HYBRID CHAOS SYNCHRONIZATION OF HYPERCHAOTIC NEWTON-LEIPNIK SYSTEMS BY SLIDING MODE CONTROL

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ABSTRACT

This paper investigates the hybrid chaos synchronization of identical hyperchaotic Newton-Leipnik systems (Ghosh and Bhattacharya, 2010) by sliding mode control. The stability results derived in this paper for the hybrid chaos synchronization of identical hyperchaotic Newton-Leipnik systems are established using Lyapunov stability theory. Hybrid synchronization of hyperchaotic Newton-Leipnik systems is achieved through the complete synchronization of first and third states of the systems and the anti-synchronization of second and fourth states of the master and slave systems. Since the Lyapunov exponents are not required for these calculations, the sliding mode control is very effective and convenient to achieve hybrid chaos synchronization of the identical hyperchaotic Newton-Leipnik systems. Numerical simulations are shown to validate and demonstrate the effectiveness of the synchronization schemes derived in this paper.

KEYWORDS

Hybrid Synchronization, Hyperchaos, Sliding Mode Control, Hyperchaotic Newton-Leipnik System.

1. INTRODUCTION

Chaotic systems are dynamical systems that are highly sensitive to initial conditions. The sensitive nature of chaotic systems is commonly called as the *butterfly effect* [1].

Synchronization of chaotic systems is a phenomenon which may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem.

A hyperchaotic system is usually characterized as a chaotic system with more than one positive Lyapunov exponent implying that the dynamics expand in more than one direction giving rise to “thicker” and “more complex” chaotic dynamics. There has been significant interest in the research on hyperchaotic systems in the literature. Hyperchaotic systems have important applications in engineering in areas like secure communication, data encryption, etc.

In most of the chaos synchronization approaches, the *master-slave* or *drive-response* formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

Since the pioneering work by Pecora and Carroll ([2], 1990), chaos synchronization problem has been studied extensively and intensively in the literature [2-17]. Chaos theory has been applied to a variety of fields such as physical systems [3], chemical systems [4], ecological systems [5], secure communications [6-8], etc.

In the last two decades, various schemes have been successfully applied for chaos synchronization such as PC method [2], OGY method [9], active control method [10-12], adaptive control method [13-14], time-delay feedback method [15], backstepping design method [16], sampled-data feedback method [17], etc.

So far, many types of chaos synchronization phenomenon have been studied in the literature such as complete synchronization [2], phase synchronization [18], generalized synchronization [19], anti-synchronization [20-21], projective synchronization [22], generalized projective synchronization [23], etc. Complete synchronization (CS) is characterized by the equality of state variables evolving in time, while anti-synchronization (AS) is characterized by the disappearance of the sum of relevant state variables evolving in time. Projective synchronization (PS) is characterized by the fact that the master and slave systems could be synchronized up to a scaling factor, whereas in generalized projective synchronization (GPS), the responses of the synchronized dynamical systems synchronize up to a constant scaling matrix α. It is easy to see that complete synchronization and anti-synchronization are special cases of the generalized projective synchronization where the scaling matrix $\alpha = I$ and $\alpha = -I$, respectively.

In hybrid synchronization of chaotic systems [24-25], one part of the system is completely synchronized and the other part is anti-synchronized so that complete synchronization (CS) and anti-synchronization (AS) co-exist in the systems. The coexistence of CS and AS is very useful in applications such as secure communication, chaotic encryption schemes, etc.

In this paper, we derive new results based on the sliding mode control [26-28] for the hybrid chaos synchronization of identical hyperchaotic Newton-Leipnik systems (Ghosh and Bhattacharya, [29], 2010). In robust control systems, the sliding mode control method is often adopted due to its inherent advantages of easy realization, fast response and good transient performance as well as its insensitivity to parameter uncertainties and external disturbances.

This paper has been organized as follows. In Section 2, we describe the problem statement and our methodology using sliding mode control (SMC). In Section 3, we discuss the global chaos synchronization of identical hyperchaotic Newton-Leipnik systems. In Section 4, we summarize the main results obtained in this paper.

2. Problem Statement and Our Methodology using SMC

In this section, we describe the problem statement for the global chaos synchronization for identical chaotic systems and our methodology using sliding mode control (SMC).

Consider the chaotic system described by

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in R^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : R^n \to R^n$ is the nonlinear part of the system.

We consider the system (1) as the *master* or *drive* system.

As the *slave* or *response* system, we consider the following chaotic system described by the dynamics

$$\dot{y} = Ay + f(y) + u \tag{2}$$

where $y \in R^n$ is the state of the system and $u \in R^m$ is the controller to be designed.

We define the *hybrid synchronization error* as

$$e_i = \begin{cases} y_i - x_i, & \text{if } i \text{ is odd} \\ y_i + x_i, & \text{if } i \text{ is even} \end{cases} \tag{3}$$

Then the error dynamics can be obtained in the form

$$\dot{e} = Ae + \eta(x, y) + u, \tag{4}$$

The objective of the global chaos synchronization problem is to find a controller u such that

$$\lim_{t \to \infty} \left\| e(t) \right\| = 0 \quad \text{for all } e(0) \in R^n. \tag{5}$$

To solve this problem, we first define the control u as

$$u = -\eta(x, y) + Bv \tag{6}$$

where B is a constant gain vector selected such that (A, B) is controllable.

Substituting (6) into (4), the error dynamics simplifies to

$$\dot{e} = Ae + Bv \tag{7}$$

which is a linear time-invariant control system with single input v.

Thus, the original global chaos synchronization problem can be replaced by an equivalent problem of stabilizing the zero solution $e = 0$ of the system (7) by a suitable choice of the sliding mode control. In the sliding mode control, we define the variable

$$s(e) = Ce = c_1 e_1 + c_2 e_2 + \cdots + c_n e_n \tag{8}$$

where $C = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}$ is a constant vector to be determined.

In the sliding mode control, we constrain the motion of the system (7) to the sliding manifold defined by

$$S = \left\{ x \in R^n \mid s(e) = 0 \right\}$$

which is required to be invariant under the flow of the error dynamics (7).

When in sliding manifold S, the system (7) satisfies the following conditions:

$$s(e) = 0 \tag{9}$$

which is the defining equation for the manifold S and

$$\dot{s}(e) = 0 \tag{10}$$

which is the necessary condition for the state trajectory $e(t)$ of (7) to stay on the sliding manifold S.

Using (7) and (8), the equation (10) can be rewritten as

$$\dot{s}(e) = C\left[Ae + Bv \right] = 0 \tag{11}$$

Solving (11) for v, we obtain the equivalent control law

$$v_{\rm eq}(t) = -(CB)^{-1} CA\ e(t) \tag{12}$$

where C is chosen such that $CB \neq 0$.

Substituting (12) into the error dynamics (7), we obtain the closed-loop dynamics as

$$\dot{e} = \left[I - B(CB)^{-1} C \right] Ae \tag{13}$$

The row vector C is selected such that the system matrix of the controlled dynamics $\left[I - B(CB)^{-1} C \right] A$ is Hurwitz, *i.e.* it has all eigenvalues with negative real parts. Then the controlled system (13) is globally asymptotically stable.

To design the sliding mode controller for (7), we apply the constant plus proportional rate reaching law

$$\dot{s} = -q\, \text{sgn}(s) - k\ s \tag{14}$$

where $\text{sgn}(\cdot)$ denotes the sign function and the gains $q > 0,\ \ k > 0$ are determined such that the sliding condition is satisfied and sliding motion will occur.

From equations (11) and (14), we can obtain the control $v(t)$ as

$$v(t) = -(CB)^{-1} \left[C(kI + A)e + q\, \text{sgn}(s) \right] \tag{15}$$

which yields

$$v(t) = \begin{cases} -(CB)^{-1} \left[C(kI + A)e + q \right], & \text{if } s(e) > 0 \\ -(CB)^{-1} \left[C(kI + A)e - q \right], & \text{if } s(e) < 0 \end{cases} \tag{16}$$

Theorem 1. *The master system (1) and the slave system (2) are globally and asymptotically hybrid synchronized for all initial conditions* $x(0), y(0) \in R^n$ *by the feedback control law*

$$u(t) = -\eta(x, y) + Bv(t) \tag{17}$$

where $v(t)$ *is defined by (15) and* B *is a column vector such that* (A, B) *is controllable. Also, the sliding mode gains* k, q *are positive.*

Proof. First, we note that substituting (17) and (15) into the error dynamics (4), we obtain the closed-loop error dynamics as

$$\dot{e} = Ae - B(CB)^{-1} \left[C(kI + A)e + q\, \text{sgn}(s) \right] \tag{18}$$

To prove that the error dynamics (18) is globally asymptotically stable, we consider the candidate Lyapunov function defined by the equation

$$V(e) = \frac{1}{2} s^2(e) \tag{19}$$

which is a positive definite function on R^n.

Differentiating V along the trajectories of (18) or the equivalent dynamics (14), we get

$$\dot{V}(e) = s(e)\dot{s}(e) = -ks^2 - q\, \text{sgn}(s) s \tag{20}$$

which is a negative definite function on R^n.

This calculation shows that V is a globally defined, positive definite, Lyapunov function for the error dynamics (18), which has a globally defined, negative definite time derivative $\dot{V}$.

Thus, by Lyapunov stability theory [22], it is immediate that the error dynamics (18) is globally asymptotically stable for all initial conditions $e(0) \in R^n$.

This means that for all initial conditions $e(0) \in R^n$, we have

$$\lim_{t \to \infty} \|e(t)\| = 0$$

This shows that the hybrid synchronization error vector $e(t)$ decays to zero asymptotically with time.Hence, it follows that the master system (1) and the slave system (2) are globally and asymptotically hybrid synchronized for all initial conditions $x(0), y(0) \in R^n$.

This completes the proof. ■

3. Hybrid Chaos Synchronization of Identical Hyperchaotic Newton-Leipnik Systems using Sliding Mode Control

3.1 Theoretical Results

In this section, we apply the sliding mode control results derived in Section 2 for the hybrid chaos synchronization of identical hyperchaotic Newton-Leipnik systems (Ghosh and Bhattacharya, [29], 2010). In hybrid synchronization of master and slave systems, the first and third states of the two systems are completely synchronized, while the second and fourth states of the two systems are anti-synchronized.

Thus, the master system is described by the hyperchaotic Newton-Leipnik dynamics

$$\begin{aligned} \dot{x}_1 &= -ax_1 + x_2 + 10x_2x_3 + x_4 \\ \dot{x}_2 &= -x_1 - 0.4x_2 + 5x_1x_3 \\ \dot{x}_3 &= bx_3 - 5x_1x_2 \\ \dot{x}_4 &= -cx_1x_3 + dx_4 \end{aligned} \tag{21}$$

where x_1, x_2, x_3, x_4 are state variables and a, b, c, d are positive, constant parameters of the system.The slave system is described by the controlled hyperchaotic Newton-Leipnik dynamics

$$\begin{aligned} \dot{y}_1 &= -ay_1 + y_2 + 10y_2y_3 + y_4 + u_1 \\ \dot{y}_2 &= -y_1 - 0.4y_2 + 5y_1y_3 + u_2 \\ \dot{y}_3 &= by_3 - 5y_1y_2 + u_3 \\ \dot{y}_4 &= -cy_1y_3 + dy_4 + u_4 \end{aligned} \tag{22}$$

where y_1, y_2, y_3, y_4 are state variables and u_1, u_2, u_3, u_4 are the controllers to be designed.

The systems (21) and (22) are hyperchaotic when

$$a = 0.4,\ b = 0.175,\ c = 0.8 \ \text{ and } \ d = 0.01.$$

Figure 1 illustrates the state portrait of the hyperchaotic Newton-Leipnik system (21).

The chaos synchronization error is defined by

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 + x_2 \\ e_3 &= y_3 - x_3 \\ e_4 &= y_4 + x_4 \end{aligned} \tag{23}$$

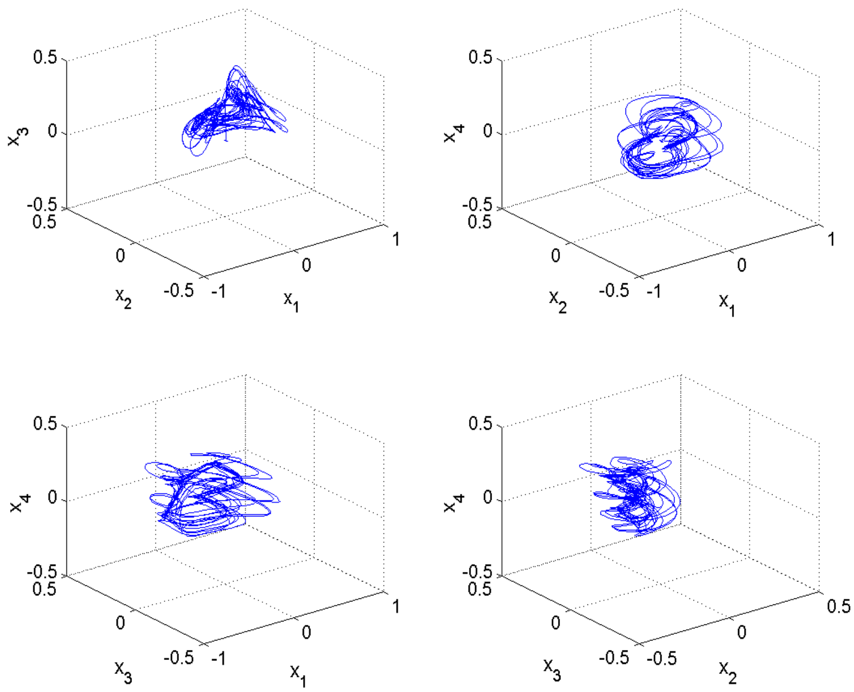

Figure 1. State Orbits of the Hyperchaotic Newton-Leipnik System

The error dynamics is easily obtained as

$$\begin{aligned} \dot{e}_1 &= -ae_1 + e_2 + e_4 - 2x_2 - 2x_4 + 10(y_2 y_3 - x_2 x_3) + u_1 \\ \dot{e}_2 &= -e_1 - 0.4e_2 - 2x_1 + 5(y_1 y_3 - x_1 x_3) + u_2 \\ \dot{e}_3 &= be_3 - 5(y_1 y_2 - x_1 x_2) + u_3 \\ \dot{e}_4 &= de_4 - c(y_1 y_3 - x_1 x_3) + u_4 \end{aligned} \tag{24}$$

We write the error dynamics (24) in the matrix notation as

$$\dot{e} = Ae + \eta(x, y) + u \tag{25}$$

where

$$A = \begin{bmatrix} -a & 1 & 0 & 1 \\ -1 & -0.4 & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & d \end{bmatrix}, \quad \eta(x, y) = \begin{bmatrix} -2x_2 - 2x_4 + 10(y_2 y_3 - x_2 x_3) \\ -2x_1 + 5(y_1 y_3 + x_1 x_3) \\ -5(y_1 y_2 - x_1 x_2) \\ -c(y_1 y_3 + x_1 x_3) \end{bmatrix} \text{ and } u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}. \tag{26}$$

The sliding mode controller design is carried out as detailed in Section 2.

First, we set u as

$$u = -\eta(x, y) + Bv \tag{27}$$

where B is chosen such that (A, B) is controllable.

We take B as

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}. \tag{28}$$

In the hyperchaotic case, the parameter values are

$$a = 0.4,\ b = 0.175,\ c = 0.8 \ \text{ and } \ d = 0.01.$$

The sliding mode variable is selected as

$$s = Ce = \begin{bmatrix} 0.4 & 4 & 8 & 0.1 \end{bmatrix} e = 0.4e_1 + 4e_2 + 8e_3 + 0.1e_4 \tag{29}$$

which makes the sliding mode state equation asymptotically stable.

We choose the sliding mode gains as $k = 5$ and $q = 0.1$.

We note that a large value of k can cause chattering and an appropriate value of q is chosen to speed up the time taken to reach the sliding manifold as well as to reduce the system chattering.

From Eq. (15), we can obtain $v(t)$ as

$$v(t) = 0.1728\ e_1 - 1.5040e_2 - 3.3120e_3 - 0.0721e_4 - 0.0080\ \text{sgn}(s) \tag{30}$$

Thus, the required sliding mode controller is obtained as

$$u = -\eta(x, y) + Bv \tag{31}$$

where $\eta(x, y), B$ and $v(t)$ are defined as in the equations (26), (28) and (30).

By Theorem 1, we obtain the following result.

Theorem 2. *The identical hyperchaotic Newton-Leipnik systems (21) and (22) are globally and asymptotically hybrid synchronized for all initial conditions with the sliding mode controller* u *defined by (31).* ■

3.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the hyperchaotic Newton-Leipnik systems (21) and (22) with the sliding mode controller u given by (31) using MATLAB.

The sliding mode gains are chosen as $k = 5$ and $q = 0.1$.

The initial values of the master system (21) are taken as $x(0) = (6, 5, 14, 7)$ and the initial values of the slave system (22) are taken as $y(0) = (3, 12, 18, 16)$.

Figure 2 illustrates the hybrid synchronization of the identical hyperchaotic Newton-Leipnik systems (21) and (22).

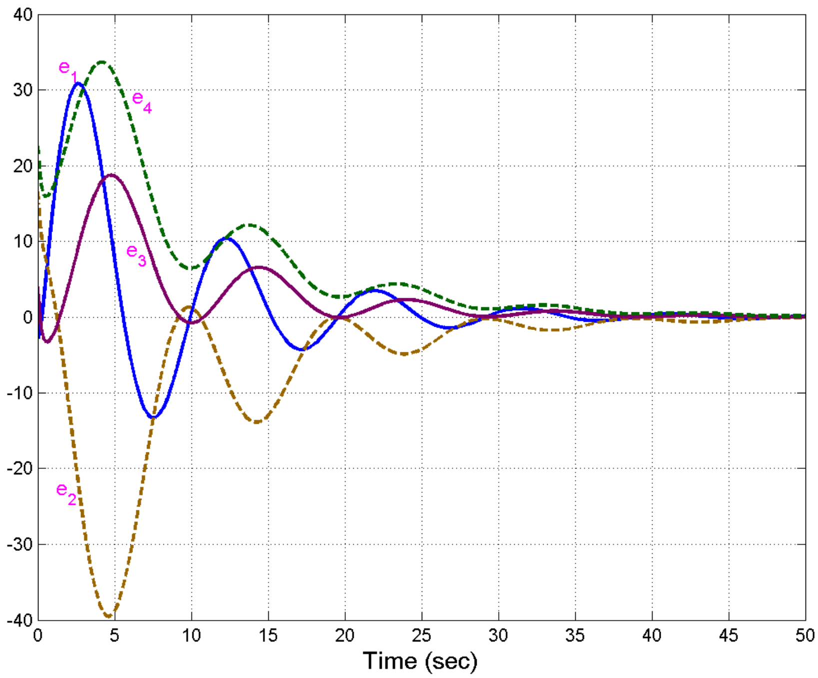

Figure 2. Hybrid Synchronization of Identical Hyperchaotic Newton-Leipnik Systems

4. CONCLUSIONS

In this paper, we have deployed sliding mode control (SMC) to achieve hybrid chaos synchronization for the identical hyperchaotic Newton-Leipnik systems (2010). Our hybrid synchronization results for the identical hyperchaotic Newton-Leipnik systems have been proved using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the sliding mode control method is very effective and convenient to achieve hybrid chaos synchronization for the identical hyperchaotic Newton-Leipnik systems. Numerical simulations are also shown to illustrate the effectiveness of the hybrid synchronization results derived in this paper using the sliding mode control.

REFERENCES

[1] Alligood, K.T., Sauer, T. & Yorke, J.A. (1997) *Chaos: An Introduction to Dynamical Systems*, Springer, New York.

[2] Pecora, L.M. & Carroll, T.L. (1990) "Synchronization in chaotic systems", *Phys. Rev. Lett.*, Vol. 64, pp 821-824.

[3] Lakshmanan, M. & Murali, K. (1996) *Nonlinear Oscillators: Controlling and Synchronization*, World Scientific, Singapore.

[4] Han, S.K., Kerrer, C. & Kuramoto, Y. (1995) "Dephasing and burstling in coupled neural oscillators", *Phys. Rev. Lett.,* Vol. 75, pp 3190-3193.

[5] Blasius, B., Huppert, A. & Stone, L. (1999) "Complex dynamics and phase synchronization in spatially extended ecological system", *Nature*, Vol. 399, pp 354-359.

[6] Cuomo, K.M. & Oppenheim, A.V. (1993) "Circuit implementation of synchronized chaos with applications to communications," *Physical Review Letters*, Vol. 71, pp 65-68.

[7] Kocarev, L. & Parlitz, U. (1995) "General approach for chaotic synchronization with applications to communication," *Physical Review Letters*, Vol. 74, pp 5028-5030.

[8] Tao, Y. (1999) "Chaotic secure communication systems – history and new results," *Telecommun. Review*, Vol. 9, pp 597-634.

[9] Ott, E., Grebogi, C. & Yorke, J.A. (1990) "Controlling chaos", *Phys. Rev. Lett.*, Vol. 64, pp 1196-1199.

[10] Ho, M.C. & Hung, Y.C. (2002) "Synchronization of two different chaotic systems using generalized active network," Physics Letters A, Vol. 301, pp 424-428.

[11] Huang, L., Feng, R. & Wang, M. (2005) "Synchronization of chaotic systems via nonlinear control," *Physical Letters A*, Vol. 320, pp 271-275.

[12] Chen, H.K. (2005) "Global chaos synchronization of new chaotic systems via nonlinear control," *Chaos, Solitons & Fractals*, Vol. 23, pp 1245-1251.

[13] Lu, J., Wu, X., Han, X. & Lü, J. (2004) "Adaptive feedback synchronization of a unified chaotic system," *Physics Letters A*, Vol. 329, pp 327-333.

[14] Chen, S.H. & Lü, J. (2002) "Synchronization of an uncertain unified system via adaptive control," *Chaos, Solitons & Fractals*, Vol. 14, pp 643-647.

[15] Park, J.H. & Kwon, O.M. (2003) "A novel criterion for delayed feedback control of time-delay chaotic systems," *Chaos, Solitons & Fractals*, Vol. 17, pp 709-716.

[16] Wu, X. & Lü, J. (2003) "Parameter identification and backstepping control of uncertain Lü system," *Chaos, Solitons & Fractals*, Vol. 18, pp 721-729.

[17] Zhao, J. & J. Lu (2006) "Using sampled-data feedback control and linear feedback synchronization in a new hyperchaotic system," *Chaos, Solitons & Fractals*, Vol. 35, pp 376-382.

[18] Ge, Z.M. & Chen, C.C. (2004) "Phase synchronization of coupled chaotic multiple time scales systems," *Chaos, Solitons & Fractals*, Vol. 20, pp 639-647.

[19] Wang, Y.W. & Guan, Z.H. (2006) "Generalized synchronization of continuous chaotic systems," *Chaos, Solitons & Fractals*, Vol. 27, pp 97-101.

[20] Zhang, X. & Zhu, H. (2008) "Anti-synchronization of two different hyperchaotic systems via active and adaptive control," *International J. Nonlinear Science*, Vol. 6, pp 216-223.

[21] Chiang, T., Lin, J., Liao, T. & Yan, J. (2008) "Anti-synchronization of uncertain unified chaotic systems with dead-zone nonlinearity," *Nonlinear Analysis*, Vol. 68, pp 2629-2637.

[22] Qiang, J. (2007) "Projective synchronization of a new hyperchaotic Lorenz system," *Physics Letters A*, Vol. 370, pp 40-45.

[23] Jian-Ping, Y. & Chang-Pin, L. (2006) "Generalized projective synchronization for the chaotic Lorenz system and the chaotic Chen system," *J. Shanghai University*, Vol. 10, pp 299-304.

[24] Li, R.H. (2008) "A special full-state hybrid projective synchronization in symmetrical chaotic systems," *Applied Math. Computation*, Vol. 200, pp 321-329.

[25] Sundarapandian, V. (2011) "Hybrid synchronization of Lorenz and Pehlivan systems by active nonlinear control," *International J. Advances in Science and Technology*, Vol. 2, No. 6, pp 10-20.

[26] Utkin, V.I. (1993) "Sliding mode control design principles and applications to electric drives," *IEEE Trans. Industrial Electronics*, Vol. 40, pp 23-36, 1993.

[27] Sundarapandian, V. (2011) "Sliding mode controller design for synchronization of Shimizu-Morioka chaotic systems," *International Journal of Information Sciences and Techniques*, Vol. 1, No. 1, pp 20-29.

[28] Sundarapandian, V. (2011) "Global chaos synchronization of Pehlivan systems by sliding mode control," *International Journal on Control Science and Engineering*, Vol. 3, No. 5, pp 2163-2169.

[29] Ghosh, D. & Bhattacharya, S. (2010) "Projective synchronization of new hyperchaotic system with fully unknown parameters," Nonlinear Dynamics, Vol. 61, pp 11-21.

[30] Hahn, W. (1967) *The Stability of Motion*, Springer, New York.

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