ANTI-SYNCHRONIZATION OF HYPERCHAOTIC PANG AND HYPERCHAOTIC WANG-CHEN SYSTEMS VIA ACTIVE CONTROL

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ABSTRACT

Hyperchaotic systems are chaotic systems having more than one positive Lyapunov exponent and they have important applications in secure data transmission and communication. This paper applies active control method for the synchronization of identical and different hyperchaotic Pang systems (2011) and hyperchaotic Wang-Chen systems (2008). Main results are proved with the stability theorems of Lypuanov stability theory and numerical simulations are plotted using MATLAB to show the synchronization of hyperchaotic systems addressed in this paper.

KEYWORDS

Anti-Synchronization, Active Control, Chaos, Hyperchaos, Hyperchaotic Systems.

1. INTRODUCTION

Hyperchaotic systems have a lot of important applications in several fields in Science and Engineering. They are chaotic systems with more than one positive Lyapunov exponent. The hyperchaotic systemwas first found by O.E.Rössler ([1], 1979).

Hyperchaotic systems have attractive features like high security, high capacity and high efficiency and they find miscellaneous applications in several areas like neural networks [2],oscillators [3], secure communication [4-5], encryption [6], synchronization [7], etc.
There are many important methods available in the literature for synchronization and anti-synchronization like PC method [8], OGY method [9], backstepping method [10-12], sliding control method [13-15], active control method [16-18], adaptive control method [19-20], sampled-data feedback control method [21], time-delay feedback method [22], etc.

The anti-synchronization problem deals witha pair of chaotic systems called the master and slave systems, where the design goal is to anti-synchronize their states, i.e. the sum of the states of the master and slave systems approach to zero asymptotically.

This paper focuses upon active controller design for the anti-synchronization of hyperchaotic Pang systems ([23], 2011) and hyperchaotic Wang-Chen systems ([24], 2008). The main results derived in this paper were proved using Lyapunov stability theory [25].

Using active control method, new results have been derived for the anti-synchronization of identical hyperchaotic Pang systems, identical hyperchaotic Wang-Chen systems and non-identical hyperchaotic Pang and hyperchaotic Wang-Chen systems. Numerical simulations were shown using MATLAB to illustrate the main results derived in this paper.

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2. Problem Statement

The *master system* is described by the chaotic dynamics

$$\dot{x} = Ax + f(x) \tag{1}$$

where A is the $n \times n$ matrix of the system parameters and $f : R^n \to R^n$ is the nonlinear part. The *slave system* is described by the chaotic dynamics

$$\dot{y} = By + g(y) + u \tag{2}$$

where B is the $n \times n$ matrix of the system parameters, $g : R^n \to R^n$ is the nonlinear part and $u \in R^n$ is the active controller to be designed.

For the pair of chaotic systems (1) and (2), the anti-synchronization problem aims to design a feedback controller u, which *anti-synchronizes* their states for all $x(0), y(0) \in R^n$.

The*anti-synchronization error*is defined as

$$e = y + x, \tag{3}$$

Theerror dynamics is obtained as

$$\dot{e} = By + Ax + g(y) + f(x) + u \tag{4}$$

The design goal is to find a feedback controller u so that

$$\lim_{t \to \infty} \left\| e(t) \right\| = 0 \text{ for all } e(0) \in R^{\boldsymbol{n}} \tag{5}$$

Using the matrix method, we consider a candidate Lyapunov function

$$V(e) = e^T Pe, \tag{6}$$

where P is a positive definite matrix.

It is noted that $V : R^n \to R$ is a positive definite function.
If we find a feedback controller u so that

$$\dot{V}(e) = -e^T Qe, \tag{7}$$

where Q is a positive definite matrix, then $\dot{V} : R^n \to R$ is a negative definite function.
Thus, by Lyapunov stability theory [15], the error dynamics (4) is globally exponentially stable.

Hence, the states of the chaotic systems (1) and (2) will be globally and exponentially anti-synchronized for all initial conditions $x(0), y(0) \in R^n$.

3. HYPERCHAOTIC SYSTEMS

The hyperchaotic Pang system ([23], 2011) is given by

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= cx_2 - x_1x_3 + x_4 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \\ \dot{x}_4 &= -d(x_1 + x_2)\end{aligned} \tag{8}$$

where a, b, c, d are constant, positive parameters of the system.

The Pang system (8) exhibits a hyperchaotic attractor for the parametric values

$$a = 36, \quad b = 3, \quad c = 20, \quad d = 2 \tag{9}$$

The Lyapunov exponents of the system (8) for the parametric values in (9) are

$$\lambda_1 = 1.4106, \quad \lambda_2 = 0.1232, \quad \lambda_3 = 0, \quad \lambda_4 = -20.5339 \tag{10}$$

Since there are two positive Lyapunov exponents in (10), the Pang system (8) is hyperchaotic for the parametric values (9).

The phase portrait of the hyperchaotic Pang system is described in Figure 1.

The hyperchaotic Wang-Chen system ([24], 2008) is given by

$$\begin{aligned}\dot{x}_1 &= \alpha(x_2 - x_1) + x_2x_3 \\ \dot{x}_2 &= \gamma x_1 - x_1x_3 - x_2 - 0.5x_4 \\ \dot{x}_3 &= -3x_3 + x_1x_2 \\ \dot{x}_4 &= \beta x_4 + 0.5x_1x_3\end{aligned} \tag{11}$$

where α, β, γ are constant, positive parameters of the system.
The Wang system (11) exhibits a hyperchaotic attractor for the parametric values

$$\alpha = 40, \quad \beta = 1.7, \quad \gamma = 88 \tag{12}$$

The Lyapunov exponents of the system (9) for the parametric values in (12) are

$$\lambda_1 = 3.2553, \quad \lambda_2 = 1.4252, \quad \lambda_3 = 0, \quad \lambda_4 = -46.9794 \tag{13}$$

Since there are two positive Lyapunov exponents in (13), the Wang-Chen system (11) is hyperchaotic for the parametric values (12).

The phase portrait of the hyperchaotic Wang-Chen system is described in Figure 2.

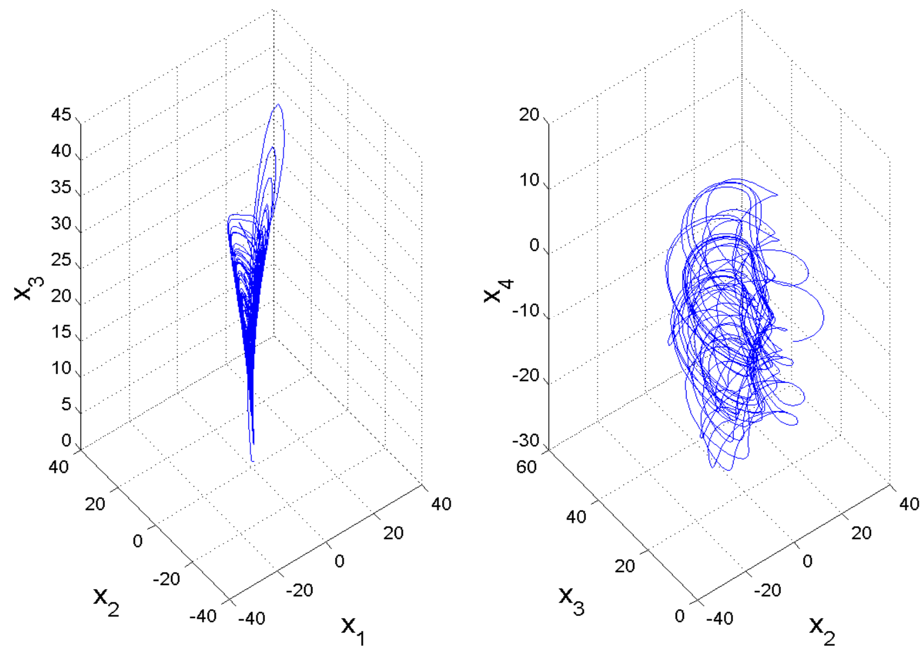

Figure 1. The Phase Portrait of the Hyperchaotic Pang System

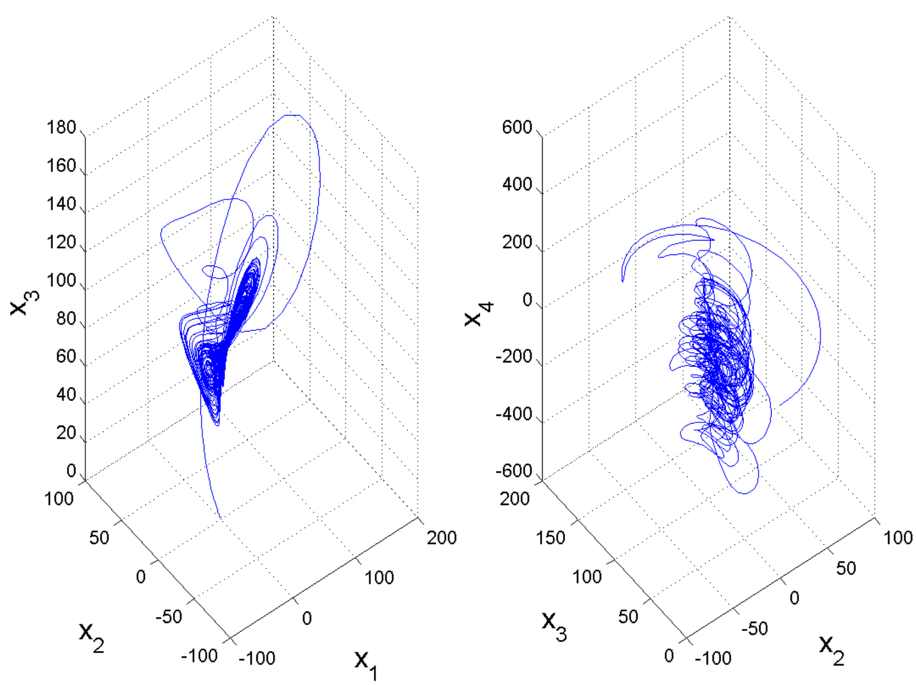

Figure 2. The Phase Portrait of the Hyperchaotic Wang-Chen System

4. ANTI-SYNCHRONIZATION OF HYPERCHAOTIC PANG SYSTEMS VIA ACTIVE CONTROL

In this section, we investigate the problem of anti-synchronization of two identical hyperchaotic Pang systems (2011) and derive new results via active control.

The master system is the hyperchaotic Pangsystem given by

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) \\
\dot{x}_2 &= cx_2 - x_1 x_3 + x_4 \\
\dot{x}_3 &= -bx_3 + x_1 x_2 \\
\dot{x}_4 &= -d(x_1 + x_2)
\end{aligned} \tag{14}$$

where a, b, c, d are positive parameters of the system and $x \in R^4$ is the state.

The slave system is the controlled hyperchaotic Pangsystem given by

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + u_1 \\
\dot{y}_2 &= cy_2 - y_1 y_3 + y_4 + u_2 \\
\dot{y}_3 &= -by_3 + y_1 y_2 + u_3 \\
\dot{y}_4 &= -d(y_1 + y_2) + u_4
\end{aligned} \tag{15}$$

where $y \in R^4$ is the state and u_1, u_2, u_3, u_4 are the active controllers to be designed.
For the anti-synchronization, the error e is defined as

$$e_i = y_i + x_i, \quad (i = 1, 2, 3, 4) \tag{16}$$

We obtain the error dynamics as

$$\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) + u_1 \\
\dot{e}_2 &= ce_2 + e_4 - y_1 y_3 - x_1 x_3 + u_2 \\
\dot{e}_3 &= -be_3 + y_1 y_2 + x_1 x_2 + u_3 \\
\dot{e}_4 &= -d(e_1 + e_2) + u_4
\end{aligned} \tag{17}$$

The active controller to achieve anti-synchronization is chosen as

$$\begin{aligned}
u_1 &= -a(e_2 - e_1) - k_1 e_1 \\
u_2 &= -ce_2 - e_4 + y_1 y_3 + x_1 x_3 - k_2 e_2 \\
u_3 &= be_3 - y_1 y_2 - x_1 x_2 - k_3 e_3 \\
u_4 &= d(e_1 + e_2) - k_4 e_4
\end{aligned} \tag{18}$$

where $k_i, \ (i = 1, 2, 3, 4)$ are positive gains.

By the substitution of (18) into (17), the error dynamics is simplified as

$$\dot{e}_i = -k_i e_i, \quad (i = 1,2,3,4) \tag{19}$$

Thus, we obtain the following result.

Theorem 4.1 The nonlinear controller defined by Eq. (18) achieves global and exponential anti-synchronization of the identical hyperchaotic Pang systems (14) and (15) for all initial conditions $x(0), y(0) \in R^4$.

Proof. The proof is via Lyapunov stability theorem [25] for global exponential stability.

We take the quadratic Lyapunov function

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{20}$$

which is a positive definite function on R^4.

When we differentiate (18) along the trajectories of (17), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \tag{21}$$

which is a negative definite function on R^4. Hence, the error dynamics (19) is globally exponentially stable for all $e(0) \in R^4$. This completes the proof.

Next, we illustrate our anti-synchronization results with MATLAB simulations.The 4thorder Runge-Kutta method with time-step $h = 10^{-8}$ has been applied to solve the hyperchaotic Pang systems (14) and (15) with the active nonlinear controller defined by (18).

The feedback gains in the active controller (18) are taken as $k_i = 5, \ (i = 1,2,3,4)$.

The parameters of the hyperchaotic Pang systems are taken as in the hyperchaotic case, *i.e.*

$$a = 36, \quad b = 3, \quad c = 20, \quad d = 2$$

For simulations, the initial conditions of the master system (14) are taken as

$$x_1(0) = 12, \quad x_2(0) = 20, \quad x_3(0) = -14, \quad x_4(0) = -27$$

Also, the initial conditions of the slave system (15) are taken as

$$y_1(0) = 32, \quad y_2(0) = 15, \quad y_3(0) = 20, \quad y_4(0) = -10$$

Figure 3 depicts the anti-synchronization of the identical hyperchaotic Pang systems.

Figure 4 depicts the time-history of the anti-synchronization errors e_1, e_2, e_3, e_4.

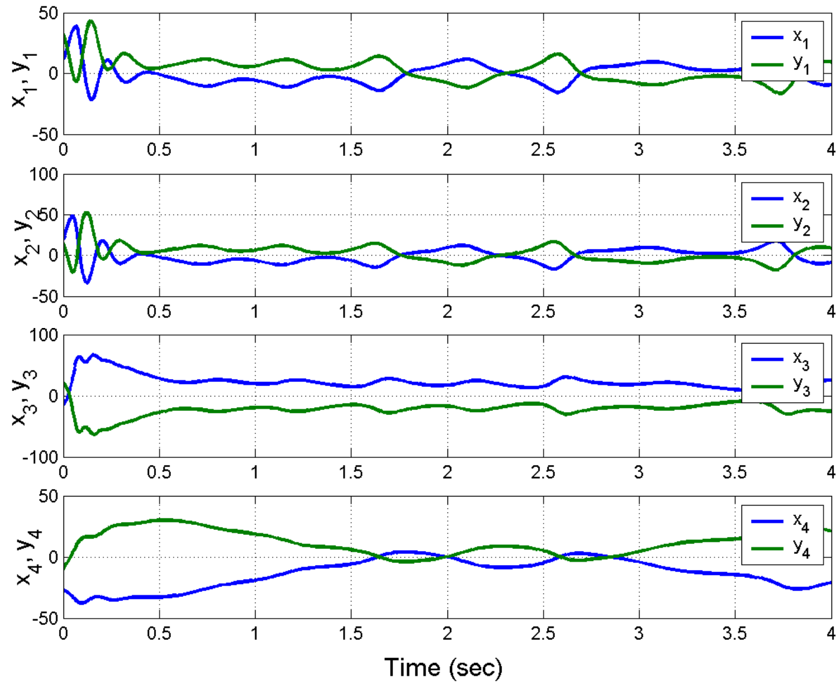

Figure 3. Anti-Synchronization of Identical Hyperchaotic Pang Systems

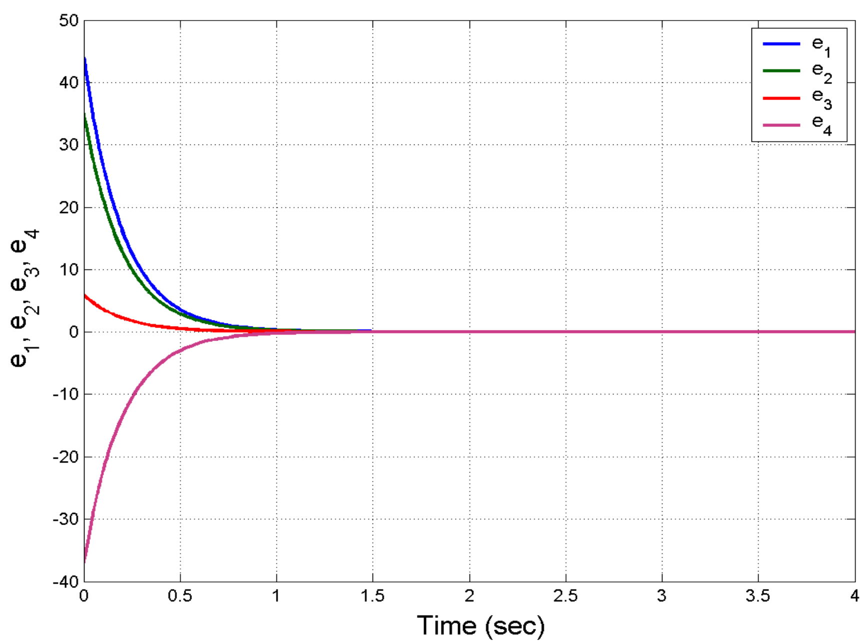

Figure 4. Time-History of the Anti-Synchronization Errors e_1, e_2, e_3, e_4

5. ANTI-SYNCHRONIZATION OF HYPERCHAOTIC WANG-CHEN SYSTEMS VIA ACTIVE CONTROL

In this section, we investigate the problem of anti-synchronization of two identical hyperchaotic Wang-Chen systems (2008) and derive new results via active control.

The master system is the hyperchaotic Wang-Chensystem given by

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 &= \gamma x_1 - x_1 x_3 - x_2 - 0.5 x_4 \\ \dot{x}_3 &= -3x_3 + x_1 x_2 \\ \dot{x}_4 &= \beta x_4 + 0.5 x_1 x_3 \end{aligned} \tag{22}$$

where α, β, γ are positive parameters of the system and $x \in R^4$ is the state.

The slave system is the controlled hyperchaotic Wang-Chensystem given by

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + y_2 y_3 + u_1 \\ \dot{y}_2 &= \gamma y_1 - y_1 y_3 - y_2 - 0.5 y_4 + u_2 \\ \dot{y}_3 &= -3y_3 + y_1 y_2 + u_3 \\ \dot{y}_4 &= \beta y_4 + 0.5 y_1 y_3 + u_4 \end{aligned} \tag{23}$$

where $y \in R^4$ is the state and u_1, u_2, u_3, u_4 are the active controllers to be designed.
For the anti-synchronization, the error e is defined as

$$e_i = y_i + x_i, \quad (i = 1, 2, 3, 4) \tag{24}$$

We obtain the error dynamics as

$$\begin{aligned} \dot{e}_1 &= \alpha(e_2 - e_1) + y_2 y_3 + x_2 x_3 + u_1 \\ \dot{e}_2 &= \gamma e_1 - e_2 - 0.5 e_4 - y_1 y_3 - x_1 x_3 + u_2 \\ \dot{e}_3 &= -3e_3 + y_1 y_2 + x_1 x_2 + u_3 \\ \dot{e}_4 &= \beta e_4 + 0.5(y_1 y_3 + x_1 x_3) + u_4 \end{aligned} \tag{25}$$

The active controller to achieve anti-synchronization is chosen as

$$\begin{aligned} u_1 &= -\alpha(e_2 - e_1) - y_2 y_3 - x_2 x_3 - k_1 e_1 \\ u_2 &= -\gamma e_1 + e_2 + 0.5 e_4 + y_1 y_3 + x_1 x_3 - k_2 e_2 \\ u_3 &= 3e_3 - y_1 y_2 - x_1 x_2 - k_3 e_3 \\ u_4 &= -\beta e_4 - 0.5(y_1 y_3 + x_1 x_3) - k_4 e_4 \end{aligned} \tag{26}$$

where $k_i, \; (i = 1, 2, 3, 4)$ are positive gains.

By the substitution of (26) into (25), the error dynamics is simplified as

$$\dot{e}_i = -k_i e_i, \quad (i = 1, 2, 3, 4) \tag{27}$$

Thus, we obtain the following result.

Theorem 5.1 The nonlinear controller defined by Eq. (26) achieves global and exponential anti-synchronization of the identical hyperchaotic Wang-Chensystems (22) and (23) for all initial conditions $x(0), y(0) \in R^4$.

Proof. The proof is via Lyapunov stability theorem [25] for global exponential stability.

We take the quadratic Lyapunov function

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{28}$$

which is a positive definite function on R^4.

When we differentiate (26) along the trajectories of (25), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \tag{29}$$

which is a negative definite function on R^4. Hence, the error dynamics (27) is globally exponentially stable for all $e(0) \in R^4$. This completes the proof.

Next, we illustrate our anti-synchronization results with MATLAB simulations. The 4$^{\text{th}}$order Runge-Kutta method with time-step $h = 10^{-8}$ has been applied to solve the hyperchaotic Wang-Chen systems (22) and (23) with the active nonlinear controller defined by (26).

The feedback gains in the active controller (26) are taken as $k_i = 5, \ (i = 1, 2, 3, 4)$.

The parameters of the hyperchaotic WCsystems are taken as in the hyperchaotic case, *i.e.*

$$\alpha = 40, \quad \beta = 1.7, \quad \gamma = 88$$

For simulations, the initial conditions of the master system (22) are taken as

$$x_1(0) = -7, \quad x_2(0) = 12, \quad x_3(0) = -15, \quad x_4(0) = 17$$

Also, the initial conditions of the slave system (23) are taken as

$$y_1(0) = 30, \quad y_2(0) = 25, \quad y_3(0) = -18, \quad y_4(0) = -5$$

Figure 5depicts the anti-synchronization of the identical hyperchaotic Wang-Chen systems.
Figure 6depicts the time-history of the anti-synchronization errors e_1, e_2, e_3, e_4.

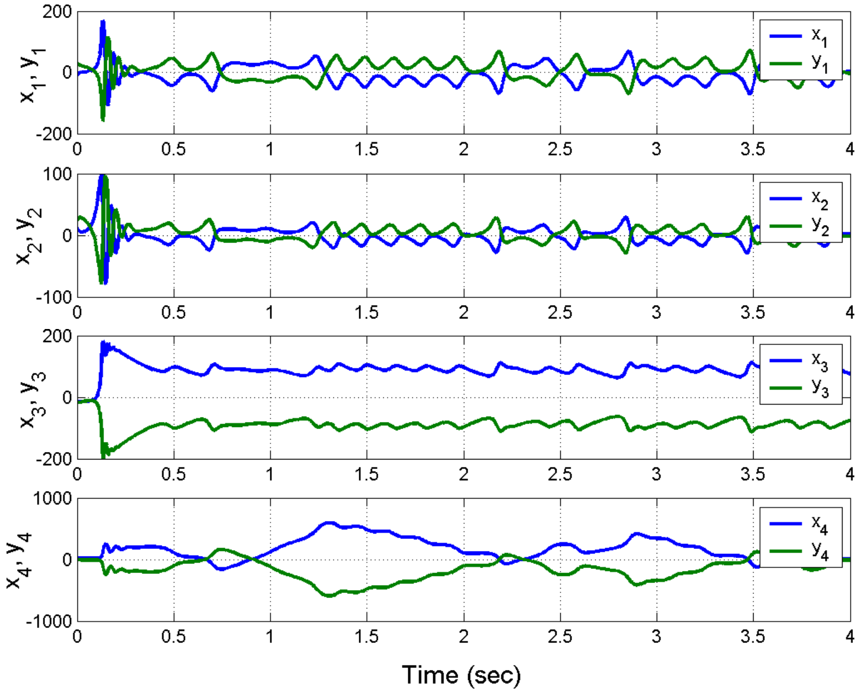

Figure 5. Anti-Synchronization of Identical Hyperchaotic Wang-Chen Systems

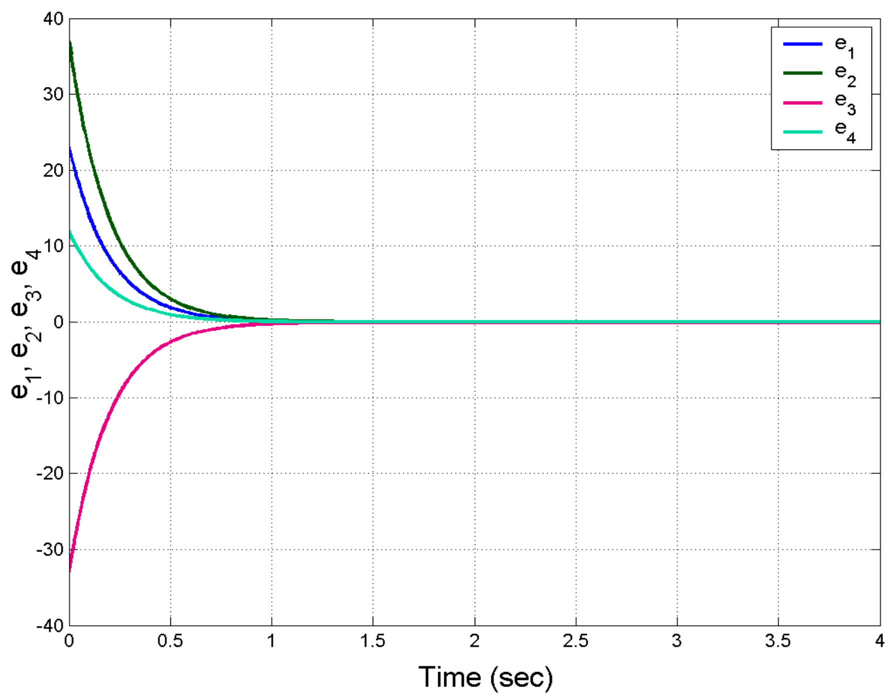

Figure 6. Time-History of the Anti-Synchronization Errors e_1, e_2, e_3, e_4

6. ANTI-SYNCHRONIZATION OF HYPERCHAOTIC PANG AND HYPERCHAOTIC WANG-CHEN SYSTEMS VIA ACTIVE CONTROL

In this section, we derive new results for the problem of anti-synchronization of hyperchaotic Pang (2011) and hyperchaotic Wang-Chen systems (2008) via active control.
The master system is the hyperchaotic Pang system given by

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1)\\ \dot{x}_2 &= cx_2 - x_1x_3 + x_4\\ \dot{x}_3 &= -bx_3 + x_1x_2\\ \dot{x}_4 &= -d(x_1 + x_2)\end{aligned} \tag{30}$$

where a,b,c,d are positive parameters of the system and $x \in R^4$ is the state.
The slave system is the controlled hyperchaotic Wang-Chensystem given by

$$\begin{aligned}\dot{y}_1 &= \alpha(y_2 - y_1) + y_2y_3 + u_1\\ \dot{y}_2 &= \gamma y_1 - y_1y_3 - y_2 - 0.5y_4 + u_2\\ \dot{y}_3 &= -3y_3 + y_1y_2 + u_3\\ \dot{y}_4 &= \beta y_4 + 0.5y_1y_3 + u_4\end{aligned} \tag{31}$$

where α,β,γ are positive parameters, $y \in R^4$ is the state and u_1,u_2,u_3,u_4 are the active controllers to be designed.
For the anti-synchronization, the error e is defined as

$$e_i = y_i + x_i, \quad (i = 1,2,3,4) \tag{32}$$

We obtain the error dynamics as

$$\begin{aligned}\dot{e}_1 &= \alpha(y_2 - y_1) + a(x_2 - x_1) + y_2y_3 + u_1\\ \dot{e}_2 &= \gamma y_1 - y_2 - 0.5y_4 + cx_2 + x_4 - y_1y_3 - x_1x_3 + u_2\\ \dot{e}_3 &= -3y_3 - bx_3 + y_1y_2 + x_1x_2 + u_3\\ \dot{e}_4 &= \beta y_4 - d(x_1 + x_2) + 0.5y_1y_3 + u_4\end{aligned} \tag{33}$$

The active controller to achieve anti-synchronization is chosen as

$$\begin{aligned}u_1 &= -\alpha(y_2 - y_1) - a(x_2 - x_1) - y_2y_3 - k_1e_1\\ u_2 &= -\gamma y_1 + y_2 + 0.5y_4 - cx_2 - x_4 + y_1y_3 + x_1x_3 - k_2e_2\\ u_3 &= 3y_3 + bx_3 - y_1y_2 - x_1x_2 - k_3e_3\\ u_4 &= -\beta y_4 + d(x_1 + x_2) - 0.5y_1y_3 - k_4e_4\end{aligned} \tag{34}$$

where $k_i, \ (i = 1,2,3,4)$ are positive gains.

By the substitution of (34) into (33), the error dynamics is simplified as

$$\dot{e}_i = -k_i e_i, \quad (i = 1, 2, 3, 4) \tag{35}$$

Thus, we obtain the following result.

Theorem 6.1 The nonlinear controller defined by Eq. (34) achieves global and exponential anti-synchronization of the non-identical hyperchaotic Pang system (30) and the controlled hyperchaotic Wang-Chen system (31) for all initial conditions $x(0), y(0) \in R^4$.

Proof. The proof is via Lyapunov stability theorem [25] for global exponential stability.

We take the quadratic Lyapunov function

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{36}$$

which is a positive definite function on R^4.

When we differentiate (34) along the trajectories of (33), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \tag{37}$$

which is a negative definite function on R^4. Hence, the error dynamics (35) is globally exponentially stable for all $e(0) \in R^4$. This completes the proof.

Next, we illustrate our anti-synchronization results with MATLAB simulations. The 4thorder Runge-Kutta method with time-step $h = 10^{-8}$ has been applied to solve the hyperchaotic systems (30) and (31) with the active nonlinear controller defined by (34).

The feedback gains in the active controller (34) are taken as $k_i = 5, \ (i = 1, 2, 3, 4)$.
The parameters of the hyperchaotic Pang and hyperchaotic WC systems are taken as in the hyperchaotic case, *i.e.*

$$a = 36, \quad b = 3, \quad c = 20, \quad d = 2, \quad \alpha = 40, \quad \beta = 1.7, \quad \gamma = 88$$

For simulations, the initial conditions of the master system (30) are taken as

$$x_1(0) = 27, \ \ x_2(0) = -16, \ \ x_3(0) = -12, \ \ x_4(0) = 31$$

Also, the initial conditions of the slave system (31) are taken as

$$y_1(0) = 10, \ \ y_2(0) = 5, \ \ y_3(0) = -28, \ \ y_4(0) = -14$$

Figure 7depicts the anti-synchronization of the hyperchaotic Pang and Wang-Chen systems.

Figure 8depicts the time-history of the anti-synchronization errors e_1, e_2, e_3, e_4.

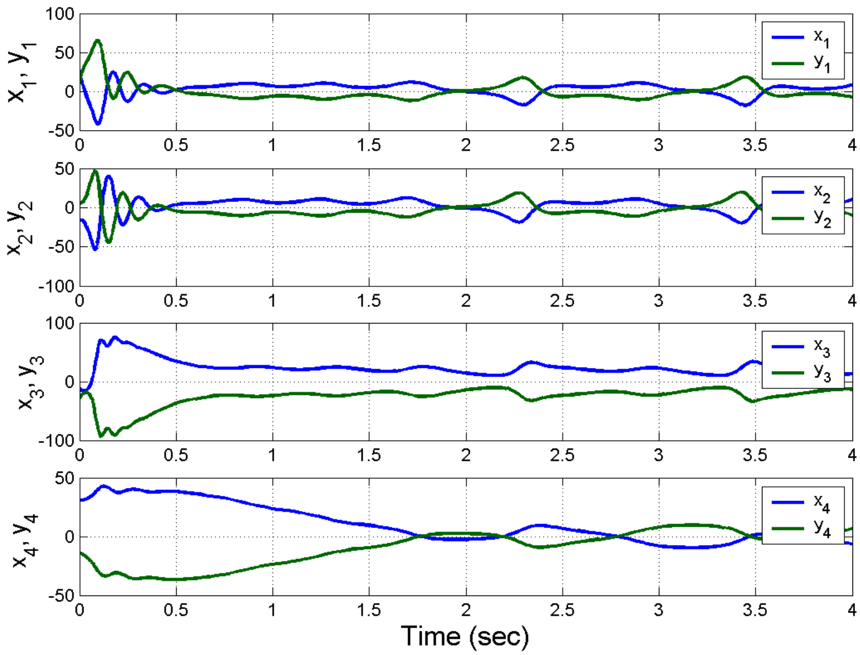

Figure 7. Anti-Synchronization of Identical Hyperchaotic Pang and Hyperchaotic WC Systems

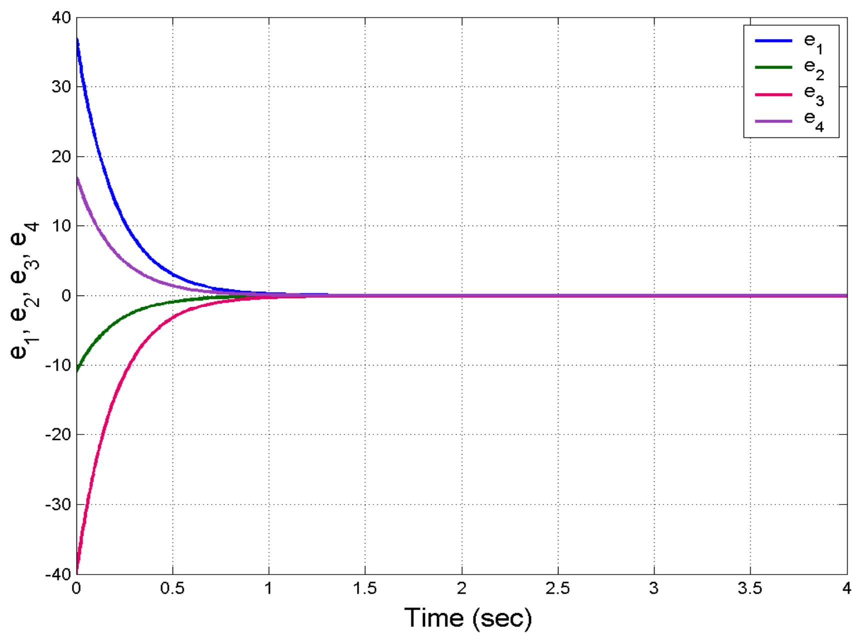

Figure 8. Time-History of the Anti-Synchronization Errors e_1, e_2, e_3, e_4

7. CONCLUSIONS

This paper derived new results for the anti-synchronization of hyperchaotic Pang systems (2011) and hyperchaotic Wang-Chen systems (2008) using active control method. Explicitly, active control laws were derived for globally anti-synchronizing the states of identical hyperchaotic Pang systems, identical hyperchaotic Wang-Chen systems and non-identical hyperchaotic Pang and Wang-Chen systems. The main results validating the anti-synchronizing active controllers were proved using Lyapunov stability theory. MATLAB simulations were shown to illustrate the anti-synchronization results derived in this paper for hyperchaotic Pang and hyperchaotic Wang-Chen systems.

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