ANTI-SYNCHRONIZATION OF 4-DIMENSIONAL HYPERCHAOTIC LI AND HYPERCHAOTIC LÜ SYSTEMS VIA ACTIVE CONTROL

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ABSTRACT

In this paper, we derive new results for the anti-synchronization of identical and non-identical hyperchaotic Li systems (Li, Tang and Chen, 2005) and hyperchaotic Lü systems (Bao and Liu, 2008). Active control method has been deployed for achieving the four-dimensional hyperchaotic systems discussed in this paper and the stability results have been proved using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the active nonlinear control method is effective and convenient to achieve anti-synchronization of the identical and different hyperchaotic Li systems and hyperchaotic Lü systems. Numerical simulations using MATLAB have been shown to illustrate the anti-synchronization controllers designed in this paper.

KEYWORDS

Anti-Synchronization, Active Control, Chaos, Hyperchaotic Li System, Hyperchaotic Lü System.

1. INTRODUCTION

Hyperchaotic system is typically defined as a chaotic system having more than one positive Lyapunov exponent (LE). It was first discovered by O.E. Rössler ([1], 1979).

Hyperchaotic systems have the characteristics of high capacity, high security and high efficiency. Hence, they find applications in several areas like electronic oscillators [2-3], secure communication [4-7], synchronization [8-9], encryption [10], etc. Thus, control and synchronization of hyperchaotic systems have become important research problems.

The anti-synchronization problem can be stated as follows. If a particular chaotic system is called the master system and another chaotic system is called the slave system, then the idea of anti-synchronization is to use the output of the master system to control the slave system so that the states of the slave system have the same amplitude but opposite signs as the states of the master system asymptotically with time.

Since the seminal work by Pecora and Carroll ([11], 1990), many impressive methods have been developed for the synchronization and anti-synchronization of chaotic systems, *viz*. OGY method [12], active control method [13-15], adaptive control method [16-20], backstepping method [21-24], sampled-data feedback synchronization method [25-26], time-delay feedback method [27-28], sliding mode control method [29-32], etc.

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In this paper, we deploy active control method to derive new results for the anti-synchronization for identical and different hyperchaotic Li systems ([33], 2005) and hyperchaotic Lü systems ([34], 2008).   .

The organization of this paper is as follows. Section 2 contains the problem statement and active control methodology, Section 3 provides a description of the hyperchaotic Li and hyperchaotic Lü systems. Section 4 contains our new results for the anti-synchronization of two identical hyperchaotic Li systems. Section 5 contains our new results for the anti-synchronization of two identical hyperchaotic Lü systems. Section 6 contains our new results for the anti-synchronization of hyperchaotic Li and hyperchaotic Lü systems. Section 7 contains a summary of the main results derived in this paper.

2. Problem Statement and Active control Methodology

As the *master* system, we consider the chaotic system described by the dynamics

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in R^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : R^n \to R^n$ is the nonlinear part of the system.

As the *slave* system, we consider the chaotic system described by the dynamics

$$\dot{y} = By + g(y) + u \tag{2}$$

where $y \in R^n$ is the state of the system, B is the $n \times n$ matrix of the system parameters, $g : R^n \to R^n$ is the nonlinear part of the system and $u \in R^n$ is the active controller of the slave system.

If $A = B$ and $f = g$, then x and y are the states of two identical chaotic systems. If $A \neq B$ or $f \neq g$, then x and y are the states of two different chaotic systems.

For the master system (1) and the slave system (2), the design goal is to build a feedback controller u, which *anti-synchronizes* their states for all initial conditions $x(0), y(0) \in R^n$.
We define the *anti-synchronization error* as

$$e = y + x, \tag{3}$$

Then a simple calculation yields the error dynamics as

$$\dot{e} = By + Ax + g(y) + f(x) + u \tag{4}$$

In the anti-synchronization problem, we wish to find a feedback controller u so that

$$\lim_{t \to \infty} \|e(t)\| = 0 \text{ for all } e(0) \in R^n \tag{5}$$

We take as a candidate Lyapunov function

$$V(e) = e^T P e, \tag{6}$$

where P is a positive definite matrix.

Note that $V : R^n \to R$ is a positive definite function by construction. We assume that the parameters of the master and slave system are known and that the states of both systems (1) and (2) are measurable.

If we find a feedback controller u so that

$$\dot{V}(e) = -e^T Q e, \tag{7}$$

where Q is a positive definite matrix, then $\dot{V} : R^n \to R$ is a negative definite function.

Thus, by Lyapunov stability theory [35], the error dynamics (4) is globally exponentially stable. Hence, it follows that the states of the master system (1) and the slave system (2) will be globally and exponentially anti-synchronized.

3. SYSTEMS DESCRIPTION

The hyperchaotic Li system ([33], 2005) is described by the 4-D dynamics

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_4 \\
\dot{x}_2 &= dx_1 - x_1 x_3 + cx_2 \\
\dot{x}_3 &= -bx_3 + x_1 x_2 \\
\dot{x}_4 &= x_2 x_3 + \eta x_4
\end{aligned} \tag{8}$$

where x_1, x_2, x_3, x_4 are the states and a, b, c, d, η are constant, positive parameters of the system. The 4-D system (8) exhibits a hyperchaotic attractor, when the parameter values are taken as

$$a = 35, \quad b = 3, \quad c = 12, \quad d = 7, \quad \eta = 0.6 \tag{9}$$

The phase portrait of the hyperchaotic Li system is shown in Figure 1.

The hyperchaotic Lü system ([34], 2008) is described by the 4-D dynamics

$$\begin{aligned}
\dot{x}_1 &= \alpha(x_2 - x_1) + x_4 \\
\dot{x}_2 &= \gamma x_2 - x_1 x_3 \\
\dot{x}_3 &= -\beta x_3 + x_1 x_2 \\
\dot{x}_4 &= \varepsilon x_1 + \delta x_2 x_3
\end{aligned} \tag{10}$$

where x_1, x_2, x_3, x_4 are the states and $\alpha, \beta, \gamma, \delta, \varepsilon$ are constant, positive parameters of the system.

The 4-D system (10) exhibits a hyperchaotic attractor, when the parameter values are taken as

$$\alpha = 36, \quad \beta = 3, \quad \gamma = 20, \quad \delta = 0.1, \quad \varepsilon = 21 \tag{11}$$

The phase portrait of the hyperchaotic Lü system is shown in Figure 2.

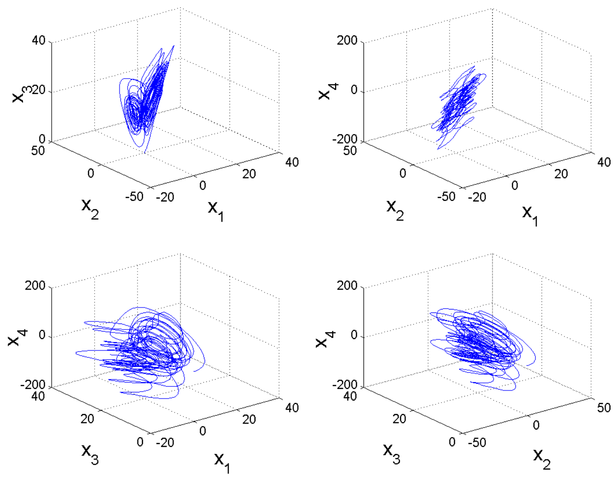

Figure 1. The Phase Portrait of the Hyperchaotic Li System

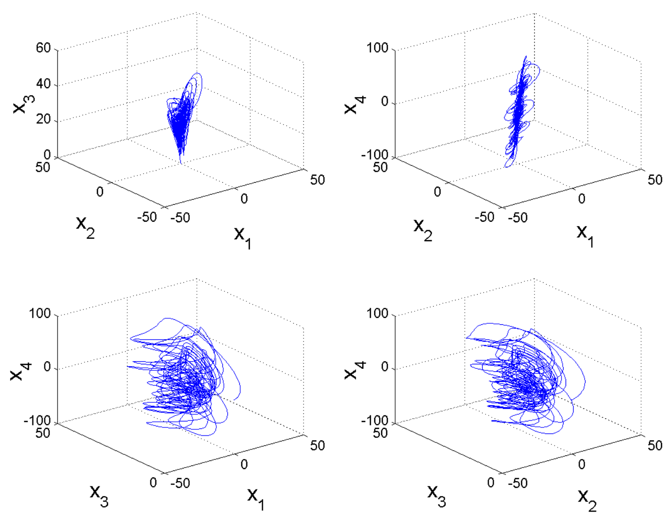

Figure 2. The Phase Portrait of the Hyperchaotic Lü System

4. Anti-Synchronization of Identical Hyperchaotic Li Systems via Active Control

4.1 Theoretical Results

In this section, we derive new results for the anti-synchronization of two identical hyperchaotic Li systems (2005) via active control.

As the master system, we take the hyperchaotic Li dynamics

$$
\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_4 \\
\dot{x}_2 &= dx_1 - x_1 x_3 + c x_2 \\
\dot{x}_3 &= -b x_3 + x_1 x_2 \\
\dot{x}_4 &= x_2 x_3 + \eta x_4
\end{aligned}
\tag{12}
$$

where x_1, x_2, x_3, x_4 are the states and a, b, c, d, η are positive parameters of the system.

As the slave system, we take the controlled hyperchaotic Li dynamics

$$
\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + y_4 + u_1 \\
\dot{y}_2 &= dy_1 - y_1 y_3 + c y_2 + u_2 \\
\dot{y}_3 &= -b y_3 + y_1 y_2 + u_3 \\
\dot{y}_4 &= y_2 y_3 + \eta y_4 + u_4
\end{aligned}
\tag{13}
$$

where y_1, y_2, y_3, y_4 are the states and u_1, u_2, u_3, u_4 are the active nonlinear controls.

The anti-synchronization error e is defined by

$$
e_i = y_i + x_i, \quad (i = 1, 2, 3, 4) \tag{14}
$$

The error dynamics is obtained as

$$
\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) + e_4 + u_1 \\
\dot{e}_2 &= de_1 + ce_2 - y_1 y_3 - x_1 x_3 + u_2 \\
\dot{e}_3 &= -be_3 + y_1 y_2 + x_1 x_2 + u_3 \\
\dot{e}_4 &= \eta e_4 + y_2 y_3 + x_2 x_3 + u_4
\end{aligned}
\tag{15}
$$

We choose the active nonlinear controller as

$$
\begin{aligned}
u_1 &= -a(e_2 - e_1) - e_4 - k_1 e_1 \\
u_2 &= -de_1 - ce_2 + y_1 y_3 + x_1 x_3 - k_2 e_2 \\
u_3 &= be_3 - y_1 y_2 - x_1 x_2 - k_3 e_3 \\
u_4 &= -\eta e_4 - y_2 y_3 - x_2 x_3 - k_4 e_4
\end{aligned}
\tag{16}
$$

where the gains $k_i, \ (i = 1, 2, 3, 4)$ are positive constants.

Substituting (16) into (15), the error dynamics simplifies to

$$
\dot{e}_i = -k_i e_i, \quad (i = 1, 2, 3, 4) \tag{17}
$$

Next, we prove the following result.

Theorem 4.1. The active nonlinear controller defined by (16) achieves the global and exponential anti-synchronization of the identical hyperchaotic Li systems (12) and (13).

Proof. Consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{18}$$

which is a positive definite function on R^4.

Differentiating (18) along the trajectories of (17), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \tag{19}$$

which is a negative definite function on R^4.

Hence, by Lyapunov stability theory [35], the error dynamics (17) is globally exponentially stable. This completes the proof.

4.2 Numerical Results

For MATLAB simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-8}$ is used to solve the differential equations (12) and (13) with the active nonlinear controller (16).
The feedback gains used in the equation (16) are chosen as

$$k_1 = 5, \;\; k_2 = 5, \;\; k_3 = 5, \;\; k_4 = 5$$

The parameters of the hyperchaotic Li systems are chosen as

$$a = 35, \quad b = 3, \;\; c = 12, \;\; d = 7, \;\; \eta = 0.6$$

The initial conditions of the master system (12) are chosen as

$$x_1(0) = 18, \;\; x_2(0) = 6, \;\; x_3(0) = -10, \;\; x_4(0) = -25$$

The initial conditions of the slave system (13) are chosen as

$$y_1(0) = 25, \;\; y_2(0) = -15, \;\; y_3(0) = 22, \;\; y_4(0) = -12$$

Figure 3 shows the anti-synchronization of the identical hyperchaotic Li systems.

Figure 4 shows the time-history of the anti-synchronization errors e_1, e_2, e_3, e_4.

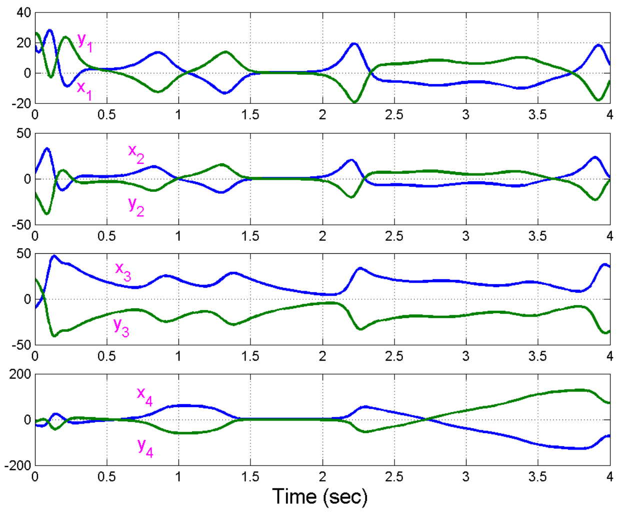

Figure 3. Anti-Synchronization of the Identical Hyperchaotic Li Systems

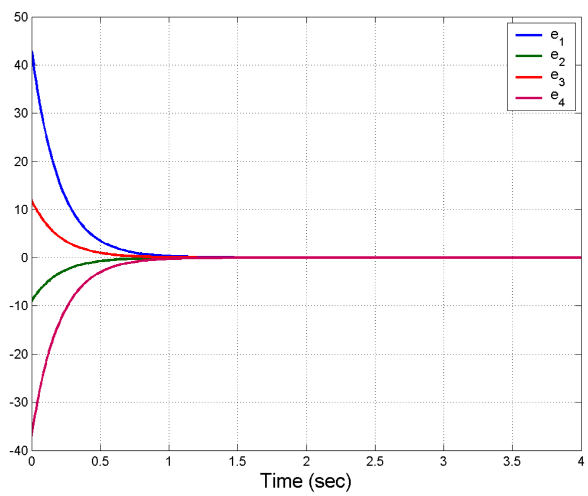

Figure 4. Time-History of the Anti-Synchronization Error

5. Anti- Synchronization of Identical Hyperchaotic Lü Systems by Active Control

5.1 Theoretical Results

In this section, we derive new results for the anti-synchronization of two identical hyperchaotic Lü systems (2008) via active control.

As the master system, we take the hyperchaotic Lü dynamics

$$\begin{aligned}
\dot{x}_1 &= \alpha(x_2 - x_1) + x_4 \\
\dot{x}_2 &= \gamma x_2 - x_1 x_3 \\
\dot{x}_3 &= -\beta x_3 + x_1 x_2 \\
\dot{x}_4 &= \varepsilon x_1 + \delta x_2 x_3
\end{aligned} \tag{20}$$

where x_1, x_2, x_3, x_4 are the states and $\alpha, \beta, \gamma, \delta, \varepsilon$ are positive parameters of the system.

As the slave system, we take the controlled hyperchaotic Lü dynamics

$$\begin{aligned}
\dot{y}_1 &= \alpha(y_2 - y_1) + y_4 + u_1 \\
\dot{y}_2 &= \gamma y_2 - y_1 y_3 + u_2 \\
\dot{y}_3 &= -\beta y_3 + y_1 y_2 + u_3 \\
\dot{y}_4 &= \varepsilon y_1 + \delta y_2 y_3 + u_4
\end{aligned} \tag{21}$$

where y_1, y_2, y_3, y_4 are the states and u_1, u_2, u_3, u_4 are the active nonlinear controls.

The anti-synchronization error e is defined by

$$e_i = y_i + x_i, \quad (i = 1, 2, 3, 4) \tag{22}$$

The error dynamics is obtained as

$$\begin{aligned}
\dot{e}_1 &= \alpha(e_2 - e_1) + e_4 + u_1 \\
\dot{e}_2 &= \gamma e_2 - y_1 y_3 - x_1 x_3 + u_2 \\
\dot{e}_3 &= -\beta e_3 + y_1 y_2 + x_1 x_2 + u_3 \\
\dot{e}_4 &= \varepsilon e_1 + \delta(y_2 y_3 + x_2 x_3) + u_4
\end{aligned} \tag{23}$$

We choose the active nonlinear controller as

$$\begin{aligned}
u_1 &= -\alpha(e_2 - e_1) - e_4 - k_1 e_1 \\
u_2 &= -\gamma e_2 + y_1 y_3 + x_1 x_3 - k_2 e_2 \\
u_3 &= \beta e_3 - y_1 y_2 - x_1 x_2 - k_3 e_3 \\
u_4 &= -\varepsilon e_1 - \delta(y_2 y_3 + x_2 x_3) - k_4 e_4
\end{aligned} \tag{24}$$

where the gains $k_i, \ (i = 1, 2, 3, 4)$ are positive constants.

Substituting (24) into (23), the error dynamics simplifies to

$$\dot{e}_i = -k_i e_i, \quad (i = 1, 2, 3, 4) \tag{25}$$

Next, we prove the following result.

Theorem 5.1. The active nonlinear controller defined by (24) achieves the global and exponential anti-synchronization of the identical hyperchaotic Lü systems (20) and (21).

Proof. Consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{26}$$

which is a positive definite function on R^4.

Differentiating (26) along the trajectories of (25), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \tag{27}$$

which is a negative definite function on R^4.

Hence, by Lyapunov stability theory [35], the error dynamics (25) is globally exponentially stable. This completes the proof.

5.2 Numerical Results

For MATLAB simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-8}$ is used to solve the differential equations (20) and (21) with the active nonlinear controller (24).

The feedback gains used in the equation (24) are chosen as

$$k_1 = 5, \quad k_2 = 5, \quad k_3 = 5, \quad k_4 = 5$$

The parameters of the hyperchaotic Lü systems are chosen as

$$\alpha = 36, \quad \beta = 3, \quad \gamma = 20, \quad \delta = 0.1, \quad \varepsilon = 21$$

The initial conditions of the master system (20) are chosen as

$$x_1(0) = 4, \quad x_2(0) = 26, \quad x_3(0) = -20, \quad x_4(0) = 15$$

The initial conditions of the slave system (21) are chosen as

$$y_1(0) = 20, \quad y_2(0) = -17 \quad y_3(0) = -12, \quad y_4(0) = 22$$

Figure 5 shows the anti-synchronization of the identical hyperchaotic Lü systems.

Figure 6 shows the time-history of the anti-synchronization errors e_1, e_2, e_3, e_4.

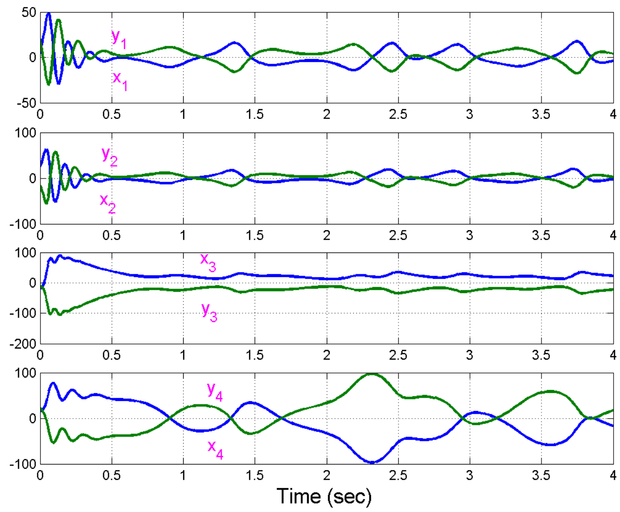

Figure 5. Anti-Synchronization of the Identical Hyperchaotic Lü Systems

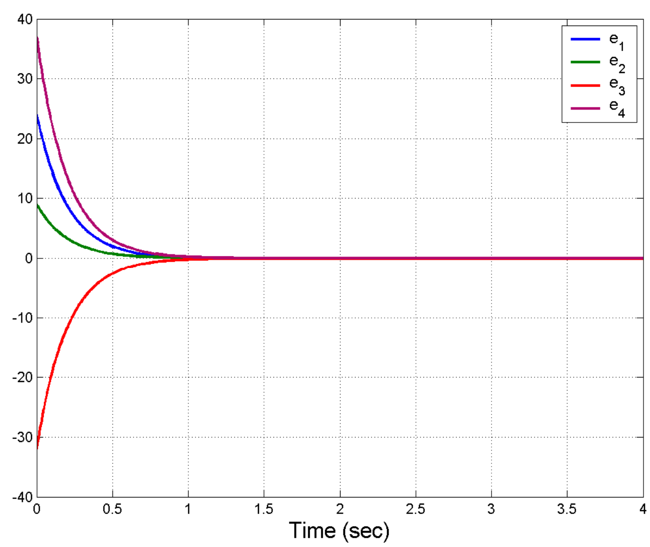

Figure 6. Time-History of the Anti-Synchronization Error

6. Anti- Synchronization of Non-Identical Hyperchaotic Li and Hyperchaotic Lü Systems via Active Control

6.1 Theoretical Results

In this section, we apply the active nonlinear control method for the anti-synchronization of the non-identical hyperchaotic Li system (2005) and hyperchaotic Lü system (2008).

As the master system, we take the hyperchaotic Li dynamics

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + x_4 \\ \dot{x}_2 &= dx_1 - x_1x_3 + cx_2 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \\ \dot{x}_4 &= x_2x_3 + \eta x_4\end{aligned} \tag{28}$$

where x_1, x_2, x_3, x_4 are the states and a, b, c, d, η are positive parameters of the system.

As the slave system, we take the controlled hyperchaotic Lü dynamics

$$\begin{aligned}\dot{y}_1 &= \alpha(y_2 - y_1) + y_4 + u_1 \\ \dot{y}_2 &= \gamma y_2 - y_1y_3 + u_2 \\ \dot{y}_3 &= -\beta y_3 + y_1y_2 + u_3 \\ \dot{y}_4 &= \varepsilon y_1 + \delta y_2y_3 + u_4\end{aligned} \tag{29}$$

where y_1, y_2, y_3, y_4 are the states, $\alpha, \beta, \gamma, \delta, \varepsilon$ are positive parameters and u_1, u_2, u_3, u_4 are the active nonlinear controls. The anti-synchronization error e is defined by

$$e_i = y_i + x_i, \quad (i = 1, 2, 3, 4) \tag{30}$$

The error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= \alpha(y_2 - y_1) + a(x_2 - x_1) + e_4 + u_1 \\ \dot{e}_2 &= \gamma y_2 + dx_1 + cx_2 - y_1y_3 - x_1x_3 + u_2 \\ \dot{e}_3 &= -\beta y_3 - bx_3 + y_1y_2 + x_1x_2 + u_3 \\ \dot{e}_4 &= \varepsilon y_1 + \eta x_4 + \delta y_2y_3 + x_2x_3 + u_4\end{aligned} \tag{31}$$

We choose the active nonlinear controller as

$$\begin{aligned}u_1 &= -\alpha(y_2 - y_1) - a(x_2 - x_1) - e_4 - k_1e_1 \\ u_2 &= -\gamma y_2 - dx_1 - cx_2 + y_1y_3 + x_1x_3 - k_2e_2 \\ u_3 &= \beta y_3 + bx_3 - y_1y_2 - x_1x_2 - k_3e_3 \\ u_4 &= -\varepsilon y_1 - \eta x_4 - \delta y_2y_3 - x_2x_3 - k_4e_4\end{aligned} \tag{32}$$

where the gains $k_i, \ (i = 1, 2, 3, 4)$ are positive constants.

Substituting (32) into (31), the error dynamics simplifies to

$$\dot{e}_i = -k_ie_i, \quad (i = 1, 2, 3, 4) \tag{33}$$

Next, we prove the following result.

Theorem 6.1. The active nonlinear controller defined by (32) achieves the global and exponential anti-synchronization of the hyperchaotic Li system (28) and hyperchaotic Lü system (29).

Proof. Consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{34}$$

which is a positive definite function on R^4.

Differentiating (34) along the trajectories of (44), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \tag{35}$$

which is a negative definite function on R^4.

Hence, by Lyapunov stability theory [35], the error dynamics (33) is globally exponentially stable. This completes the proof.

6.2 Numerical Results

For MATLAB simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-8}$ is used to solve the differential equations (28) and (28) with the active nonlinear controller (24).

The feedback gains used in the equation (32) are chosen as $k_i = 5, (i = 1, 2, 3, 4)$.

The parameters of the hyperchaotic Li and hyperchaotic Lü systems are chosen as

$$a = 35, \;\; b = 3, \; c = 12, \; d = 7, \; \eta = 0.6, \; \alpha = 36, \; \beta = 3, \; \gamma = 20, \; \delta = 0.1, \; \varepsilon = 21$$

The initial conditions of the master system (28) are chosen as

$$x_1(0) = 8, \;\; x_2(0) = 7, \;\; x_3(0) = -10, \;\; x_4(0) = 12$$

The initial conditions of the slave system (29) are chosen as

$$y_1(0) = 32, \;\; y_2(0) = -17, \;\; y_3(0) = -22, \;\; y_4(0) = 4$$

Figure 7 shows the anti-synchronization of the hyperchaotic Li and hyperchaotic Lü systems.

Figure 8 shows the time-history of the anti-synchronization errors e_1, e_2, e_3, e_4.

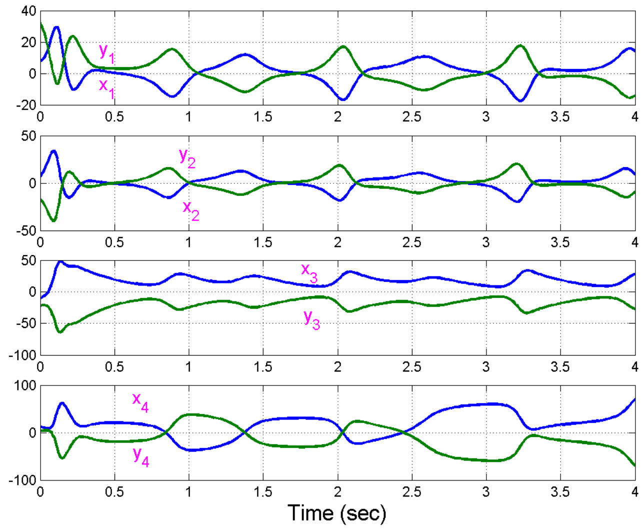

Figure 7. Anti-Synchronization of the Hyperchaotic Li and Hyperchaotic Lü Systems

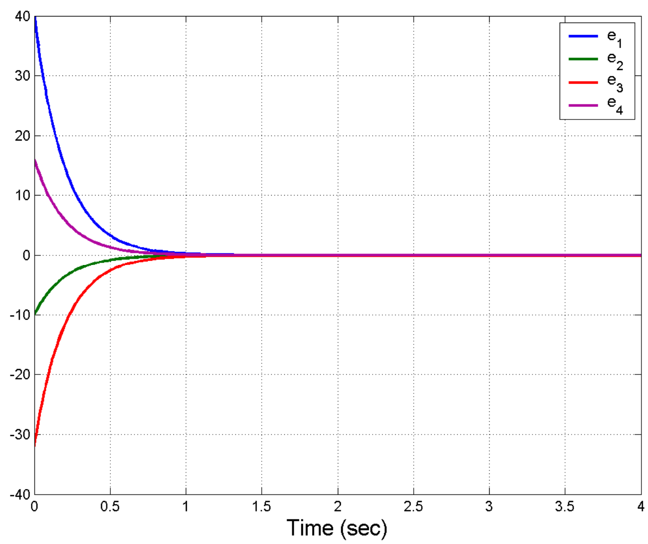

Figure 8. Time-History of the Anti-Synchronization Error

7. CONCLUSIONS

In this paper, active control method was applied to derive anti-synchronization results for the identical hyperchaotic Li systems (2005), identical hyperchaotic Lü systems (2008), and non-identical hyperchaotic Li and hyperchaotic Lü systems. The stability results validating the anti-synchronizing controllers have been proved using Lyapunov stability theory. Numerical simulations using MATLAB were presented to validate and demonstrate the efficiency of the anti-synchronization schemes derived for the hyperchaotic Li and hyperchaotic Lü systems.

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