PERFORMANCES OF MODEL REDUCTION USING FACTOR DIVISION ALGORITHM AND EIGEN SPECTRUM ANALYSIS

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ABSTRACT

The easy and simplicity of manipulation and the multiple applications provided by approximate reduced-order systems, in comparison with original high order systems, make the topic of model reduction a subject with big importance; it allows us to get models that approached in their behavior, by satisfactory way, the high order original systems.

In this paper we study one of recent reduction methods, which combine the algorithm of factor division and eigen spectrum analysis, and we compare it in the time domain (Schur method), and in the frequency domain (Routh approximation approach), in order to determine its performances.

KEYWORDS

High order systems, Model reduction, Factor division, Eigen spectrum analysis, Schur method, Routh approximation.

1. INTRODUCTION

In recent years, the problem of simplifying the model has been the subject of much research because the study of direct practice such systems necessarily leads to costs as well as extremely high computation time, this is due to the use of numerical algorithms whose convergence is sometimes difficult and often impossible when the size of the process to be studied is large. Therefore, the reduction becomes a tool more than necessary. The reduction technique is to define a model of reduced dimension, retaining the main physical aspects of the initial system and its features, such as stability. The approximation of complex model is a large area, several studies were performed, each algorithm can be applied to a given problem [1,2]. In our work, we study a new technique for order reduction of large systems that combines factor division algorithm and spectral analysis to obtain a reduced order. Therefore, by comparing this approach with existing others, interesting results are obtained, as it will be shown by simulations performed in MATLAB® 7.1.

2. ROUTH APPROXIMATION METHOD (RAM)

It is simple and direct method, based on the Routh stability criterion. The reduced order transfer function is determined directly from elements in the Routh stability arrays of high-

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order denominator and numerator.

Model reduction Procedure **[8]**

Input : The strictly proper high-order system n represented by

$$H(s) = \frac{b_{11}s^m + b_{21}s^{m-1} + b_{12}s^{m-2} + b_{22}s^{m-3} + ...}{a_{11}s^n + a_{21}s^{n-1} + a_{12}s^{n-2} + a_{22}s^{n-3} + ...}, \quad (1)$$

Step1 : The Routh stability array for the numerator and denominator polynomials of $H(s)$ are shown below in Tables1 and 2, respectively.

Table 1. Numerator Stability Array

$$\begin{array}{lllll}
b_{11} & b_{12} & b_{13} & b_{14} & \cdots \\
b_{21} & b_{22} & b_{23} & b_{24} & \cdots \\
\hline
b_{31} & b_{32} & b_{33} & \cdots & \\
b_{41} & b_{42} & b_{43} & \cdots & \\
. & & & & \\
. & & & & \\
b_{m,1} & & & & \\
b_{m+1,1} & & & &
\end{array}$$

Table 2. Denominator Stability Array

$$\begin{array}{lllll}
a_{11} & a_{12} & a_{13} & a_{14} & \cdots \\
a_{21} & a_{22} & a_{23} & a_{24} & \cdots \\
\hline
a_{31} & a_{32} & a_{33} & \cdots & \\
a_{41} & a_{42} & a_{43} & \cdots & \\
. & & & & \\
. & & & & \\
a_{n,1} & & & & \\
a_{n+1,1} & & & &
\end{array}$$

Step2: The tables coefficients are calculated by

$$C_{ij} = C_{i-2,j+1} - \left(C_{i-2,1}.C_{i-1,j+1}\right)/\left(C_{i-1,1}\right),\ \mathrm{i} \geq 3, 1 \leq \mathrm{j} \leq (\mathrm{n} - \mathrm{i} + 3)/2, \quad (2)$$

Output: Construction of the reduced order $H_r(s)$ transfer-function of a system with reduced order:

$$H_r(s)=\frac{b_{(m+2-r),1}s^{r-1}+b_{(m+3-r),1}s^{r-2}+\ldots}{a_{(n+1-r),1}s^{r}+a_{(n+2-r),1}s^{r-1}+\ldots} \quad (3)$$

End of procedure.

3. SCHUR METHOD (NONMINIMAL MODELS)

The Schur method can be computed the Moore reduced model directly without passing by balanced state space realization, via projections defined in terms of arbitrary bases for left and right eigenspaces associated with the large eigenvalues of the product of the controllability and observability gramians $(W_c W_o)$.

***Procedure* [9]**

Input: The complete order system *n* have state space realization (*A, B, C, D, n*).

Step1: Resolve Lyapunov equations to get the controllability and observability gramians W_c, W_o.

Step 2: Compute an orthogonal real matrix V, such that ($V.W_c.W_o.V^T$) is upper triangular, i.e., put $(W_c.W_o)$ into Schur form.

Step3 : Using orthogonal rotations for compute orthogonal real transformations which order the Schur forms in ascending and descending order, respectively such that

$$V_D^T.W_c.W_o.V_D=\begin{bmatrix} \lambda_{D1} & \times & \times & & .. & \times & \times \\ 0 & \lambda_{D2} & \times & & & & \times \\ & 0 & & \times & & & \times \\ & & 0 & & \times & & \\ & & & 0 & & & \\ & & & & 0 & & \times \\ 0 & 0 & & & 0 & 0 & \lambda_{Dn} \end{bmatrix}, \quad (4)$$

$$V_A^T.W_c.W_o.V_A=\begin{bmatrix} \lambda_{A_n} & * & * & . & .. & * & * \\ 0 & \lambda_{A_{n-1}} & * & . & . & . & * \\ . & 0 & . & * & . & . & * \\ . & . & 0 & . & * & . & . \\ . & . & . & 0 & . & . & . \\ . & . & . & . & 0 & . & * \\ 0 & 0 & . & . & 0 & 0 & \lambda_{A_1} \end{bmatrix}, \quad (5)$$

Where $\lambda_{Ai}=\lambda_{Di}=\sigma_i^2 \quad (i=\overline{1,r})$ (6)

$$\lambda_{Ai} = \lambda_{Di} = \sigma_i^2 \quad (i = \overline{r+1, n}) \tag{7}$$

Step 4: Partition of V_A and V_D for obtained the matrix $V_{g,g}$ and $V_{d,g}$

$$V_A = \left[\overbrace{V_{d,p}}^{n-r} \overbrace{V_{g,g}}^{r} \right], V_D = \left[\overbrace{V_{d,g}}^{r} \overbrace{V_{g,p}}^{n-r} \right] \tag{8}$$

The columns of $V_{d,g}$ and $V_{d,p}$ form, respectively, les orthonormal bases for the right eigenspaces of $(W_c.W_o)$ associated with big eigenvalues $\sigma_i^2 (i = \overline{1, r})$ and the small eigenvalues $\sigma_i^2 (i = \overline{r+1, n})$, also, the columns of ($V_{g,g}$) and ($V_{g,p}$) provide an analogous decomposition of the left eigenspaces.

Step 5: Form the projection

$$E_g = V_{g,g}^T V_{d,g}, \tag{9}$$

and compute its singular value decomposition (SVD)

$$E_g = U_{E,g} . \sum\nolimits_{E,g} . V_{E,g}^T, \tag{10}$$

Step 6: Form the matices

$$S_{g,g} = V_{g,g} . U_{E,g} . \sum\nolimits_{E,g}^{-1/2} \in \Re^{n \times r}, \tag{11}$$

$$S_{d,g} = V_{d,g} . U_{E,g} . \sum\nolimits_{E,g}^{-1/2} \in \Re^{n \times r}. \tag{12}$$

Output: Construction of the low-order model $[A_r, B_r, C_r, D]$,

$$\begin{bmatrix} A_r & B_r \\ C_r & D_r \end{bmatrix} = \begin{bmatrix} [S_{g,g}^T . A . S_{d,g}] & [S_{g,g}^T . B] \\ [C . S_{d,g}] & [D] \end{bmatrix} \tag{13}$$

End of procedure.

4. FACTOR DIVISION METHOD

Consider a n^{th} order transfer function given by

$$G_n(s) = \frac{b_0 + b_1 s + b_2 s^2 + \dots + b_{n-1} s^{n-1}}{(s+p_1)(s+p_2)\dots(s+p_n)}, \tag{14}$$

where ($-p_i$, $i = 1, 2, 3, \dots, n$) are poles, and $p_1 < p_2 < p_3 \dots < p_n$.

A reduced transfer function which retains the $(n-1)$ dominant poles and the first time moments of G(s) is obtained by dividing the factor $(s + P_n)$ into the numerator of G(s) from the constant term first. This gives the reduced model

$$G_{n-1}(s) = \frac{d_0 + d_1 s + d_2 s^2 + \dots + d_{n-2} s^{n-2}}{(s+p_1)(s+p_2)\dots(s+p_{n-1})}, \tag{15}$$

where

$$d_0 = \frac{b_0}{p_n},$$

and

$$d_i = \frac{(b_i - d_{i-1})}{p_n}, \tag{16}$$

For a lower-order model, the procedure may be repeated on $G_{n-1}(s)$, and so on until the desired order is reached.

Remark: For an r^th order model successive of d_i, $i = \overline{0, r-1}$, only need be calculated from eq. (16). Alternatively, an algorithm can be used for reduction to an r-order.

Factor Division Procedure

Input: Given a full n^th order system represented by a transfer-function $G_n(s)$.

Step1: Divide the (n - r) undesirable poles, verifying

$(s+p_{r+1})(s+p_{r+2})\dots(s+p_n) = e_0 + e_1 s + \dots + e_{n-r} s^{n-r}$, directly into the numerator.

Step 2: The numerator of the reduced transfer function $G_r(s)$ is thus given by the series expansion of the expression $A(s)$,

$$A(s) = \frac{b_0 + b_1 s + b_2 s^2 + \dots + b_{n-1} s^{n-1}}{e_0 + e_1 s + e_2 s^2 + \dots + e_{n-r} s^{n-r}},$$

Step 3: Compute the other coefficients in the following table [3] by using b_i and e_i coefficients:

$$\alpha_0 = \frac{b_0}{e_0} \quad \begin{array}{l} \nearrow \; b_0 \quad b_1 \quad b_2 \ldots\ldots b_{r-1} \\ \searrow \; e_0 \quad e_1 \quad e_2 \ldots\ldots e_{r-1} \end{array}$$

$$\alpha_1 = \frac{q_0}{e_0} \quad \begin{array}{l} \nearrow \; q_0 \quad q_1 \ldots\ldots q_{r-2} \\ \searrow \; e_0 \quad e_1 \ldots\ldots e_{r-2} \end{array}$$

$$\alpha_2 = \frac{r_0}{e_0} \quad \begin{array}{l} \nearrow \; r_0 \quad r_1 \ldots\ldots r_{r-3} \\ \searrow \; e_0 \quad e_1 \ldots\ldots e_{r-3} \end{array}$$

$$\vdots$$

$$\alpha_{r-2} = \frac{u_0}{e_0} \quad \begin{array}{l} \nearrow \; u_0 \quad u_1 \\ \searrow \; e_0 \quad e_1 \end{array}$$

$$\alpha_{r-1} = \frac{v_0}{e_0} \quad \begin{array}{l} \nearrow \; v_0 \\ \searrow \; e_0 \end{array} \tag{17}$$

Where

$$q_i = b_{i+1} - \alpha_0 e_{i+1}, \; i = 0,1,.., \; r-2,$$
$$r_i = q_{i+1} - \alpha_1 e_{i+1}, i = 0,1,....., \; r-3,$$
$$.$$
$$.$$
$$v_0 = u_1 - \alpha_{r-2} e_1.$$

Output: The low order model is given by

$$G_r(s) = \frac{\alpha_0 + \alpha_1 s + \alpha_2 s^2 + ... + \alpha_{r-1} s^{r-1}}{(s+p_1)(s+p_2).....(s+p_r)}.$$

End of procedure.

5. FACTOR DIVISION ALGORITHM AND EIGEN SPECTRUM ANALYSIS

A large number of methods are available in the literature for order reduction of linear continuous systems in time domain as well as in frequency domain [4]. Further, several methods have also been suggested by combining the features of two deferent methods [5]. This technique is based onto combined factor division algorithm to obtain the zeroes of the reduced order model, and eigen spectrum analysis to obtain its poles [7]. Given the transfer function of the high- order n system (HOS),

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{b_0 + b_1 s + b_2 s^2 + ... + b_{n-1} s^{n-1}}{(s + p_1)(s + p_2)....(s + p_n)} \tag{18}$$

where $-p_1 > -p_2 > ..... > -p_n$ are poles of HOS

The transfer function low- order system (LOS) of order *r* is:

$$G_r(s) = \frac{\tilde{N}(s)}{\tilde{D}(s)} = \frac{\alpha_0 + \alpha_1 s + ... + \alpha_{r-1} s^{r-1}}{(s + p'_1)(s + p'_2)....(s + p'_r)}, \tag{19}$$

where $-p'_1 > -p'_2 > ..... > -p'_r$ are poles of LOS.

To construct $G_r(s)$, the following procedure is proposed.

Procedure [7]

Input *:* Given n[th] *order system represent by transfer function* $G_n(s)$.

Step 1: Fixing of eigen spectrum zone of **HOS** (Fig. 1)

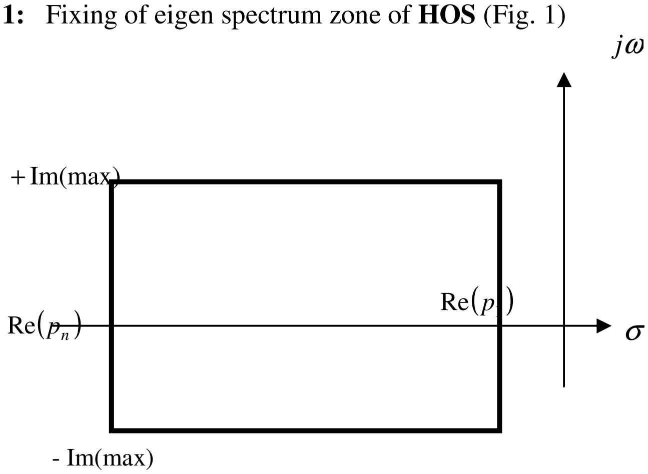

Fig.1. Eigen spectrum zone of HOS

Step 2: Quantification of pole centroid p_m de HOS:

$$p_m = \frac{\sum_{i=1}^{n} \mathrm{Re}(p_i)}{n} \tag{20}$$

and the pole stiffness p_s of HOS is

$$p_s = \frac{\mathrm{Re}\,(p_1)}{\mathrm{Re}\,(p_n)} \tag{21}$$

Step 3: Compute p'_m and p'_s where $p'_m = p_m$ and $p'_s = p_s$

$$\begin{aligned} p'_s &= \frac{\mathrm{Re}(p'_1)}{\mathrm{Re}(p'_r),} = p_s \\ p'_m &= \frac{\mathrm{Re}(p'_1)+\mathrm{Re}(p'_2)+\ldots+\mathrm{Re}(p'_r)}{r} = p_m, \end{aligned} \tag{22}$$

Step 4 : Compute the r poles of reduce model and M (see Fig. 2) solutions of

$$\begin{bmatrix} p_s(r-1)+1 & Q \\ (1-p_s) & (1-r) \end{bmatrix} \begin{bmatrix} \mathrm{Re}(p'_r) \\ M \end{bmatrix} = \begin{bmatrix} N \\ 0 \end{bmatrix}, \tag{23}$$

where $N = P_m r$ and $Q = r-2$,

and M is the distance between two successive poles.

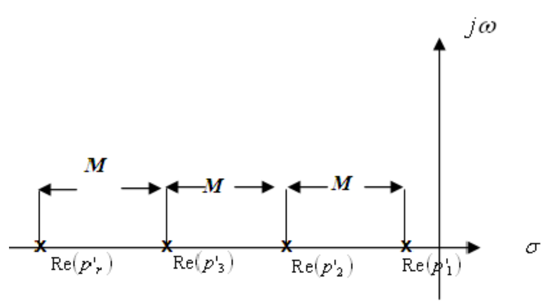

Fig.2. Eigen spectral poles of LOS

Step 5: Using factor division algorithm [6] for calculate $\tilde{N}(S)$ of equation (19), can be given by:

$$\frac{N(s)}{D(s)} = \frac{\tilde{N}(s)}{\tilde{D}(s)} \tag{24}$$

Output : The low order approximant is given by

$$G_r(s) = \frac{\tilde{N}(s)}{\tilde{D}(s)}$$

6. NUMERICAL APPLICATION

Consider the partial deferential equation (PDE), representing a continuous, stable system of complete order $n = 84$ [12],

$\frac{\partial x}{\partial t} = \frac{\partial^2 x}{\partial z^2} + \frac{\partial^2 x}{\partial v^2} + 20\frac{\partial x}{\partial z} - 180x + f(v,z)u(t)$, where, x is a function of time (t), vertical position (v) and horizontal position (z).

The transfer functions of second order models obtained through proposed and other methods are

$$G_{(\text{Proposed})}(s) = \frac{6831\,s + 1.914\,e\,005}{s^2 + 703.5\,s + 1.767\,e\,004}$$

$$G_{(\text{Routh})} = \frac{2018s + 5.903e\,004}{s^2 + 208.8\,s + 5449}$$

$$G_{(\text{Schur})}(s) = \frac{2822s + 2.006e\,006}{s^2 + 1030\,s + 1.85e\,005}$$

After reduction (r = 2), we see that the impulse response (Fig. 3), and the step response (Fig. 4) of low order approximant fitted original model. Also, key properties of the initial system are preserved such as stability (Fig. 5) in the reduced order system. To well appreciate the performances of the proposed approach comparing with the others, the distance between original complete order and its approximants is measured in term of integral-square-error (ISE) index error (Table 3):

Table 3 – Comparison of integral-squared-error (ISE)

Reduced order Models from:	ISE
Schur method	8.191e-005
Proposed method	5.441e-003
Routh method	1.01e-002

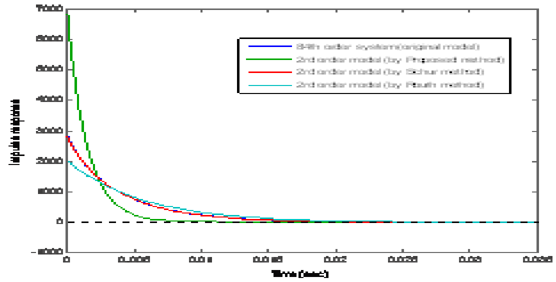

Fig. 3. Impulse responses of original system and its approximants

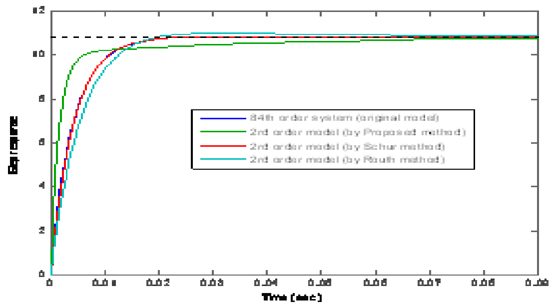

Fig. 4. Step responses of original system and its pproximants

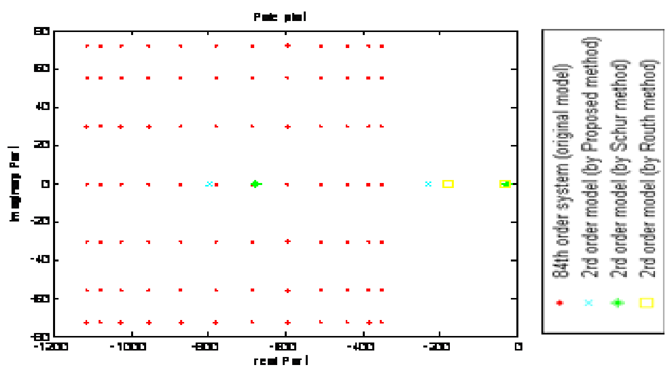

Fig.5 Poles of original and reduced order models

For the previous example, we have shown the frequency responses by plotting the amplitude spectra (dB) and the phase spectra (Fig. 6), and the singular values distribution (Fig. 7), according to the low order r. The graphs show that the approximants closely follow the global behavior of the initial system. It should be noted as even a slight deviation; these errors are due to a substantial reduction of order.

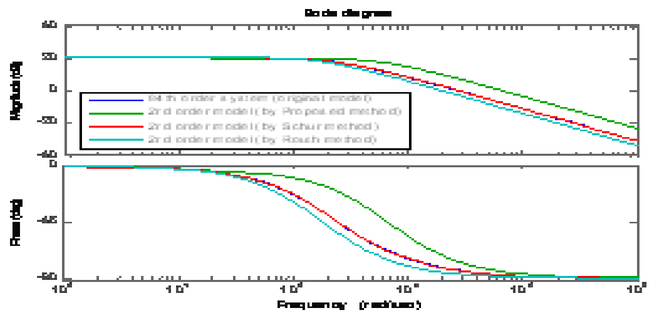

Fig. 6. Frequency responses (dB) of original system and its approximants

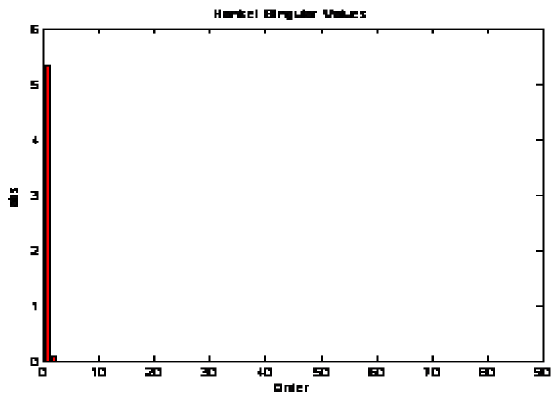

Fig.7. Hankel singular values

To appreciate the distance between initial model and its approximants from the model order reduction, an integral index error is calculated. It reveals that the proposed technique is interesting but not the best.

6. CONCLUSION

This work is based on the approximation of high order system, using the factor division algorithm and eigen spectrum analysis.A simulation is performed and compared to other techniques mentioned in this paper. We note that the reduced order approximation by proposed method keeps the key properties of the original system and from the error criterion ISE and frequency and time responses, our approximation yields to a good model rather than Routh technique, but once again, the Schur technique prove its superiority [13].

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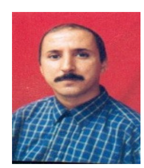

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