } \ \varepsilon = 4.$$

Figure 1 illustrates the four-wing chaotic attractor of the Qi system (21).

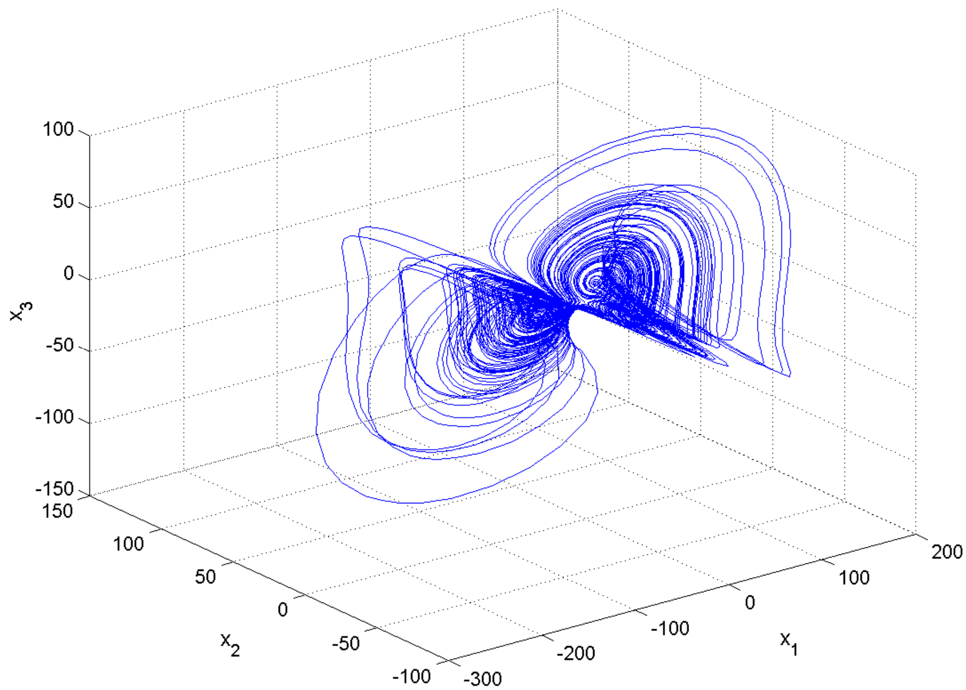

Figure 1. State Orbits of the Qi Chaotic System

The chaos synchronization error is defined by

$$e_i = y_i - x_i, \ (i = 1, 2, 3) \tag{23}$$

The error dynamics is easily obtained as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + \varepsilon(y_2 y_3 - x_2 x_3) + u_1 \\ \dot{e}_2 &= ce_1 + de_2 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= -be_3 + y_1 y_2 - x_1 x_2 + u_3 \end{aligned} \tag{24}$$

We write the error dynamics (24) in the matrix notation as

$$\dot{e} = Ae + \eta(x, y) + u \tag{25}$$

where

$$A = \begin{bmatrix} -a & a & 0 \\ c & d & 0 \\ 0 & 0 & -b \end{bmatrix}, \quad \eta(x, y) = \begin{bmatrix} \varepsilon(y_2 y_3 - x_2 x_3) \\ -y_1 y_3 + x_1 x_3 \\ y_1 y_2 - x_1 x_2 \end{bmatrix} \quad \text{and} \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}. \tag{26}$$

The sliding mode controller design is carried out as detailed in Section 2.

First, we set u as

$$u = -\eta(x, y) + Bv \tag{27}$$

where B is chosen such that (A, B) is controllable.

We take B as

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{28}$$

In the chaotic case, the parameter values are

$$a = 14,\ b = 43,\ c = -1,\ d = 16 \ \text{ and } \ \varepsilon = 4.$$

The sliding mode variable is selected as

$$s = Ce = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} e = e_1 + e_2 + e_3 \tag{29}$$

which makes the sliding mode state equation asymptotically stable.

We choose the sliding mode gains as $k = 5$ and $q = 0.1$.

We note that a large value of k can cause chattering and an appropriate value of q is chosen to speed up the time taken to reach the sliding manifold as well as to reduce the system chattering.

From Eq. (15), we can obtain $v(t)$ as

$$v(t) = 3.3333\ e_1 - 11.6667 e_2 + 12.6667 e_3 - 0.0333\ \text{sgn}(s) \tag{30}$$

Thus, the required sliding mode controller is obtained as

$$u = -\eta(x, y) + Bv \tag{31}$$

where $\eta(x, y), B$ and $v(t)$ are defined as in the equations (26), (28) and (30).

By Theorem 1, we obtain the following result.

Theorem 2. *The identical Qi four-wing chaotic systems (21) and (22) are globally and asymptotically synchronized for all initial conditions with the sliding mode controller* u *defined by (31).* ■

3.2 Numerical Results

In this section For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the Qi four-wing chaotic systems (21) and (22) with the sliding mode controller u given by (31) using MATLAB.

In the chaotic case, the parameter values are

$$a = 14,\ b = 43,\ c = -1,\ d = 16 \ \text{ and } \ \varepsilon = 4.$$

The sliding mode gains are chosen as

$$k = 5 \ \text{ and } \ q = 0.1.$$

The initial values of the master system (21) are taken as

$$x_1(0) = 20,\ x_2(0) = 15,\ x_3(0) = 12.$$

The initial values of the slave system (22) are taken as

$$y_1(0) = 4,\ y_2(0) = 10,\ y_3(0) = 15.$$

Figure 2 illustrates the complete synchronization of the identical Qi four-wing chaotic systems (21) and (22).

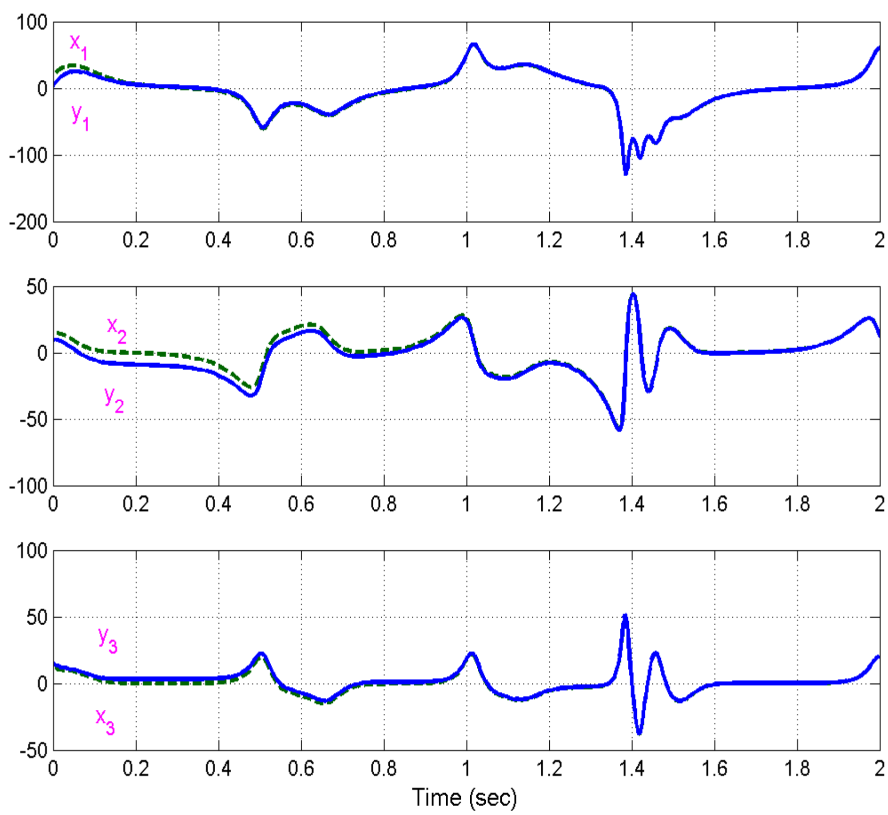

Figure 2. Complete Synchronization of Identical Qi Four-Wing Chaotic Systems

4. GLOBAL CHAOS SYNCHRONIZATION OF IDENTICAL LIU FOUR-WING SYSTEMS USING SLIDING MODE CONTROL

4.1 Theoretical Results

In this section, we apply the sliding mode control results derived in Section 2 for the global chaos synchronization of identical Liu four-wing chaotic systems ([28], Liu, 2009).

Thus, the master system is described by the Liu dynamics

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) + x_2 x_3^2 \\ \dot{x}_2 &= \beta(x_1 + x_2) - x_1 x_3^2 \\ \dot{x}_3 &= -\gamma x_3 + \delta x_2 + x_1 x_2 x_3 \end{aligned} \tag{32}$$

where x_1, x_2, x_3 are state variables and $\alpha, \beta, \gamma, \delta$ are real, positive, constant parameters of the system.

The Liu system (32) is chaotic when

$$\alpha = 50, \ \beta = 13, \ \gamma = 13 \ \text{and} \ \delta = 6.$$

Figure 3 illustrates the four-wing chaotic attractor of the Liu system (32).

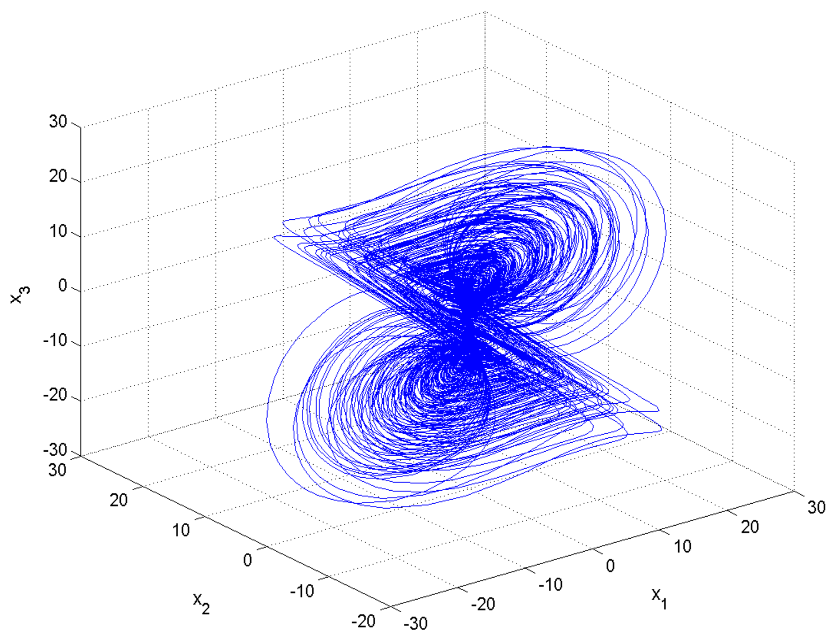

Figure 3. State Orbits of the Liu Four-Wing Chaotic System

The slave system is described by the controlled Liu dynamics

$$\begin{aligned}\dot{y}_1 &= \alpha(y_2 - y_1) + y_2 y_3^2 + u_1 \\ \dot{y}_2 &= \beta(y_1 + y_2) - y_1 y_3^2 + u_2 \\ \dot{y}_3 &= -\gamma y_3 + \delta y_2 + y_1 y_2 y_3 + u_3\end{aligned} \tag{33}$$

where y_1, y_2, y_3 are state variables and u_1, u_2, u_3 are the controllers to be designed.

The chaos synchronization error is defined by

$$e_i = y_i - x_i, \ (i = 1, 2, 3) \tag{34}$$

The error dynamics is easily obtained as

$$\begin{aligned}\dot{e}_1 &= \alpha(e_2 - e_1) + y_2 y_3^2 - x_2 x_3^2 + u_1 \\ \dot{e}_2 &= \beta(e_1 + e_2) - y_1 y_3^2 + x_1 x_3^2 + u_2 \\ \dot{e}_3 &= -\gamma e_3 + \delta e_2 + y_1 y_2 y_3 - x_1 x_2 x_3 + u_3\end{aligned} \tag{35}$$

We write the error dynamics (35) in the matrix notation as

$$\dot{e} = Ae + \eta(x, y) + u \tag{36}$$

where

$$A = \begin{bmatrix} -\alpha & \alpha & 0 \\ \beta & \beta & 0 \\ 0 & \delta & -\gamma \end{bmatrix}, \ \eta(x, y) = \begin{bmatrix} y_2 y_3^2 - x_2 x_3^2 \\ -y_1 y_3^2 + x_1 x_3^2 \\ y_1 y_2 y_3 - x_1 x_2 x_3 \end{bmatrix} \text{ and } u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}. \tag{37}$$

The sliding mode controller design is carried out as detailed in Section 2.

First, we set u as

$$u = -\eta(x, y) + Bv \tag{38}$$

where B is chosen such that (A, B) is controllable.

We take B as

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{39}$$

In the chaotic case, the parameter values are

$\alpha = 50, \ \beta = 13, \ \gamma = 13$ and $\delta = 6$.

The sliding mode variable is selected as

$$s = Ce = \begin{bmatrix} 1 & 3 & 1 \end{bmatrix} e = e_1 + 3e_2 + e_3 \tag{40}$$

which makes the sliding mode state equation asymptotically stable.

We choose the sliding mode gains as $k = 5$ and $q = 0.1$.

We note that a large value of k can cause chattering and an appropriate value of q is chosen to speed up the time taken to reach the sliding manifold as well as to reduce the system chattering.

From Eq. (15), we can obtain $v(t)$ as

$$v(t) = 1.2\, e_1 - 22e_2 + 1.6e_3 - 0.02\ \mathrm{sgn}(s) \tag{41}$$

Thus, the required sliding mode controller is obtained as

$$u = -\eta(x, y) + Bv \tag{42}$$

where $\eta(x, y), B$ and $v(t)$ are defined as in the equations (37), (39) and (41).

By Theorem 1, we obtain the following result.

Theorem 3. *The identical Liu four-wing chaotic systems (32) and (33) are globally and asymptotically synchronized for all initial conditions with the sliding mode controller* u *defined by (42).* ■

4.2 Numerical Results

In this section For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the Liu four-wing chaotic systems (32) and (33) with the sliding mode controller u given by (42) using MATLAB.

In the chaotic case, the parameter values are

$$\alpha = 50,\ \beta = 13,\ \gamma = 13 \text{ and } \delta = 6.$$

The sliding mode gains are chosen as $k = 5$ and $q = 0.1$.

The initial values of the master system (32) are taken as

$$x_1(0) = 12,\ x_2(0) = 18,\ x_3(0) = 25.$$

The initial values of the slave system (32) are taken as

$$y_1(0) = 24,\ y_2(0) = 20,\ y_3(0) = 6.$$

Figure 4 illustrates the complete synchronization of the identical Liu four-wing chaotic systems (32) and (33).

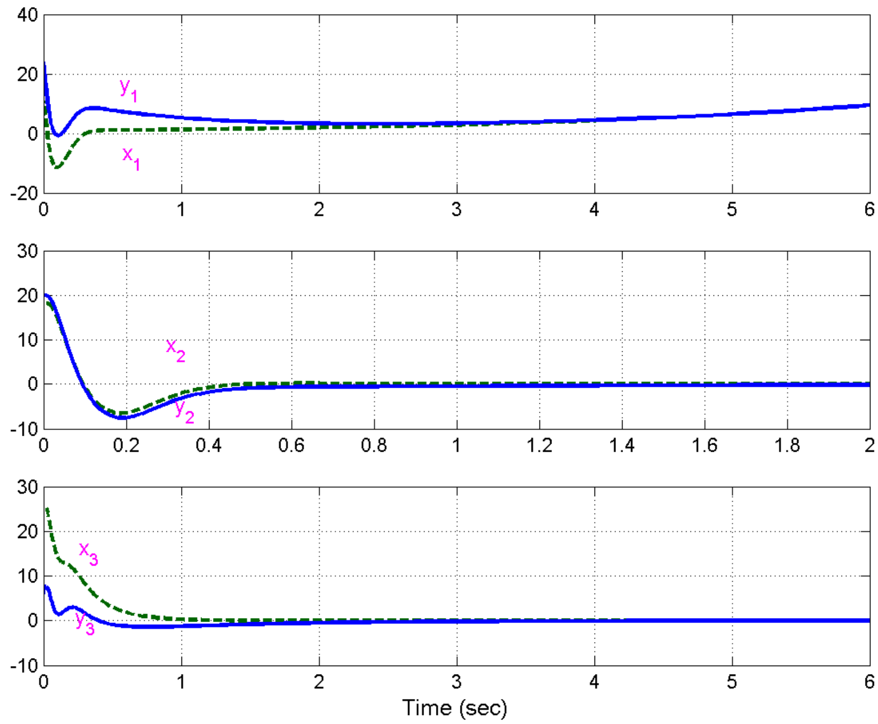

Figure 4. Complete Synchronization of Identical Liu Four-Wing Chaotic Systems

5. Global chaos Synchronization of Identical Wang Four-Wing Systems using Sliding Mode Control

5.1 Theoretical Results

In this section, we apply the sliding mode control results derived in Section 2 for the global chaos synchronization of identical Wang four-wing chaotic systems ([29], Wang *et al*., 2009).

Thus, the master system is described by the Wang dynamics

$$\begin{aligned}\dot{x}_1 &= ax_1 + cx_2x_3 \\ \dot{x}_2 &= bx_1 + dx_2 - x_1x_3 \\ \dot{x}_3 &= \varepsilon x_3 + fx_1x_2\end{aligned} \tag{43}$$

where x_1, x_2, x_3 are state variables and a, b, d, ε, f are real, positive, constant parameters of the system with $c > 0, f < 0$ and $a + d + \varepsilon < 0.$

The Wang system (43) is chaotic when

$$a = 0.2,\ b = -0.01,\ c = 1,\ d = -0.4,\ \varepsilon = -1 \text{ and } f = -1.$$

Figure 5 illustrates the four-wing chaotic attractor of the Wang system (43).

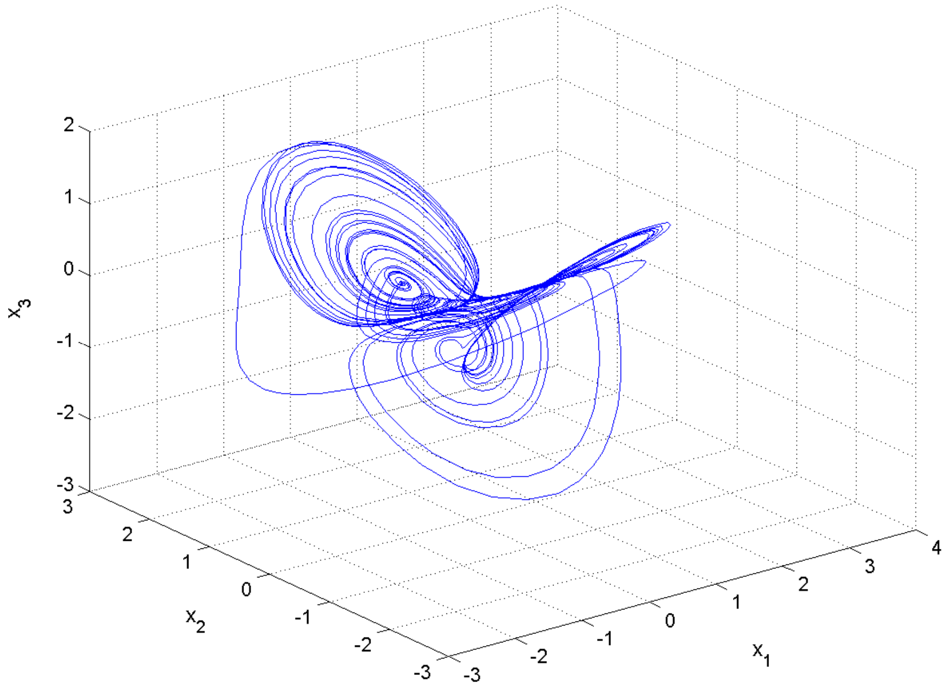

Figure 5. State Orbits of the Wang Four-Wing Chaotic System

The slave system is described by the controlled Wang dynamics

$$\begin{aligned}\dot{y}_1 &= ay_1 + cy_2 y_3 + u_1 \\ \dot{y}_2 &= by_1 + dy_2 - y_1 y_3 + u_2 \\ \dot{y}_3 &= \varepsilon y_3 + f y_1 y_2 + u_3\end{aligned} \tag{44}$$

where y_1, y_2, y_3 are state variables and u_1, u_2, u_3 are the controllers to be designed.

The chaos synchronization error is defined by

$$e_i = y_i - x_i, \ (i = 1, 2, 3) \tag{45}$$

The error dynamics is easily obtained as

$$\begin{aligned}\dot{e}_1 &= ae_1 + \varepsilon(y_2 y_3 - x_2 x_3) + u_1 \\ \dot{e}_2 &= be_1 + de_2 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= \varepsilon e_3 + f(y_1 y_2 - x_1 x_2) + u_3\end{aligned} \tag{46}$$

We write the error dynamics (46) in the matrix notation as

$$\dot{e} = Ae + \eta(x, y) + u \tag{47}$$

where

$$A = \begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ 0 & 0 & \varepsilon \end{bmatrix}, \quad \eta(x, y) = \begin{bmatrix} c(y_2 y_3 - x_2 x_3) \\ -y_1 y_3 + x_1 x_3 \\ f(y_1 y_2 - x_1 x_2) \end{bmatrix} \text{ and } \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}. \tag{48}$$

The sliding mode controller design is carried out as detailed in Section 2.

First, we set u as

$$u = -\eta(x, y) + Bv \tag{49}$$

where B is chosen such that (A, B) is controllable.

We take B as

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{50}$$

In the chaotic case, the parameter values are

$$a = 0.2, \; b = -0.01, \; c = 1, \; d = -0.4, \; \varepsilon = -1 \text{ and } f = -1.$$

The sliding mode variable is selected as

$$s = Ce = \begin{bmatrix} 8 & -4 & -3 \end{bmatrix} e = 8e_1 - 4e_2 - 3e_3 \tag{51}$$

which makes the sliding mode state equation asymptotically stable.

We choose the sliding mode gains as $k = 5$ and $q = 0.1$.

We note that a large value of k can cause chattering and an appropriate value of q is chosen to speed up the time taken to reach the sliding manifold as well as to reduce the system chattering.

From Eq. (15), we can obtain $v(t)$ as

$$v(t) = -41.64\, e_1 + 18.4\, e_2 + 12\, e_3 - 0.1 \operatorname{sgn}(s) \tag{52}$$

Thus, the required sliding mode controller is obtained as

$$u = -\eta(x, y) + Bv \tag{53}$$

where $\eta(x, y), B$ and $v(t)$ are defined as in the equations (48), (50) and (52).

By Theorem 1, we obtain the following result.

Theorem 4. *The identical Wang four-wing chaotic systems (43) and (44) are globally and asymptotically synchronized for all initial conditions with the sliding mode controller* u *defined by (53).* ■

4.2 Numerical Results

In this section For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the Wang four-wing chaotic systems (43) and (44) with the sliding mode controller u given by (53) using MATLAB.

In the chaotic case, the parameter values are $a = 0.2,\ b = -0.01,\ c = 1,\ d = -0.4,\ \varepsilon = -1$ and $f = -1$. The sliding mode gains are chosen as $k = 5$ and $q = 0.1$.

The initial values of the master system (43) are taken as

$$x_1(0) = 21,\ x_2(0) = 4,\ x_3(0) = 15.$$

The initial values of the slave system (44) are taken as

$$y_1(0) = 11,\ y_2(0) = 10,\ y_3(0) = 26.$$

Figure 6 illustrates the complete synchronization of the identical Liu four-wing chaotic systems (43) and (44).

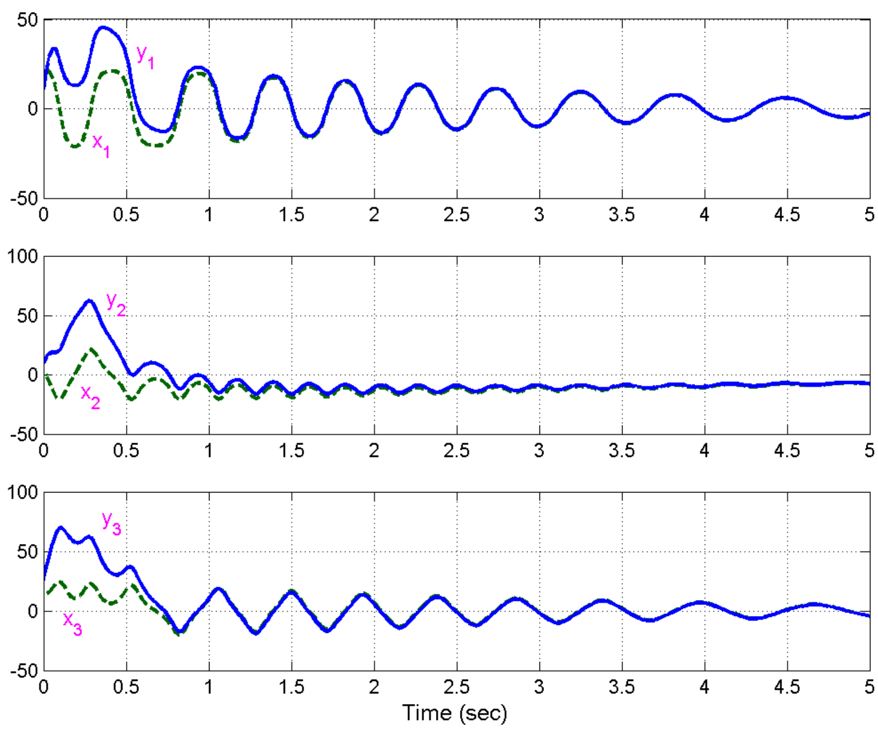

Figure 6. Complete Synchronization of Identical Wang Four-Wing Chaotic Systems

5. CONCLUSIONS

In this paper, we have deployed sliding mode control (SMC) to achieve global chaos synchronization for the identical Qi four-wing chaotic systems (2008), identical Liu four-wing chaotic systems (2009) and identical Wang four-wing chaotic systems (2009). Since the Lyapunov exponents are not required for these calculations, the sliding mode control method is very effective and convenient to achieve global chaos synchronization for the identical four-wing chaotic systems discussed in this paper. Numerical simulations are also shown to illustrate the effectiveness of the synchronization results using the sliding mode control for the identical four-wing chaotic systems discussed in this paper.

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