TAKAGI-SUGENO MODEL FOR QUADROTOR MODELLING AND CONTROL USING NONLINEAR STATE FEEDBACK CONTROLLER

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ABSTRACT

In this paper we present a Takagi-Sugeno (T-S) model for Quadrotor modelling. This model is developed using multiple model approach, composed of three locally accurate models valid in different region of the operating space. It enables us to model the global nonlinear system with some degree of accuracy. Once the T-S model has been defined it is claimed to be relatively straightforward to design a controller with the same strategy of T-S model. A nonlinear state feedback controller based on Linear Matrix Inequality (LMI), and PDC technique with pole placement constraint is synthesized. The requirements of stability and pole-placement in LMI region are formulated based on the Lyapunov direct method. By recasting these constraints into LMIs, we formulate an LMI feasibility problem for the design of the nonlinear state feedback controller. This controller is applied to a nonlinear Quadrotor system, which is one of the most complex flying systems that exist. A comparative study between controller with stability constraints and controller with pole placement constrains is made. Simulation results show that the controller with pole placement constrains yields good tracking performance. The designed T-S model is validated using Matlab Simulink.

KEYWORDS

Linear Matrix Inequality (LMI), Multiple Model Approach (MMA), Parallel Disturbance Compensation (PDC), Pole Placement, Quadrotor, Takagi-Sugeno model.

1. INTRODUCTION

In everyday life, the strategy how to solve a complex problem is called divide & conquer. The problem is divided into simpler parts, which are solved independently and together yields the solution to the whole problem. The same strategy can be used for modelling and control of non-linear systems, where the non-linear plant is substituted by locally valid set of linear sub models [1]. The model should be simple enough so that it can be easy understood. The accurate model that characterizes important aspects of the system being controlled is a necessary prerequisite for design of a controller.

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The idea of approximation based on Multiple Model Approach (MMA) is not new. Since the publication of Johansen and Foss, the multiple-models approach knew an unquestionable interest. The Multiple Model approach appears in the literature under many different names, including Takagi-Sugeno (T-S) model [2], local model networks or operating regime decomposition.

During the last years, many works have been carried out to investigate the stability analysis and the design of state feedback controller of T-S systems. Using a quadratic Lyapunov function and PDC (Parallel Disturbance Compensation) technique, sufficient conditions for the stability and stabilisability have been established [3] [4]. The stability depends on the existence of a common positive definite matrix guarantying the stability of all local subsystems. The PDC control is a nonlinear state feedback controller. The gain of this controller can be expressed as the solution of a linear matrix inequality (LMIs) set [5].

Quadrotor Helicopter is considered as one of the most popular UAV platform. This kind of helicopters are dynamically unstable, and therefore suitable control methods was used to make them stable, as back-stepping and sliding-mode techniques [6] [7].

The main contribution of this paper is the proposition of a T-S model for modelling and control of Quadrotor, using multiple model approach with state feedback controller. This approach, in spite of its oldness it's never applied to Quadrotor, except two works, the first one by Bouabdallah [8], which used a single linear model and applied LQ controller, and the second work by Kostas [9], which used the multiple-model concept for only the control of Quadrotor attitude, moreover he was not validate the model.

2. QUADROTOR DYNAMICAL MODEL

The Quadrotor is a four rotor helicopter (Figure 1); each rotor consists of an electrical motor, a drive gear and propeller. The two pairs of propellers (1, 3) and (2, 4) turn in opposite directions. Forward motion is accomplished by increasing the speed of the rear rotor while simultaneously and right motion work in the same way. Yaw command is accomplished by accelerating the two clockwise turning rotors while decelerating the counter-clockwise turning rotors. This helicopter is one of the most complex flying systems that exist. This is due partly to the number of physical effects (Aerodynamic effects, gravity, gyroscopic, friction and inertial counter torques) acting on the system.

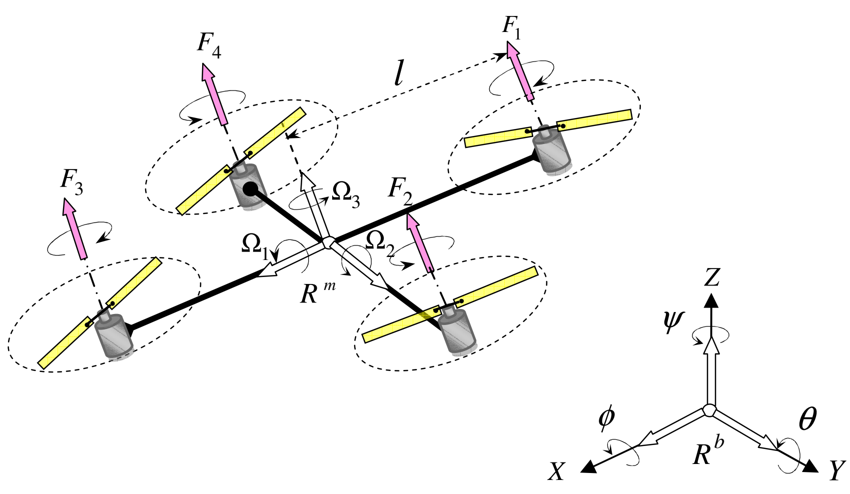


Figure 1. Quadrotor Architecture

The first step before the control development is an adequate dynamic system modelling, especially for lightweight flying systems. Let us consider earth fixed frame R^b and body fixed frame R^m, as seen in Figure 1. Using Euler angles parameterization, the airframe orientation in space is given by a rotation matrix R from R^b to R^m.

$$R = \begin{bmatrix} c\psi c\theta & s\phi s\theta c\psi - s\psi c\phi & c\phi s\theta c\psi + s\psi s\phi \\ s\psi c\theta & s\phi s\theta s\psi + c\psi c\theta & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \quad (1)$$

With $s(.)$ and $c(.)$ represent $\sin(.)$ and $\cos(.)$ respectively.

The Quadrotor dynamic model describing the roll, pitch and yaw rotations, and (x, y, z) translation; contains then four terms which are the gyroscopic effect resulting from the rigid body rotation, the gyroscopic effect resulting from the propeller rotation coupled with the body rotation, aerodynamics frictions and finally the actuators action [10]:

$$\begin{cases} \ddot{\phi} = \dot{\theta}\dot{\psi}\dfrac{(I_y - I_z)}{I_x} - \dfrac{I_r}{I_x}\Omega_r\dot{\theta} - \dfrac{K_{fax}}{I_x}\dot{\phi}^2 + \dfrac{1}{I_x}u_1 \\ \ddot{\theta} = \dot{\phi}\dot{\psi}\dfrac{(I_z - I_x)}{I_y} + \dfrac{I_r}{I_y}\Omega_r\dot{\phi} - \dfrac{K_{fay}}{I_y}\dot{\theta}^2 + \dfrac{1}{I_y}u_2 \\ \ddot{\psi} = \dot{\theta}\dot{\phi}\dfrac{(I_x - I_y)}{I_z} - \dfrac{K_{faz}}{I_z}\dot{\psi}^2 + \dfrac{1}{I_z}u_3 \\ \ddot{x} = -\dfrac{K_{ftx}}{m}\dot{x} + \dfrac{1}{m}(c\phi s\theta c\psi + s\phi s\psi)u_4 \\ \ddot{y} = -\dfrac{K_{fty}}{m}\dot{y} + \dfrac{1}{m}(c\phi s\theta s\psi - s\phi c\psi)u_4 \\ \ddot{z} = -\dfrac{K_{ftz}}{m}\dot{z} - g + \dfrac{1}{m}(c\phi c\theta)u_4 \end{cases} \quad (2)$$

The inputs of the system are u_1, u_2, u_3, u_4, and Ω_r as a disturbance, obtaining:

$$\begin{cases} u_1 = bl\left(\omega_4^2 - \omega_2^2\right) \\ u_2 = bl\left(\omega_3^2 - \omega_1^2\right) \\ u_3 = d\left(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2\right) \\ u_4 = b\left(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2\right) \\ \Omega_r = \omega_1 - \omega_2 + \omega_3 - \omega_4 \end{cases} \quad (3)$$

3. QUADROTOR TAKAGI-SUGENO MODEL DESIGN

3.1. Multiple model approach

The multiple model approach is one promising class of modelling approaches with interpolation, wherein a small number of relatively simple dynamic systems are, in some sense, blended together. It employs the divide-and-conquer strategy of dividing a complex system into several simpler sub-problems, whose individual solution combine to give the solution to the original problem by interpolation, associated with a corresponding set of weighting functions that defines the validity of the local models. Typically, each simple system is a local linear model or affine model, which describes the dynamics of the non-linear system in some small region of the operating space. The role of blending is to provide smooth interpolation, in some sense, between the local models, with the aim of achieving an accurate representation with only a small number of local models [11].

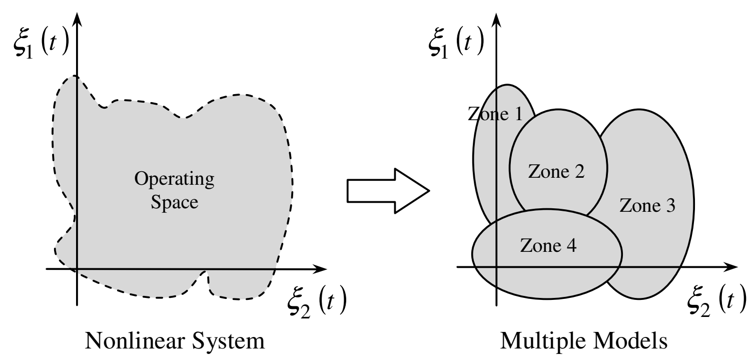


Figure 2. Multiple model approach

3.2. Takagi-Sugeno model

A T-S model is based on the interpolation between several LTI (linear time invariant) local models as follow:

$$\dot{x}_m(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\left(A_i x_i(t) + B_i u(t)\right) \tag{4}$$

Where r is the number of sub-models, $x_m(t) \in \mathbb{R}^p$ is the state vector, $u(t) \in \mathbb{R}^h$ is the input vector, $A_i \in \mathbb{R}^{p\times p}$, $B_i \in \mathbb{R}^{p\times h}$ and $\xi(t) \in \mathbb{R}^q$ is the decision variable vector.

The variable $\xi(t)$ may represent measurable states and/or inputs and the form of this variable may leads to different class of systems: if $\xi(t)$ is known functions than the model (4) represents a nonlinear system and if there are unknown we consider that this leads to linear differential inclusion (LDI). This variable can also be a function of the measurable outputs of the system.
The normalized activation function $\mu_i(\xi(t))$ in relation with the ith sub-model is such that:

$$\begin{cases} \sum_{i=1}^{r} \mu_i(\xi(t)) = 1 \\ 0 \le \mu_i(\xi(t)) \le 1 \end{cases} \tag{5}$$

According to the zone where evolves the system, this function indicates the more or less important contribution of the local model corresponding in the global model (T-S model). The global output of T-S model is interpolated as follows:

$$y_m(t) = \sum_{i=1}^{r} \mu_i\left(\xi(t)\right)\left(C_i x_i(t) + D_i u(t)\right) \tag{6}$$

Where $y_m(t) \in \mathbb{R}^l$ is the output vector and $C_i \in \mathbb{R}^{l\times p}$, $D_i \in \mathbb{R}^{l\times h}$. More detail about this type of representation can be found in [2].

3.3. Quadrotor Takagi-Sugeno model

The behaviour of a nonlinear system near an operating point (x_i, u_i), can be described by a linear time-invariant system (LTI). Using Taylor series about (x_i, u_i) and keeping only the linear terms yields:

$$\dot{x}(t) = A_i\left(x(t) - x_i\right) + B_i\left(u(t) - u_i\right) + f\left(x_i, u_i\right) \tag{7}$$

Which can written as

$$\dot{x}(t) = A_i x(t) + B_i u(t) + d_i \tag{8}$$

With:

$$A_i = \left.\frac{\partial f(x,u)}{\partial x}\right|_{\substack{x=x_i\\ u=u_i}},\ B_i = \left.\frac{\partial f(x,u)}{\partial u}\right|_{\substack{x=x_i\\ u=u_i}},\ f(x,u) = \dot{x}(t),\ d_i = f(x_i, u_i) - A_i x_i - B_i u_i$$

d_i : Normalization constant.

After calculation we obtained:

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_1 & 0 & a_2\dot{\psi}+a_3 & 0 & a_2\dot{\theta} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_5+a_4\dot{\psi} & 0 & a_6 & 0 & a_4\dot{\phi} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_7\dot{\theta} & 0 & a_7\dot{\phi} & 0 & a_8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ a_9 & 0 & a_{10} & 0 & a_{11} & 0 & 0 & a_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ a_{13} & 0 & a_{14} & 0 & a_{15} & 0 & 0 & 0 & 0 & a_{16} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ a_{17} & 0 & a_{18} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{19} \end{bmatrix}, B_i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ b_1 & b_2 & b_3 & b_4 \\ 0 & 0 & 0 & 0 \\ b_5 & b_6 & b_7 & b_8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_9 \\ 0 & 0 & 0 & 0 \\ b_{10} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ b_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ b_{12} & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

By blended local linear models with coupled structure and Gaussian activation function we describe the dynamic model of the Quadrotor by the form:

$$\begin{cases} \dot{x}_m(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x_m(t) + B_i u(t) + d_i) \\ y_m(t) = C x_m(t) \end{cases} \tag{10}$$

With: $\mu_i(\xi(t)),\ i = 1, \dots, r\ \ (r = 3)$ are the normalized activation function, and

$$\mu_i(\xi(t)) = \frac{\omega_i(\xi(t))}{\sum_{j=1}^{3} \omega_i(\xi(t))},\quad \omega_i(\xi(t)) = \prod_{j=1}^{3} \exp\left(-\frac{(\xi_j(t) - c_{i,j})^2}{2\sigma_{i,j}^2}\right)$$

- The vector of decision variables $\xi(t) = \begin{bmatrix} \dot{\phi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T$

- The parameters of activations functions $(c_{i,j}, \sigma_{i,j})$ are given as:

- The centres $c_{i,j}$ are defined according to the operation point.
- The Dispersions $\sigma_{i,j}$ are defined by optimization of a criterion, which represent the quadratic error between Takagi-Sugeno model outputs and nonlinear system outputs, using Particle Swarm Optimisation algorithm (PSO) [12].

-The operating points are chosen to cover maximum space of the operating space, with small number of local models. The attitude of Quadrotor (roll, pitch, and yaw) has a limited bound ($-\pi/2 \leq \phi \leq \pi/2, -\pi/2 \leq \theta \leq \pi/2, -\pi \leq \psi \leq \pi$), for this reason we use three local models to cover this space. Linear local model are defined in this table as follow:

Table 1. Operation Points Parameters.

N° O.P	Parameters	d_i
1	$\dot{\phi} = \dot{\theta} = \dot{\psi} = -0.523\ rad/s$	$[0\ \ 0.1964\ \ 0\ \ -0.1964\ \ 0\ \ 0]^T$
2	$\dot{\phi} = \dot{\theta} = \dot{\psi} = 0\ rad/s$	$[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]^T$
3	$\dot{\phi} = \dot{\theta} = \dot{\psi} = 0.523\ rad/s$	$[0\ \ 0.2771\ \ 0\ \ -0.2771\ \ 0\ \ 0]^T$

3.4. Quadrotor Takagi-Sugeno model validation

The input signals (rotors velocities) most appropriated for the local models network validation, and exit all dynamic of the system in this case is the Pseudo-Random Binary Signal (SBPA) due to different causes:

- SBPA has a null mean and a variance that close to one, which allows the excitation of very good frequency range (dynamics system) without moving away too much the system from the operating point.
- The SBPA is periodic deterministic signal white-noise-like properties very adapted for identification and validation tasks.

The amplitude of the SBPA can be very low, but it must be higher than the level of the residual noise. A typical value of the amplitude is from 0.5% to 5% from the value of the operating point to which the SBPA is applied, in this case the amplitude of the SBPA is given by:

$$A_{SBPA} = \omega_{eq} \pm 0.005 * \omega_{eq}\,, \quad \omega_{eq} = \sqrt{\frac{mg}{4b}}\,.$$

To validate the synthesized Takagi-Sugeno model a SBPA (input signal) is used, for Quadrotor nonlinear system and the T-S model. We simulate the two systems in parallel and we compare the resulting curves.

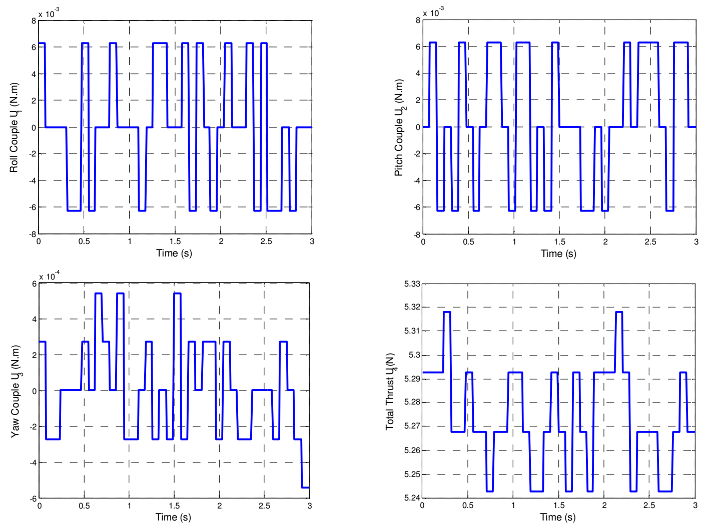


Figure 3. Validation input signals

Figure 3 present the input signals of Quadrotor, which are SBPA signals with variable amplitude. This SBPA excite all dynamic of the system.

Figure 4 present the attitude of Quadrotor and corresponding output of T-S model. We show the resemblance between the output of T-S model and Quadrotor nonlinear system. These results prove the quality of the approximation of a nonlinear system by a T-S model.

Figure 5 present attitude acceleration errors, which are close to a white-noise with null mean and a variance that close to one. Saw the designing T-S model give good approximation of the Quadrotor nonlinear system for a specific region of the operating space.

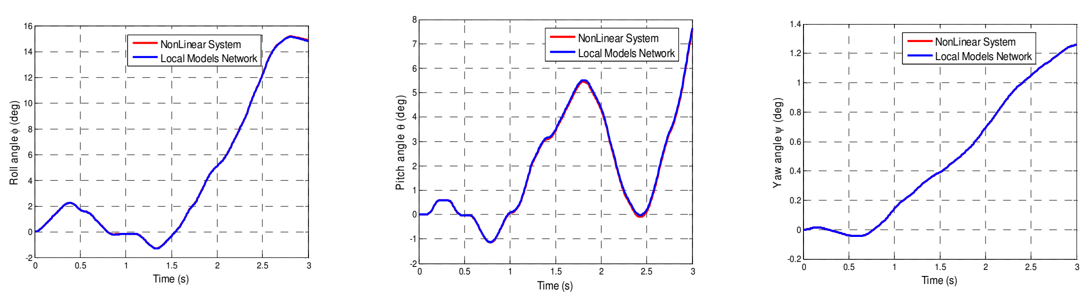


Figure 4. Takagi-Sugeno model and Quadrotor's outputs

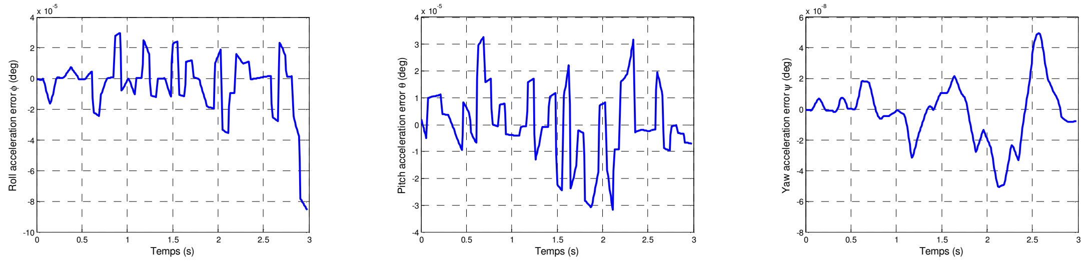


Figure 5. Attitude acceleration errors

4. Controller Design

The concept of PDC, following the terminology [3], is utilized to design state-feedback controller on the basis of the T-S model (10). Linear control theory can be used to design the control law, because T-S model is described by linear state equations. If we compute the local control input to be

The controller law is a convex linear combination of the local controller associated with the corresponding local sub-model. It can present as:

$$U(t) = -\sum_{i=1}^{r} \mu_i(\xi(t)) u_i(t) = -\sum_{i=1}^{r} \mu_i(\xi(t)) K_i x(t) \tag{11}$$

Where K_i is r vector of feedback gains.
It should be noted that the designed controller shares the same models sets with T-S models, and resulting controller (11) is nonlinear in general since the coefficient of the controller depends nonlinearly on the system input and output via the weighting functions. Substituting (11) into (10), the closed-loop T-S model can be represented by:

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\xi(t)) \mu_j(\xi(t)) (A_i - B_i K_j) x(t) \tag{12}$$

The constant d_i was neglected in this formulate, because the control law can compensate the effect of this bias term. To determine state feedback controller described by (12) the following design requirements are considered in this study.

- *Stabilisation:* Design a controller such that the closed-loop T-S model is asymptotically stable:

$$\lim_{t \to \infty} x(t) = 0 \tag{13}$$

- *Pole placement:* Design a controller such that the closed-loop Eigen values of T-S model are located in a prescribed sub-region (D) in the left half plane to prevent too fast controller dynamics and achieve desired transient behaviour:

$$\sigma(A_i - B_i K_j) \subset D \tag{14}$$

4.1. Stabilisation

A sufficient quadratic stability condition derived by Tanaka and Sugeno [13] for ensuring stability of (12) is given as follows:

Theorem 1: The closed-loop T-S model (12) is quadratic-ally stable for some stable feedback K_i (via PDC scheme) if there exists a common positive definite matrix P such that:

$$\begin{gathered} G_{ii}^T P + PG_{ii} < 0 \quad \forall i \in I_r \\ \left(\frac{G_{ij}+G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij}+G_{ji}}{2}\right) < 0, \quad \forall (i,j) \in I_r^2, i < j \end{gathered} \tag{15}$$

With: $G_{ii} = A_i - B_i K_j$, $\mu_i(\xi(t))\mu_j(\xi(t)) \neq 0$.

Which is an LMI in P when K_i are predetermined. However, our objective is to design the gain matrix K_i such that conditions (15) are satisfied. That is, K_i are not pre-determined matrices any longer, but matrix variables. This is the quadratic stability problem and can be recast as an LMI feasibility problem. With linear fractional transformation $X = P^{-1}$ and $N_i = K_i X$, we may rewrite (15) as an LMI problem in N_i, X and S_{ij} [14]:

$$\begin{gathered} X > 0 \\ XA_i^T + A_i X - N_i^T B_i^T - B_i N_i + S_{ii} < 0, \quad \forall i \in I_r \\ XA_i^T + A_i X + XA_j^T + A_j X - N_j^T B_i^T - B_i N_j \\ -N_i^T B_j^T - B_j N_i + 2S_{ij} \le 0, \quad \forall (i,j) \in I_r^2, i < j \\ \begin{pmatrix} S_{11} & \cdots & S_{1n} \\ \vdots & \ddots & \vdots \\ S_{1n} & \cdots & S_{nn} \end{pmatrix} > 0 \end{gathered} \tag{16}$$

With: $S_{ij} = XQ_{ij}X, \forall i \in \{1, \dots, r\}$, Q_{ij} are symmetric matrix.

4.2. Pole placement

In the synthesis of control system, meeting some desired performances should be considered in addition to stability. Generally, stability conditions (Theorem 1) does not directly deal with the transient responses of the closed-loop system. In contrast, a satisfactory transient response of a system can be guaranteed by confining its poles in a prescribed region. This section discusses a Lyapunov characterization of pole clustering regions in terms of LMIs. For this purpose, we introduce the following LMI-based representation of stability regions [15].

Motivated by Chilali [16] and Gutman's theorem for LMI region [17], we consider circle LMI region D

$$D_{q,\rho} = \left\{ x + jy \in \mathbb{C} : (x+q)^2 + y^2 < \rho \right\} \tag{17}$$

Centred at $(-q, 0)$ and with radius $\rho > 0$, where the characteristic function is given by:

$$f_D(z) = \begin{pmatrix} -\rho & z^* + q \\ z + q & -\rho \end{pmatrix} \tag{18}$$

As shown in Figure 6, if $\lambda = \zeta\omega_n \mp j\omega_d$ is a complex pole lying in $D_{q,\rho}$ with damping ratio ζ, undamped natural frequency ω_n, damped natural frequency ω_d, then $\zeta = \sqrt{1 - (\rho^2/q^2)}$, $\omega_n < q + \rho$ and $\omega_d < \rho$. Therefore, this circle region puts a lower bound on both exponential decay rate and the damping ratio of the closed-loop response, and thus is very common in practical control design.

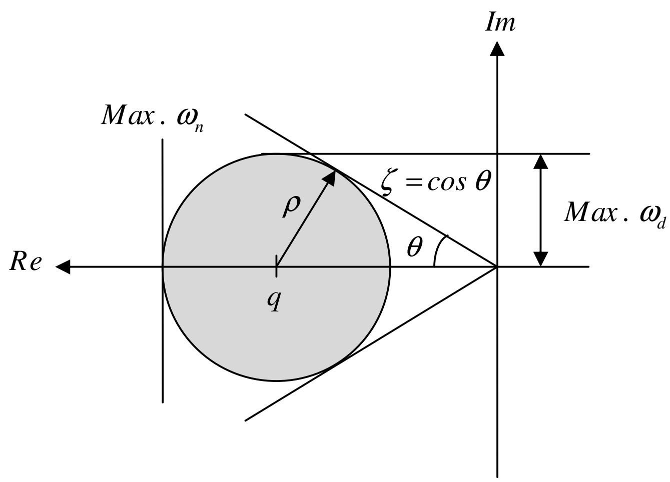


Figure 6. Circular region (D) for pole location

An extended Lyapunov Theorem for the closed loop T-S model (12) is developed with above definition of an LMI-based circular pole region as below [16].

Theorem 2: The closed loop T-S model (12) is D-stable (all the complex poles lying in LMI region *D*) for some state feedback K_i if and only if there exists a positive symmetric matrix X such that

$$\begin{pmatrix} -\rho X & qX + X\left(A_i + B_i K_j\right)^T \\ qX + \left(A_i + B_i K_j\right)X & -\rho X \end{pmatrix} \tag{19}$$

These inequalities are not convex; a simple change of variables $N_i = K_i X$ yields a convex LMI in N_i and X. This pole placement design problem can be recast as an LMI feasibility problem.

$$\begin{pmatrix} -\rho X & qX + XA_i^T + N_i^T B_i^T \\ qX + A_i X + B_i N_i & -\rho X \end{pmatrix} < 0, \quad i = j \tag{20}$$

By combining Theorems 1 and 2 leads to the following LMI formulation of two objectives state-feedback synthesis problem [15].

Theorem 3: The closed loop T-S model (12) is stabilizable in the specified region D if and only if there exists a common positive symmetric matrix X and N_i such that the following LMI condition holds

$$\begin{aligned}
& X > 0 \\
& XA_i^T + A_i X - N_i^T B_i^T - B_i N_i + S_{ii} < 0, \quad \forall i \in I_r \\
& XA_i^T + A_i X + XA_j^T + A_j X - N_j^T B_i^T - B_i N_j \\
& -N_i^T B_j^T - B_j N_i + 2S_{ij} \le 0, \quad \forall (i,j) \in I_r^2, i < j \\
& \begin{pmatrix} -\rho X & qX + XA_i^T + N_i^T B_i^T \\ qX + A_i X + B_i N_i & -\rho X \end{pmatrix} < 0, \quad \forall i \in I_r \\
& \begin{pmatrix} S_{11} & \cdots & S_{1n} \\ \vdots & \ddots & \vdots \\ S_{1n} & \cdots & S_{nn} \end{pmatrix} > 0
\end{aligned} \tag{21}$$

With: $S_{ij} = XQ_{ij}X, K_i = N_i X,\ \forall i \in \{1, \dots, r\}$

By solving these two kinds of LMI constraints directly leads to a state feedback controller, such that the resulting controller meets both the global stability and the desired transient performance simultaneously.

4.3. Quadrotor controller design

Quadrotor is an under-actuated system (2) (3), it has six outputs and four inputs, for this reason we use tow virtual control inputs (u_x, u_y) addition to four control inputs of the Quadrotor (u_1, u_2, u_3, u_4) for the control of this system.

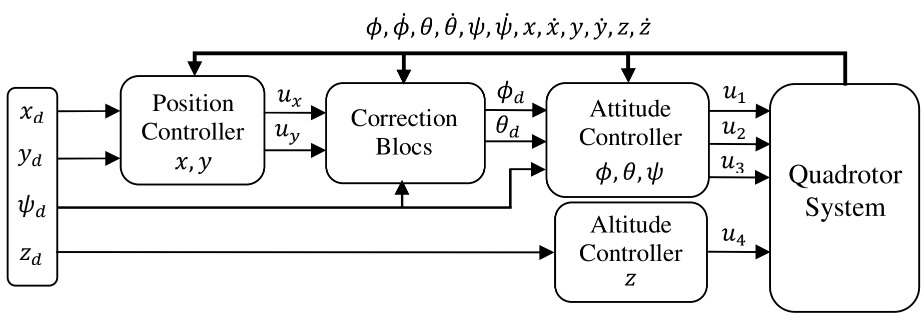


Figure 7. Control schema

The control scheme advocated for the overall system is then logically divided in an attitude controller, an altitude controller and position controller as schematized in Figure 7. Correction blocs give the relation between virtual control inputs (u_x, u_y) and desired angles (ϕ_d, θ_d) as below:

$$\begin{cases} \theta_d = atan\left(\dfrac{u_x c\psi + u_y s\psi}{g}\right) \\ \phi_d = atan\left(\dfrac{u_x s\psi - u_y c\psi}{g} c\theta_d\right) \end{cases} \tag{22}$$

Using Theorem 3, we can design a nonlinear state feedback controller that guarantees global stability while provides desired transient behaviour by constraint the closed-loop poles in D. The stability region D is a circle of centre $(q, 0)$ and radius ρ and the LMI synthesis is performed for a set of values $(q, \rho) = (2, 0.5)$.

Then the LMI region has the following characteristic function:

$$f_D(z) = \begin{pmatrix} -0.5 & z^* + 2 \\ z + 2 & -0.5 \end{pmatrix} \tag{23}$$

This circle region puts a lower bound on both exponential decay rate $q - \rho = 1.5\, rad/s$ and damping ratio $\zeta = \sqrt{1 - (\rho^2/q^2)} = 0.97$ of the closed-loop response. By solving LMI feasibility problem of Theorem 3, we can obtain a positive symmetric matrix X (by interior-point method in Matlab LMI-toolbox), and stat feedback Matrix K_i as:

$$K_1 = \begin{bmatrix} 2.75 & 0 & 2.75 & 0 & 0 & 0 & 7.12 & 3.72 & -4.54 & 0 & 0 & 0 \\ 0.04 & 0.021 & 0 & -0.001 & 0.011 & 0.004 & 0 & 0 & 0 & -0.026 & 0 & -0.007 \\ 0 & 0.001 & 0.04 & 0.021 & -0.01 & -0.004 & 0 & 0 & 0 & 0 & -0.026 & 0.007 \\ 0 & 0 & 0 & 0 & 0.084 & 0.04 & 0 & 0 & 0 & 0 & 0 & -0.053 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -2.75 & 0 & -2.75 & 0 & 0 & 0 & 7.12 & 3.72 & -4.54 & 0 & 0 & 0 \\ 0.04 & 0.021 & 0 & -0.001 & 0.012 & 0.004 & 0 & 0 & 0 & -0.027 & 0 & -0.008 \\ 0 & 0.001 & 0.042 & 0.021 & -0.012 & -0.004 & 0 & 0 & 0 & 0 & -0.026 & 0.008 \\ 0 & 0 & 0 & 0 & 0.084 & 0.043 & 0 & 0 & 0 & 0 & 0 & -0.054 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 5.34 & 2.79 & -3.41 & 0 & 0 & 0 \\ 0.042 & 0.021 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.042 & 0.021 & 0 & 0 & 0 & 0 & 0 & 0 & -0.026 & 0 \\ 0 & 0 & 0 & 0 & 0.084 & 0.043 & 0 & 0 & 0 & 0 & 0 & -0.053 \end{bmatrix}$$

4.4. Simulation results

The controller described above was simulated for the nonlinear Quadrotor system. Simulations are made for initial values equal to zero. The values of the model parameters used for simulations are the following: $m = 0.486g$, $l = 0.225$, $g = 0.9.81 m/s^2$, $d = 3.23\times10^{-7}$, $b = 2.98\times10^{5} N/(rad.s)$, $I_x = I_y = 3.82\times10^{-3} Kg.m^2$, $I_z = 7.65\times10^{-3}$, $K_{fax} = K_{fay} = 5{,}5670\times10^{-4}$, $K_{faz} = 6{,}3540\times10^{-4}$, $K_{ftx} = K_{fay} = 0.032$, $K_{ftz} = 0.048$. The applied control law can be summarized as follows:

- $K_{Stabilisation}$ Considers only stability conditions (25);

- $K_{Placement}$ Considers both stability and pole placement conditions (30).

Simulation results for Quadrotor positions and rotations control are given in figures (8, 9, 10 and 11).

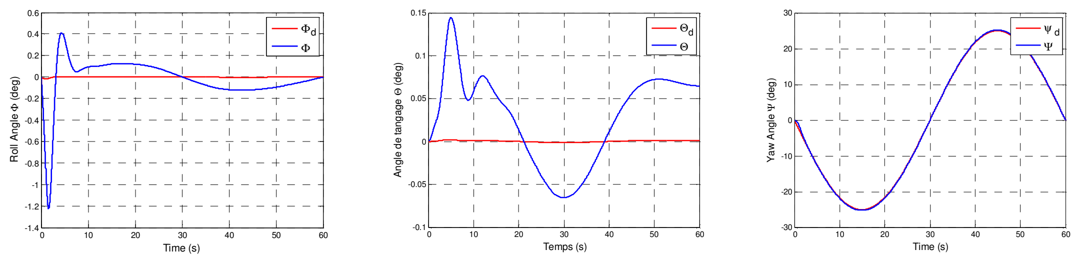


Figure 8. Quadrotor Attitude (ϕ, θ, ψ)

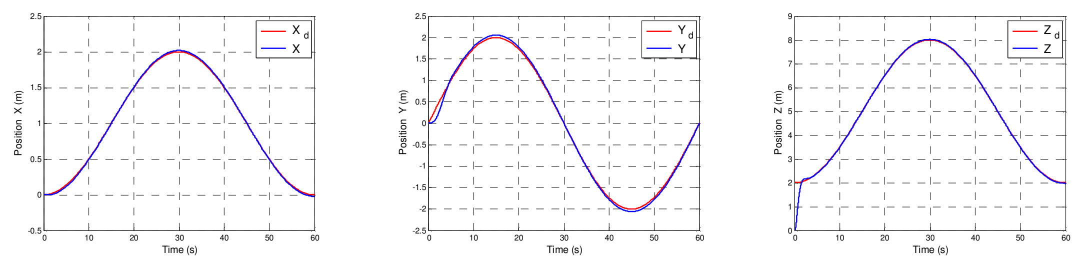
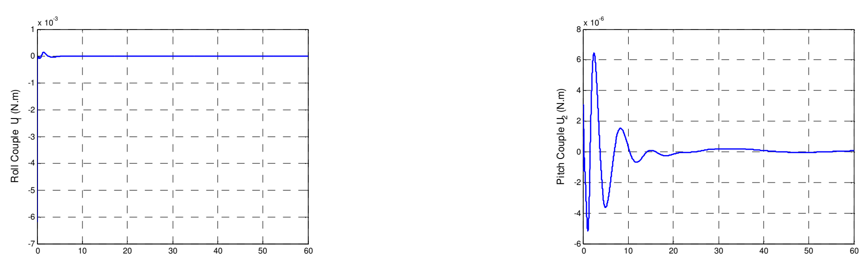


Figure 9. Quadrotor Positions (x, y, z)

Figure 8, 9 represents Quadrotor outputs (position and rotation) with sinusoidal trajectory for $K_{Placement}$ Controller, we show that this controller give good trajectory tracking performances. In Figure 8 we show that the roll and pitch angle does not follow its reference trajectory and that because our controller is linear, applied to a nonlinear system (Quadrotor). But it was a small error; more of these two angles are used for the control of x ad y position. This is our goal, Of course, in addition to z position and Yaw angle. Figure 10 represent Quadrotor controls inputs; we show that control inputs are smoother and realizable.



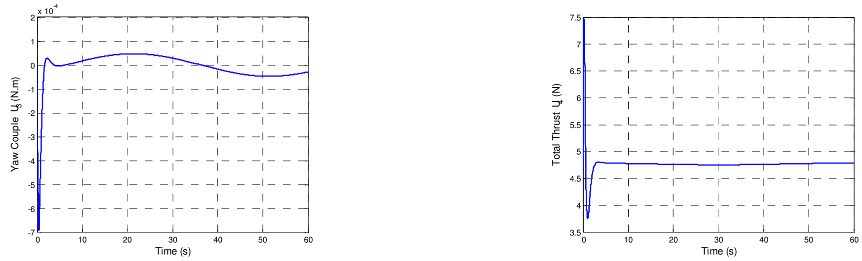


Figure 10. Quadrotor control inputs (u_1, u_2, u_3, u_4)

For the comparison purpose, we present in figure 11, 3D position for $K_{Stabilisation}$ controller, when constraint for the pole placement is neglected (considering only stability condition, Theorem 1) and for $K_{Placement}$ controller (Theorem 3) in figure 12. It can be also noticed that $K_{Placement}$ provides better tracking performance than those of $K_{Stabilisation}$.

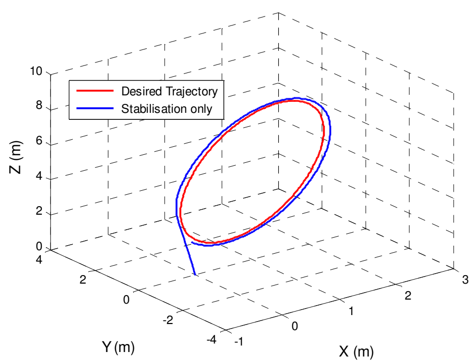


Figure 11. 3D position for $K_{Stabilisation}$

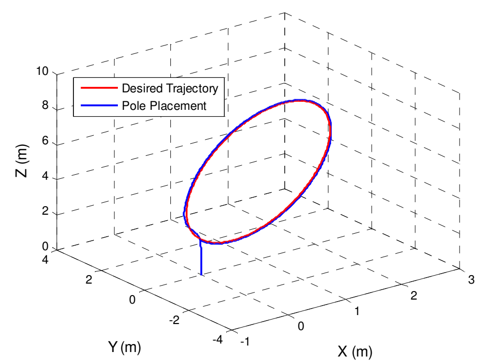


Figure 12. 3D position for $K_{Placement}$

5. CONCLUSIONS

In this paper, we propose a Takagi-Sugeno model of Quadrotor, which is developed using multiple model approach, and successfully validated using SBPA input signals, simulation results confirmed effectiveness its similarity with the nonlinear model. A systematic design methodology for the control of Quadrotor system with guaranteed stability and pre-specified transient performance is presented. The framework is based on T-S model and parallel distributed compensation (PDC) technique. Simulation results showed that the multi-objective nonlinear controller (calculated from pole placement conditions) yields not only maximized stability boundary but also better tracking performance than single objective controller (calculated from only stabilizations conditions), and theses controllers are applied with successful to one of the most complex flying, and under-actuator systems.

APPENDIX

Parameters of A_i and B_i matrix

$$
\begin{cases}
a_1 = -\dfrac{K_{fax}}{I_x} \\
a_2 = \dfrac{\left(I_y - I_z\right)}{I_x} \\
a_3 = -\dfrac{J_r}{I_x}\Omega_r \\
a_4 = \dfrac{\left(I_z - I_x\right)}{I_y} \\
a_5 = \dfrac{J_r}{I_y}\Omega_r \\
a_6 = -\dfrac{K_{fay}}{I_y} \\
a_7 = \dfrac{\left(I_x - I_y\right)}{I_z}
\end{cases}
\quad
\begin{cases}
a_8 = -\dfrac{K_{faz}}{I_z} \\
a_9 = \dfrac{u_1}{m}\left(-s\phi c\psi s\theta + s\phi s\psi\right) \\
a_{10} = \dfrac{u_1}{m}\left(c\phi c\psi c\theta\right) \\
a_{11} = \dfrac{u_1}{m}\left(-c\phi s\psi s\theta + s\phi c\psi\right) \\
a_{12} = -\dfrac{K_{ftx}}{m} \\
a_{13} = \dfrac{u_1}{m}\left(-s\phi s\psi s\theta - c\phi c\psi\right) \\
a_{14} = \dfrac{u_1}{m}\left(c\phi s\psi c\theta\right) \\
a_{15} = \dfrac{u_1}{m}\left(c\phi c\psi s\theta + s\phi s\psi\right)
\end{cases}
\quad
\begin{cases}
a_{16} = -\dfrac{K_{fty}}{m} \\
a_{17} = \dfrac{u_1}{m}\left(-s\phi c\theta\right) \\
a_{18} = \dfrac{u_1}{m}\left(-c\phi s\theta\right) \\
a_{19} = -\dfrac{K_{ftz}}{m} \\
b_1 = -\dfrac{J_r}{I_x}\dot{\theta}C_1 \\
b_2 = -\dfrac{J_r}{I_x}\dot{\theta}C_2 + \dfrac{1}{I_x} \\
b_3 = -\dfrac{J_r}{I_x}\dot{\theta}C_3 \\
b_4 = -\dfrac{J_r}{I_x}\dot{\theta}C_4
\end{cases}
\quad
\begin{cases}
b_5 = \dfrac{J_r}{I_y}\dot{\phi}C_1 \\
b_6 = \dfrac{J_r}{I_y}\dot{\phi}C_2 \\
b_7 = \dfrac{J_r}{I_y}\dot{\phi}C_3 + \dfrac{1}{I_y} \\
b_8 = \dfrac{J_r}{I_y}\dot{\phi}C_4 \\
b_9 = \dfrac{1}{I_z} \\
b_{10} = \dfrac{1}{m}\left(c\phi s\theta c\psi + s\phi s\psi\right) \\
b_{11} = \dfrac{1}{m}\left(c\phi s\theta s\psi - s\phi c\psi\right) \\
b_{12} = \dfrac{1}{m}\left(c\phi c\theta\right)
\end{cases}
$$

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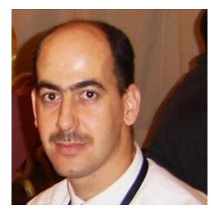

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