Efficient Decoding for Extended Alamouti Space-Time Block code

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Abstract

Space-time coding can achieve transmit diversity and power gain over spatially uncoded systems without sacrificing the bandwidth. There are various approaches in coding structures, including space-time block codes. A class of space-time block codes namely quasi-orthogonal space-time block code can achieve the full rate, but there are also interference terms resulting from neighboring signals during signal detection. This causes an increase in decoding complexity and a decrease in performance gain. In this paper, we demonstrate an efficient method to reduce decoding complexity and improved performance.

I. INTRODUCTION

Space-time coding is a coding technique designed for use with multiple transmit antennas. Coding is performed in both spatial and temporal domains to introduce correlation between signals transmitted from various antennas at various time periods. Space-time coding can achieve transmit diversity and power gain over spatially uncoded systems without sacrificing the bandwidth. A central issue in space-time coding schemes is the exploitation of multipath effects in order to achieve high spectral efficiencies and performance gains. There are various approaches in space-time coding structures, including *space-time block codes* (STBC), *space-time trellis codes* (STTC), spacetime turbo trellis codes and *layered space-time* (LST) codes. STBC gained lot of interests after their introduction by alamouti [1] and Foschini [2] and are the most popular space-time coding techniques.

The STBC proposed by alamouti for two transmit antenna provide full rate and full diversity rate and are also known as *orthogonal space time block codes* (OSTBC). Tarokh, Jafarkhani, and Calderbank proposed generalized OSTBC for more than two antennas using theory of orthogonal designs [4]–[6]. However, such schemes cannot achieve the full rate. In order to gain the advantages of OSTBCs schemes with properties close to such optimal codes but with higher data rates, so called *quasi-orthogonal space-time block codes* (QOSTBC) were proposed [7]–[10]. A QOSTBC scheme can achieve the full rate, but there are also interference terms resulting from neighboring signals during signal detection. This causes an increase in decoding complexity and a decrease in performance gain, with respect to the OSTBC schemes.

It is shown in [6], [11] that higher dimensional full rate schemes that obtain full transmit diversity exist only for binary modulation. Once complex-valued modulation is applied, OSTBC with full rate and full diversity do not exist. An alternative is to reduce the data rate [12]. It is this data rate limitation that started the interest in QOSTBCs to gain increased data rate while preserving most of the diversity advantage of OSTBC.

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In this article, we demonstrate an efficient technique for QOSTBC with four transmit antennas which can achieve the full rate and simple linear decoding. Applying matrix transformation using unitary matrix to eliminate interference terms in the detection matrix. This allows linear decoding to be applied at the receiver. The rest of the article is organized as follows: Section II will cover fundamentals of QOSTBC and define notations. In Section III, the proposed method is described in fair detail. Simulation parameters and results are presented in Section IV and conclusion follows in Section V.

II. FUNDAMENTALS OF QSTBC

The basic block diagram of an STBC encoder is given by Fig.1.The *multiple-input-single-output* (MISO) signal model for the receive vector **r** is given in

$$\mathbf{r} = \mathbf{S.h} + \mathbf{n} \tag{1}$$

where r = $[r1, \ldots, rM]^T$, is a M X 1 received signal vector, S is a N xM encoder matrix, h = $[h_1, \ldots, h_M]^T$ is a M x 1 channel vector and n = $[n_1, \ldots, n_M]^T$ is a M x 1 is a noise vector. For OSTBC alamouti code [1] for 2 x 1 MISO signal model, the eqn. (1) can be written as:

$$\begin{aligned} r_1 &= s_1 h_1 + s_2 h_2 + n_1 \\ r_2 &= s_2^* h_1 - s_1^* h_2 + n_2 \end{aligned} \tag{2}$$

Applying conjugate on both sides of eqn. (2), we get:

$$\begin{aligned} r_1 &= s_1 h_1 + s_2 h_2 + n_1 \\ r_2^* &= -s_1 h_2^* - s_2 h_1^* + n_2^* \end{aligned} \tag{3}$$

The above set of equations can be written in vector form as

$$\mathbf{Y} = \begin{pmatrix} r_1 \\ r_2^* \end{pmatrix} = \begin{pmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2^* \end{pmatrix} \tag{4}$$

In short vector notation

$$\mathbf{Y} = H.\mathbf{s} + \mathbf{n} \tag{5}$$

where s = $[s_1, s_2]^T$, is a 2 x 1 transmitted signal vector from two antennas and H is so called *equivalent virtual channel matrix* (EVCM). Notice that for OSTBC alamouti code, a Grammian matrix G, also known as detection matrix, is computed as a product given below yields:

$$G = H^H.H = h^2.I_2 \tag{6}$$

where superscript H represents hermitian transpose, $h^2 = |h_1|^2 + |h_2|^2$, I_2 is a 2 x 2 identity matrix. This property of OSTBC enables the use of simple linear decoding using *zero forcing*(ZF) decoder. ZF decoding of OSTBC is simply obtained by

$$\begin{aligned} \mathbf{s} &= (H.H^H)^{-1}.H^H.\mathbf{Y} \\ &= (H.H^H)^{-1}.H^H(H.\mathbf{s}+\mathbf{n}) \\ &= S + (H.H^H)^{-1}.H.\mathbf{n} \end{aligned} \tag{7}$$

The OSTBC scheme in [1] achieves the full rate and full diversity gain. Subsequently, Tarokh proposed OSTBC schemes to achieve full diversity for more than two transmit antennas [5]. However, such schemes cannot achieve the full rate. To solve this problem and achieve the full data rate, a quasi-orthogonal space time-block coding (QOSTBC) scheme was proposed in [7], [9]. A QOSTBC scheme can achieve the full rate, but there are also interference terms resulting from neighboring signals during signal detection. This causes an increase in decoding complexity and a decrease in performance gain, with respect to the OSTBC schemes.

QOSTBC for higher dimensions achieving the full rate in four transmit antenna system were proposed by Tirkkonen [9], Jafarkhani [7] and Papadias and Foshchini [8]. Although similar in appearance these schemes show very distinct behaviour at particular channels. For this paper we choose the Jafarkhani scheme. We demonstrate an efficient technique for Jafarkhani QOSTBC scheme which can achieve both the full rate and simple linear decoding.

The Jafarkhani scheme for sometimes also known as extended alamouti code. The encoding matrix for the extended alamouti code is denoted by S_E and is represented as

$$S_E = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3^* & s_4^* & -s_1^* & -s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix} \tag{8}$$

and corresponding H_E is given by:

$$H_E = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & h_1^* & h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix} \tag{9}$$

The Grammian matrix G_E for H_E calculated as product given by eqn.(6) yields

$$G_E = H_E^H . H_E = \begin{bmatrix} \alpha & 0 & 0 & \beta \\ 0 & \alpha & -\beta & 0 \\ 0 & -\beta & \alpha & 0 \\ \beta & 0 & 0 & \alpha \end{bmatrix} \tag{10}$$

where $\alpha = |h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2$ and $\beta = h_1^* h_4 - h_2 h_3^* - h_3 h_2^* + h_4 h_2^*$. The α term denotes the overall channel gain and β terms is the interference parameter. The interference terms causes the code to be non-orthogonal and thus simple linear decoding cannot be applied. In the next section, we demonstrate a method to transform G_E into a diagonal matrix by eliminating the interference terms and thus allowing simple linear decoding to be possible.

III. ALGORITHM FOR EFFICIENT DECODING

In this section, we apply a simple transformation to G_E that eliminates interference terms and allows linear decoding. A N x N matrix with orthogonal columns is said to be *unitary* if $U^{H.}U = U.U^H = I_N$ which implies $U^H = U^{-1}$. The inner product given by $G_E = H_E^H.\mathrm{H_E}$ remains unchanged if each vector is multiplied with an unitary matrix U. Thus applying a transformation

$$D_E = U^H.G_E.U \tag{11}$$

we can eliminate the interference terms in G_E. Few detailed steps are given here:

Since the columns of G_E are orthogonal we can write it as a linear combination

$$G_E = \alpha.I_4 + \beta.G_1 \tag{12}$$

where

$$G_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Substituting Eqn. (12) in Eqn. (11), we get

$$\begin{aligned} D_E &= U^H.(\alpha.I_4 + \beta.G_1).U \\ &= \alpha U^H I_4 U + \beta U^H G_1 U \\ &= \alpha I_4 + \beta U^H G_1 U, \end{aligned} \tag{13}$$

by selecting an appropriate U we wish to eliminate non-diagonal elements from the second term of eqn. (13).

A unitary matrix of the form given by

$$U = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

yields the desired result. Applying U in Eqn. we get G_{me} as

$$G_{me} = \begin{bmatrix} \alpha+\beta & 0 & 0 & 0 \\ 0 & \alpha-\beta & 0 & 0 \\ 0 & 0 & \alpha+\beta & 0 \\ 0 & 0 & 0 & \alpha+\beta \end{bmatrix} \tag{14}$$

The new EVCM channel matrix can be evaluated as

$$\begin{aligned} G_{me} &= U^H.G_E.U \\ &= U^H.H_E^H.H_E U \\ &= (H_E.U)^H.(H_E.U), \end{aligned} \tag{15}$$

Thus, $H_{me} = H_E.U$ and is given below:

$$\mathbf{H}_{me} = \begin{bmatrix} h_1 + h_4 & h_2 + h_3 & -h_2 + h_3 & -h1 + h_4 \\ -h_2^* + h_3^* & h_1^* - h_4^* & -h_1^* - h_4^* & h_2^* + h_3^* \\ -h_3^* + h_2^* & -h_4^* + h_1^* & h_4^* + h_1^* & h_3^* + h_2^* \\ h_4 + h_1 & -h_3 - h_2 & h_3 - h_2 & -h_4 + h_1 \end{bmatrix} \tag{16}$$

and using $R = H_{me}.S + N$, the corresponding S_{me} is given below:

$$\mathbf{S}_{me} = \begin{bmatrix} s_1 - s_4 & s_2 - s_3 & s_2 + s_3 & s_1 + s_4 \\ s_2^* - s_3^* & -s_1^* + s_3^* & s_1^* + s_3^* & -s_2^* - s_3^* \\ -s_2^* + s_3^* & s_1^* + s_4^* & -s_1^* + s_4^* & -s_2^* + s_3^* \\ s_1 + s_4 & -s_2 - s_3 & -s_2 + s_3 & s_1 - s_4 \end{bmatrix} \tag{17}$$

The new encoding matrix given in 17 is quasi-orthogonal rather than orthogonal. Nevertheless, since its channel matrix H_{me} is orthogonal ML decoding can be achieved via simple linear decoding, as given by:

$$\mathbf{s} = H_{me}^H Y = H_{me}^H + H_{me}^H N \tag{18}$$

IV. SIMULATION

A simple simulation model was developed to quantify the performance of the proposed technique for extended alamouti STBC. The wireless channel is modeled as flat Rayleigh fading channel [13]. The performance is measured in terms of symbol-error-rate (SER) versus (Signal-to-Noise Ratio) SNR. As in other conventional STBC schemes, we assumed that fading was constant over four consecutive symbol periods, and the receiver had perfect knowledge about the channel. We employed QPSK modulation and the total transmit power was equally divided by the number of transmit antennas. The number of symbol blocks over which error probability is calculated are 10^5.

It is clear from Fig. 1 that the proposed schemes achieved better performance than the conventional scheme. For four transmit antenna, the proposed scheme achieved a power gain of about 1 dB over the conventional scheme.

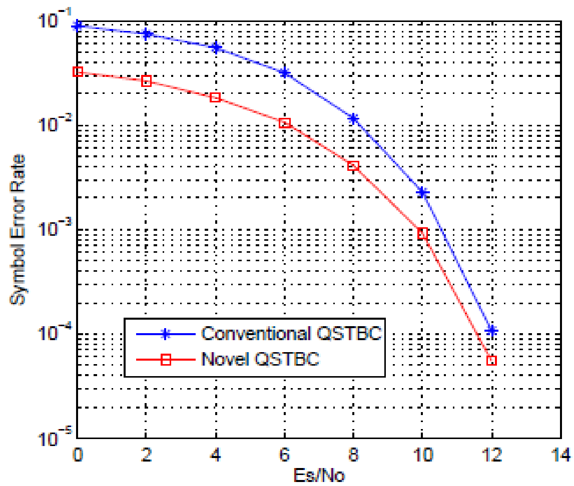

Fig. 1. Symbol error rate versus Signal to noise Ratio

V. CONCLUSION

Quasi orthogonal space time block codes can achieve full rate but there are also interference terms which results in increased decoding complexity. In this paper, we present a technique to eliminate interference terms from the detection matrix in order that linear decoding could be used thus reducing the decoding complexity. The technique mentioned in this paper could be used on other QOSTBC codes as well. The performance is improved in terms of symbol error rate versus signal-to-noise ratio; 1 dB gain is seen over the conventional QOSTBC method. Future work will include incorporating constellation rotation for improved performance in terms of lower error probability.

REFERENCES

[1] S.M. Alamouti, A simple diversity technique for wireless communications, IEEE Journal on Sel. Areas in Com., Vol. 16, No. 8, pp. 1451-1458, Oct, 1998.

[2] G.J. Foschini, M.J. Gans, On limits of wireless communications in a fading environment when using multiple antennas, Wireless Personal Communications, vol. 6, no. 3, pp. 311-335, March 1998.

[3] I.E. Telatar, Capacity of multi-antenna Gaussian channels, European Trans. Telecommunications, vol. 10, no. 6, pp. 585-595, 1999, Technical Memorandum, Bell Labs, Lucent Technologies, Oct. 1998.

[4] V. Tarokh, N. Seshadri, A.R. Calderbank, Space-time codes for high data rate wireless communication: Performance criterion and code construction, IEEE Trans. Inform. Theory, vol. 44, no. 2, pp. 744 765, March 1998.

[5] V. Tarokh, H. Jafarkhani, A.R. Calderbank, Space-time block coding for wireless communications: Performance results, IEEE Journal on Sel. Areas in Com., vol.17, no. 3, pp. 451-460, Mar. 1999.

[6] V. Tarokh, H. Jafarkhani and A.R. Calderbank, Space-time block codes from orthogonal designs, IEEE Trans. Inform. Theory, vol. 45, pp. 1456-1467, July 1999.

[7] H. Jafarkhani, A quasi orthogonal space-time block code, IEEE Trans. Comm., vol. 49, pp. 1-4, Jan. 2001.

[8] C. Papadias, G. Foschini, Capacity-Approaching Space-Time Codes for System Employing Four Transmitter Antennas, IEEE Trans. Inf. Theory, vol. 49, no. 3, pp. 726-733, March 2003.

[9] O. Tirkkonen, A. Boariu, and A. Hottinen. Minimal non-orthogonality rate 1 space-time block code for 3+ Tx antennas, in Proc. IEEE Sixth Int. Symp. on Spread Spectrum Techniques and Applications, vol. 2, pp. 429-432, Sept. 2000.

[10] C.F. Mecklenbrauker, M.Rupp, On extended Alamouti schemes for space-time coding,, WPMC02, Honolulu, pp. 115-119, Oct. 2002.

[11] B. Hochwald, T.L. Marzetta, C.B. Papadias, A transmitter diversity scheme for wideband CDMA systems based on space-time spreading, IEEE Journal on Sel. Areas in Com., vol. 19, no. 1, pp. 48-60, 2001.

[12] C. Xu, Y. Gong, K.B. Letaief, High rate complex orthogonal spacetime block codes for high number of transmit antennas, IEEE International Conference on Communications, ICC04, Paris, France, vol. 2, pp. 823 - 826, June 2004.

[13] M. El-Tarhuni, M. Hassan, A.B. Sediq, ”A Joint Power Allocation and Adaptive Coding Scheme for Image Transmission Over Wireless Channel”, IJCNC, Vol 2, No. 3, May 2010.