STRATEGIC PAYOFFS OF NORMAL DISTRIBUTIONBUMP INTO NASH EQUILIBRIUMIN 2 × 2 GAME

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ABSTRACT

In this paper we assume that strategic payoffs are Normal distribution, and discuss how the parameters of Normal distributions affect the NE payoff distribution that is also concerned by players. We find that distortions of NE payoff distributions are dominated by the distance between variances of strategic payoffs in small means cases and the variances of the dominantly strategic payoffs in large means case. We also find that the variances of strategic payoffs lead to the higher means of the NE payoff distributions, which contain risk premium and the dominantly strategic payoffs, whatever the means of strategic payoffs. However, compared with the dominant strategy that is NE of static game, our model obtains that the mean magnitudes of strategic payoffs lead to establish the different rade-off relationship between means and variances of the NE payoff distribution in the decision-making process.

KEYWORDS

Nash equilibrium, distributed payoff, uncertainty, decision-making process

1. INTRODUCTION

This paper examines payoff uncertainty in the form of Normal distribution with different variances. The previous literatures in game theory have ignored payoff uncertainty in terms of the variance effect and assume a specific distribution when the game faces payoff uncertainty. In this paper, I show that changing the values of variance and mean of Normal distribution has impacted on the payoff distribution of Nash equilibrium (NE, Nash, 1950, 1951) under the case that the players play the DS, labeled DS.

The paper simulates a 2 × 2 game where the DS payoffs are Normal distribution, the most interesting and novel results is that for different variances of Normal distribution the NE payoff distributions have different shapes and larger means whatever the DS payoffs are. But the large DS payoffs' means with different variancesleads to less changes in shape of NE payoff distributions. This result illustrates that it is important to consider the role of risk in the decision-making process of NE.

The early researches followed the game model with uncertainty use the approaches of von Neumann and Morgenstern (1944) and Savage (1972) (see Osborne and Rubinstein, 1994, p.5). Then uncertainty in game model is shown in the papers, such as that Friedman & Mezzetti (2001) and Hofbauer & Sandholm (2007) discuss how random process works in game models. Cotter (1994) follows Harsanyi (1973) and Aumann (1987) and finds that players use observations, the players' type, and a nature effect on payoffs to obtain a strategy correlated equilibrium that is better than Bayesian-Nash equilibrium because a strategy correlated equilibrium is more robust than Bayesian-Nash equilibrium that is affected by player's type only. Battigalli & Siniscalchi (2007) also follow the approach of Bayesian game with partially unknown payoff that is denoted as uncommon knowledge in the game where it has different types of players. They build an interactive-epistemology structure with complete information and slight payoff uncertainty then find that large parameter space induces in the correspondence between initial common certainty of nature and rationality then obtains weakly rationalizable strategies with complete information. Wiseman (2005) tries to use unknown payoff distribution to solve multi-armed bandit problem in a different state corresponding to a different payoff matrix in stage game. The above literatures focus on the equilibrium of strategy by the view of expected payoff and by the use of payoff distributions as probability effect without considering complete distribution effect including variances and higher-order moments. Unfortunately, the above papers did not take the complete distribution effect to present how uncertainty works in game models. It is necessary to pay attention on distribution assumption which is random draw and has more parameters interacting to show uncertainty, for example, capital asset pricing model (CAPM) shows the positive relation of means and variances (Varian, 2011) and Chamberlain (1983) and Ingersoll (1987) provide mean-variance analysis is appropriate when payoff distribution is elliptical. The paper, relative to the above papers, simultaneously manages the parameters of distributions of strategic payoffs and the decision-making process that is denoted as maximum function. The paper's advantages that we can investigate how NE payoff distributions are affected by parameters of distribution assumption and decision-making interaction.

Most specifically, players in competition with the DS and uncertain payoff usually occupy large NE payoffs from large variances when the means of strategic payoffs are fixed. Unless one of strategies has variance equaling to 1, the variances of strategic payoffs have hugely impact on the NE payoff distributions, whereas for the variance of strategic payoff that is larger than 1 causes that NE payoff distributions have different changes of each coefficient and graph. One example is that in the stock market an investor faces a huge number of common stocks and indexed stocks, whereas indexed stocks have low risk than common stocks. Other examples include companies deciding investment plans, households facing the choices of insurance portfolios, and different investing products choosing. The game model can also be used to understand the settled values of parameters have impact on NE payoff distribution. The reason to focus on the NE payoff rather than the NE strategy is that players mind which strategy can bring highest payoff, but also they want to know how the risk will make additional payoff, that is risk premium. When both variances of payoffs are high, the NE payoff has higher mean and lower variance than the payoff of the DS. One contribution of the model is that as the fluctuation of strategic payoff become more dramatic NE payoff will become less fluctuated and eventually interaction of uncertainly strategic payoffs in decision-making process will dominate.

The driving forces behind the NE payoff distributions are the values of means and variances and the decision rule. When the variances of strategic payoffs are 1, the DS payoff distribution dominates the NE payoff distribution whatever the means of strategic payoffs are large or small. In contrast, when at least one of variances of strategic payoffs is larger than 1, the norm between variances and the values of variances become important. A NE payoff benefits by getting more return and facing lower risk for players who still choose the DS. The NE payoff sare better than the DS payoffs in uncertainty, meanwhile, the environments where players only face the payoff distribution of the DS without making decision is more risky than uncertainty with decision-making. The paper is structured as follows. Section 2 describes the game model and simulation procedures. Section 3 explains the simulated results that how means and variances of strategic payoffs interactively affect the NE payoff distributions. Section 4 concludes.

2. MODEL

Consider a 2 × 2 game with the DS and the payoff matrix is illustrated in Table 1. Each player has the perfect information including the strategic payoff distribution. Player 1 has two strategies, U and D, and Player 2 has L and R. Table 1 shows static game with certain payoffs, then U is Player 1's DS with high payoff and L is Player 2's DS such that Nash equilibrium is (U, L).

Table 1. The payoff matrix of normal form game with the DS

		Player 2	
		L	R
Player 1	U	2,2	10,1
	D	1,10	5,5

Next, denote X2 that is a random variable and i.i.d.N(E(X2), Var(X2)) is represented the payoff distribution of 'U' when Player 2 chooses 'L', and X1is i.i.d. N(E(X1), Var(X2)) and represents the payoff distribution of 'D'. Normal distribution can clearly show the mean as the payoff and the variance as the risk bright from buying stock. Without loss of generalization and because of symmetric payoff setting, we only discuss the behaviour of Player 1. The decision rule is Y=MAX(X1, X2), where Y is the NE payoff distribution. The values of E(X1) and E(X2) can be divided two cases, one is small means case denoted by Case 1, E(X1)=1 and E(X2)=2, and the other is large means case denoted by Case 2, E(X1)=10 and E(X2)=20, then each case has 9 subcases with different variances and shows in Table 2.

Table 1.The distributions of 9 subcases

Case number	The distribution of X_2	The distribution of X_1
Case 1-1		N(1, 1)
Case 1-2	N(2, 1)	N(1, 25)
Case 1-3		N(1, 64)
Case 1-4		N(1, 1)
Case 1-5	N(2, 25)	N(1, 25)
Case 1-6		N(1, 64)
Case 1-7		N(1, 1)
Case 1-8	N(2, 64)	N(1, 25)
Case 1-9		N(1, 64)
Case 2-1		N(10, 1)
Case 2-2	N(20, 1)	N(10, 25)
Case 2-3		N(10, 64)
Case 2-4		N(10, 1)
Case 2-5	N(20, 25)	N(10, 25)
Case 2-6		N(10, 64)
Case 2-7		N(10, 1)
Case 2-8	N(20, 64)	N(10, 25)
Case 2-9		N(10, 64)

After the game model and parameters of distributions are constructed, the decision rule becomes transformation of probability distribution and the transformation function is maximum function that is too difficult managed by mathematics to use computer simulation. I simulate on the desktop computer with Windows 7 system and run C++ programs, which is the transformation simulator of probability distribution, to do 60 million times random draws for generating the distributions of X_1 and X_2 and then transform X_1 and X_2 by maximum into Y.

The approach of probability distribution is getting a random number (RND) from the cumulative probability function of Normal distribution, X~fx(x), Fx(x)=P(X≤x)~U(0, 1) and RND~U(0, 1) thus Fx(x)=RND, and then is using the inverse function of cumulative probability distribution to obtain the value of random variable, x=Fx-1(RND). The values of the random variable are gathered as a data set, {X1, X2, X3,…,Xn}, which can be arranged as a frequency table to form probability distribution by the law of large number. In the other words, the data set is generated randomly by Normal distribution and will approximate towards Normal distribution by the sample frequency table when n is large enough. In the simulation procedure, the simulator defines symbols as follows:

X1~N(2, 1), X2~N(2, 25), X3~N(2, 64), X4~N(1, 1), X5~N(1, 25), and X6~N(1, 64) in Case 1, while X1~N(20, 1), X2~N(20, 25), X3~N(20, 64), X4~N(10, 1), X5~N(10, 25), and X6~N(10, 64) in Case 2.

3. SIMULATED RESULTS

We look for the NE payoff distribution when players choose the DS. We simulate the game by decision rule. The results of Case 1 are summarized in Table 3 and Table 4. Table 3 shows that the simulated results are asthe same as assumptions of Case 1, that include E(X1)=E(X2)=E(X3)=2, E(X4)=E(X5)=E(X6)=1,Var(X1)=Var(X4)=1,Var(X2)=Var(X5)=25 and Var(X3)=Var(X6)=64. The i.i.d. assumption can be also shown by that the two strategic payoffs in subcases have no linear relationship.

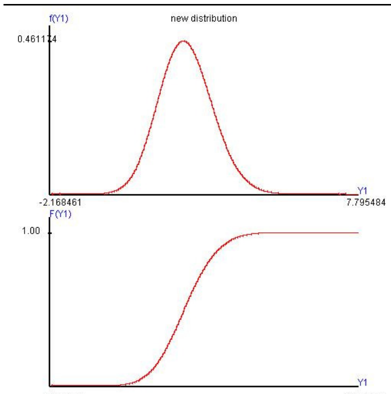
Table 1.The summarized simulation of X1 to X6 and Y1 to Y9

Case 1-1	Case 1-2	Case 1-3
X1 min= -3.749250 max=7.567529 X4 min= -4.567255 max= 6.494130 E(X1)= 2.0000, Var(X1)= 1.0001 E(X4)= 1.0000, Var(X4)= 1.0000 Cov(X1,X4)= -0.0000, X1 and X4 correlation coefficient=-0.0000. Y1 min=-2.186981 max=7.814004	X1 min=-3.488112 max=7.885902 X5 min=-27.974301max=30.070021 E(X1)= 2.0000, Var(X1)= 1.0001 E(X5)= 1.0000, Var(X5)= 25.0031 Cov(X1,X5)= 0.0001, X1 and X5 correlation coefficient=0.0000. Y2 min=-3.479289 max=27.898898	X1 min=-3.861532 max=7.567529 X6 min=-47.134888 max=43.492128 E(X1)= 2.0000, Var(X1)= 1.0001 E(X6)= 1.0000, Var(X6)= 63.9973 Cov(X1,X6)= -0.0001, X1 and X6 correlation coefficient=-0.0000. Y3 min=-3.090869 max=45.596033
Case 1-4	**Case 1-5**	**Case 1-6**
X2 min=-28.084305 max=29.872521 X4 min=-4.839419 max=6.701627 E(X2)= 2.0000, Var(X2)= 24.9991 E(X4)= 1.0000, Var(X4)= 1.0000 Cov(X2,X4)= 0.0001, X2 and X4 correlation coefficient=0.0000. Y4 min=-4.333422 max=29.872521	X2 min=-28.079255 max=29.470651 X5 min=-27.746252 max=27.898898 E(X2)= 2.0000, Var(X2)= 24.9983 E(X5)= 1.0000, Var(X5)= 25.0030 Cov(X2,X5)= 0.0006, X2 and X5 correlation coefficient=0.0000. Y5 min=-17.463845 max=29.557171	X2 min=-25.047374 max=30.508137 X6 min=-44.706943 max=45.596033 E(X2)= 2.0000, Var(X2)= 24.9989 E(X6)= 0.9999, Var(X6)= 63.9952 Cov(X2,X6)= 0.0034, X2 and X6 correlation coefficient=0.0001. Y6 min=-22.618557 max=43.492128
Case 1-7	**Case 1-8**	**Case 1-9**
X3 min=-43.994003 max=49.087219 X4 min=-5.016861 max=6.311516 E(X3)= 1.9999, Var(X3)= 64.0080 E(X4)= 1.0000, Var(X4)= 1.0000 Cov(X3,X4)= 0.0001, X3 and X4 correlation coefficient=0.0000. Y7 min=-5.015851 max=48.512033	X3 min=-44.358882 max=48.512033 X5 min=-26.440562 max=30.429512 E(X3)= 2.0002, Var(X3)= 64.0054 E(X5)= 0.9999, Var(X5)= 25.0025 Cov(X3,X5)= -0.0003, X3 and X5 correlation coefficient=-0.0000.. Y8 min=-21.376221 max=49.087219	X3 min=-43.994003 max=49.087219 X6 min=-47.126807 max=46.613019 E(X3)= 1.9999, Var(X3)= 64.0082 E(X6)= 1.0000, Var(X6)= 63.9983 Cov(X3,X6)= -0.0009, X3 and X6 correlation coefficient=-0.0000. Y9 min=-27.839514 max=45.649112

Table 4explores the graphs and coefficients of Y in 9 subcases of Case 1 given the same means of X1 and X2.If we only observe the means and variances of Y, then the graphs and coefficients may be viewed as Normal distribution in the dialog subcases, Case 1-1, 1-5 and 1-9, however, skewed coefficients that are not towards 0show that the graphs of Case 1-1, 1-5 and 1-9 are not Normal distribution. We also find that when the dialog subcases have |Var(X1) – Var(X2)|=0 the means of Y present arithmetic sequence even Var(X2) is from 1, 25 to 64.The reason is that the distributed payoffs can present uncertainty in decision rule, and lead to NE payoff distribution away from Normal distribution. Thus, players have convex market curve with twice risk premiums when they face more than twice risk. Besides, according to Case 1-1, 1-2 and 1-3 with different Var(X1) and Case 1-1, 1-4 and 1-7 with different Var(X2), the increase of |Var(X1) – Var(X2)| distorts the graphs of Y that cannot be shown by coefficients.

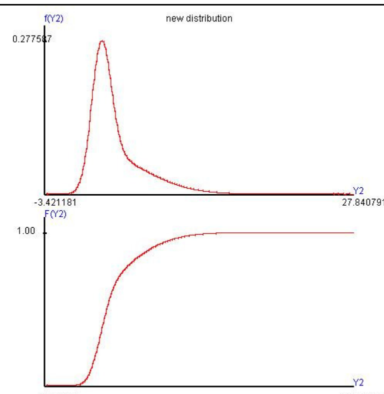
Table 1: The probability function of Nash equilibrium when E(X_1)=2 and E(X_2)=1

Case 1-1



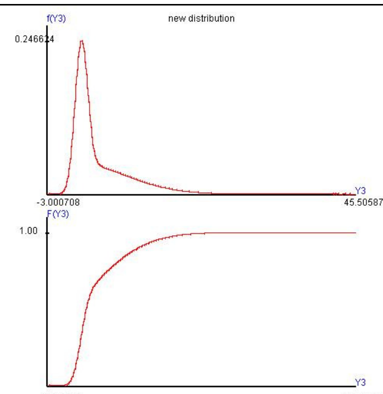
Mathematical Mean:		2.19965
Variance	:	0.76063
S.D.	:	0.87214
Skewed Coef.	:	0.17400
Kurtosis Coef.	:	3.05322

Case 1-2



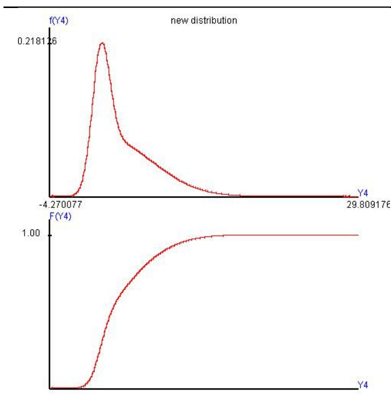
Mathematical Mean:		3.57344
Variance	:	7.08733
S.D.	:	2.66220
Skewed Coef.	:	1.62102
Kurtosis Coef.	:	6.05402

Case 1-3



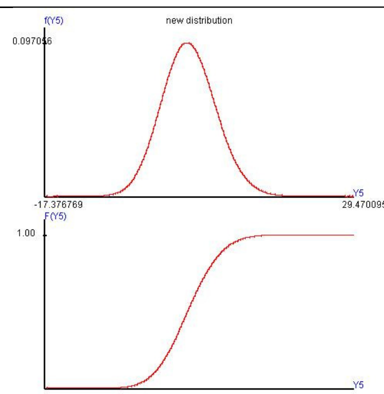
Mathematical Mean:		4.74119
Variance	:	19.13973
S.D.	:	4.37490
Skewed Coef.	:	1.71801
Kurtosis Coef.	:	6.00108

Case 1-4



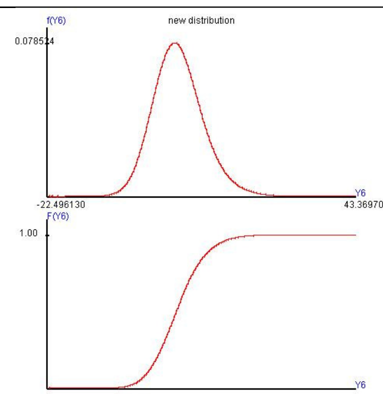
Mathematical Mean:		3.57347
Variance	:	10.81794
S.D.	:	3.28906
Skewed Coef.	:	1.21891
Kurtosis Coef.	:	4.28416

Case 1-5



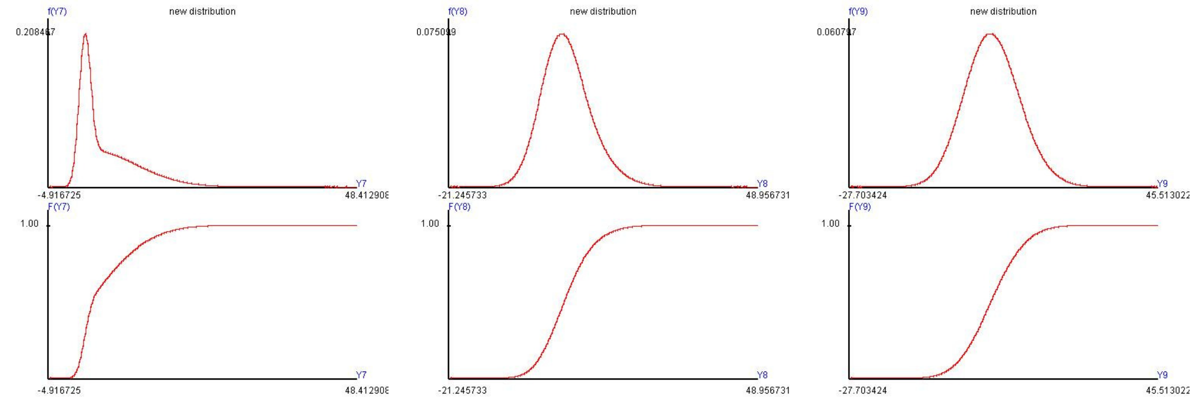
Mathematical Mean:		4.34928
Variance	:	17.13444
S.D.	:	4.13938
Skewed Coef.	:	0.13962
Kurtosis Coef.	:	3.06340

Case 1-6



Mathematical Mean:		5.28495
Variance	:	28.77757
S.D.	:	5.36447
Skewed Coef.	:	0.39348
Kurtosis Coef.	:	3.50644

Case 1-7	Case 1-8	Case 1-9



Mathematical Mean:	4.74111	Mathematical Mean:	5.28503	Mathematical Mean:	6.03146
Variance :	25.35900	Variance :	32.07070	Variance :	43.72616
S.D. :	5.03577	S.D. :	5.66310	S.D. :	6.61258
Skewed Coef. :	1.39260	Skewed Coef. :	0.45029	Skewed Coef. :	0.13819
Kurtosis Coef. :	4.66416	Kurtosis Coef. :	3.49612	Kurtosis Coef. :	3.06225

Error! Reference source not found. also shows that the values of payoff uncertainty can be the real values of firms' profit or stock prices from data analysis. It is reverent that the parameters of distributed strategic payoffs play important role in the decision process. When the means of strategic payoffs are small, the extent of distorted Y is induced from $|Var(X_1) – Var(X_2)|$. Except for the parameters of distributed strategic payoffs, the decision-making process also plays important role in Y. With comparing corresponding distributions of strategic payoffs, Y have E(Yi) > 2 and Var(Yi) < MAX(Var(X_1), Var(X_2)), i = 1, 2,…, 9, relative to the parameters of DS payoffs.

Case 2 is that the means of strategic payoffs are as large as E(X_1)=10 and E(X_2)=20,and also has 9 subcases shown in **Error! Reference source not found.**. **Error! Reference source not found.**shows that the simulated results are as the same as assumptions of Case 2, that include the simulated means of X1, X2 and X3 are 20, of X4, X5 and X6 are 10, corresponding to the simulated variances of X1 and X4 are 1, of X2 and X4 are 25, and of X3 and X6 are 64. The i.i.d. assumption can be also shown by that the two strategic payoffs in subcases have no linear relationship.**Error! Reference source not found.** also shows that the minimum and maximum of strategic payoffs determine the minimum and maximum of Y. The data from Case 2-1 to 2-3 indicates the minimum of Y is determined by X1, while maximum of Y is determined by MAX(X1, Xj), j=4, 5, 6, with random sampling from population distribution. The data from Case 2-4 to 2-6 and Case 2-7 to 2-9 indicates the minimum and maximum of Y≥ MAX(Xi, Xj), i=2, 3 and j=4, 5, 6.

Table 2.The summarized simulation of X1 to X6 and Y1 to Y9 in Case 2

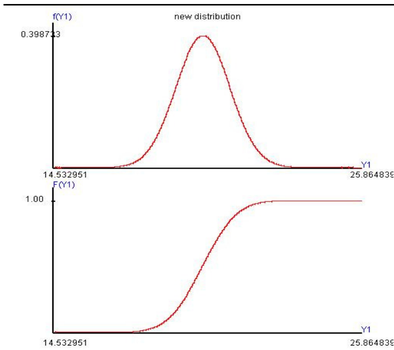
Case 2-1	Case 2-2	Case 2-3
X1 min=14.138468 max=25.567529	X1 min=14.511888 max=25.885902	X1 min=14.138468 max=25.567529
X4 min=4.320916 max=15.511434	X5 min=-18.974301 max=39.070021	X6 min=-38.134888 max=52.492128
E(X1)= 20.0000, Var(X1)= 1.0001	E(X1)= 20.0000, Var(X1)= 1.0001	E(X1)= 20.0000, Var(X1)= 1.0001
E(X4)= 10.0000, Var(X4)= 0.9999	E(X5)= 10.0001, Var(X5)= 25.0022	E(X6)= 10.0000, Var(X6)= 63.9979
Cov(X1,X4)= -0.0000	Cov(X1,X5)= 0.0000	Cov(X1,X6)= -0.0002,
X1 and X4 correlation coefficient=-0.0000.	X1 and X5 correlation coefficient=0.0000.	X1 and X6 correlation coefficient=-0.0000.
Y1 min= 14.511888 max= 25.885902	Y1 min= 14.520711 max= 37.657221	Y1 min= 14.138468 max= 52.492128
Case 2-4	**Case 2-5**	**Case 2-6**
X2 min=-8.632103 max=47.872521	X2 min=-10.079255 max=48.508137	X2 min=-8.632103 max=47.872521
X4 min=4.160581 max=15.701627	X5 min=-18.746252 max=37.662842	X6 min=-38.126807 max=54.091473
E(X2)= 20.0000, Var(X2)= 24.9985	E(X2)= 20.0000, Var(X2)= 24.9989	E(X2)= 20.0000, Var(X2)= 24.9985
E(X4)= 10.0000, Var(X4)= 1.0000	E(X5)= 10.0000,Var(X5)= 25.0030	E(X6)= 9.9998, Var(X6)= 63.9955
Cov(X2,X4)= 0.0001,	Cov(X2,X5)= 0.0006,	Cov(X2,X6)= 0.0043,
X2 and X4 correlation coefficient=0.0000.	X2 and X5 correlation coefficient=0.0000.	X2 and X6 correlation coefficient=0.0001.

Y1 min= 5.570753 max= 47.189668	Y1 min=-2.624677 max= 47.872521	Y1 min= -5.397296 max= 55.613019
Case 2-7	**Case 2-8**	**Case 2-9**
X3 min=-26.358882 max=66.512033	X3 min=-26.892258 max=64.251553	X3 min= -26.358882 max= 66.512033
X4 min=4.320916 max=15.511434	X5 min=-18.974301 max=39.070021	X6 min= -38.134888 max= 52.492128
E(X3)= 20.0001, Var(X3)= 64.0068	E(X3)= 20.0000, Var(X3)= 64.0066	E(X3)= 20.0001,Var(X3)= 64.0069
E(X4)= 10.0000, Var(X4)= 0.9999	E(X5)= 10.0001, Var(X5)= 25.0023	E(X6)= 10.0001, Var(X6)= 63.9979
Cov(X3,X4)= 0.0001,	Cov(X3,X5)= -0.0006,	Cov(X3,X6)= -0.0007,
X3 and X4 correlation coefficient=0.0000.	X3 and X5 correlation coefficient=-0.0000.	X3 and X6 correlation coefficient=-0.0000.
Y1 min=5.246439 max= 66.512033	Y1 min= -9.314740 max= 64.540232	Y1 min= -18.929956 max= 64.251553

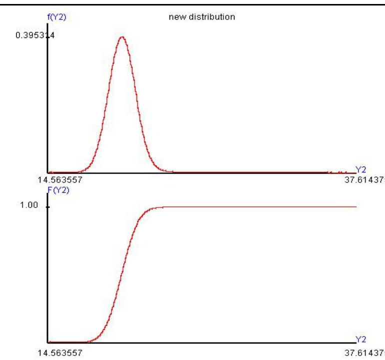
Error! Reference source not found. explores the graphs and coefficients of Y in subcases of Case 2. The graph of Case 2-1 is Normal distribution but those of other subcases are not Normal distribution according to the skewed and kurtosis coefficients. The dialog subcases of Case 2-1, 2-5 and 2-9 show that when the variances of strategic payoffs are from 1, 25 to 64 the means of Y slightly increase. Thus, players having large $E(X_1)$ and $E(X_2)$ earn less risk premium that is transferred by variance.

Table 3.The probability function in each subcase with E(X1)=10 and E(X2)=20

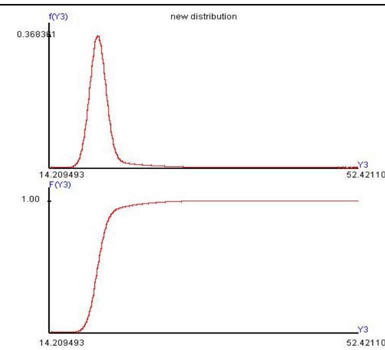
Case 2-1



Case 2-2

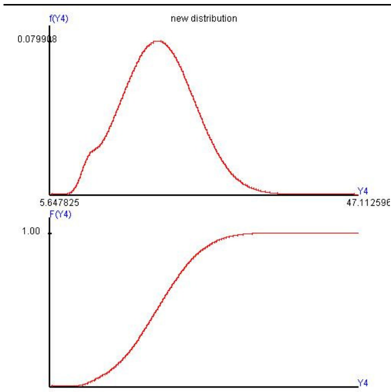


Case 2-3

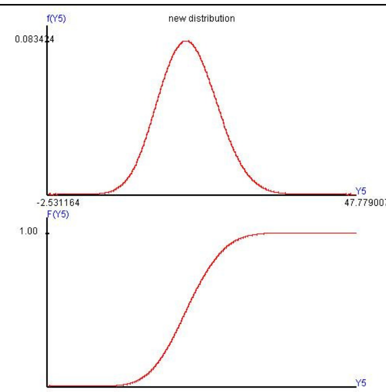


	Case 2-1	Case 2-2	Case 2-3
Mathematical Mean:	19.99989	20.04793	20.41641
Variance :	1.00034	1.11565	3.43398
S.D. :	1.00017	1.05624	1.85310
Skewed Coef. :	0.00018	0.46148	3.19896
Kurtosis Coef. :	3.00075	5.44926	20.35103

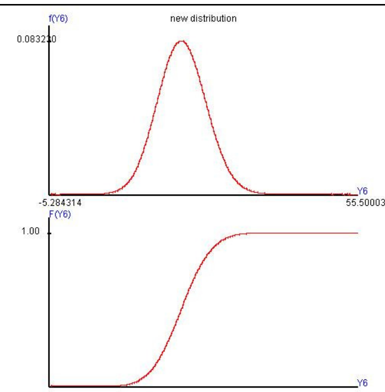
Case 2-4



Case 2-5

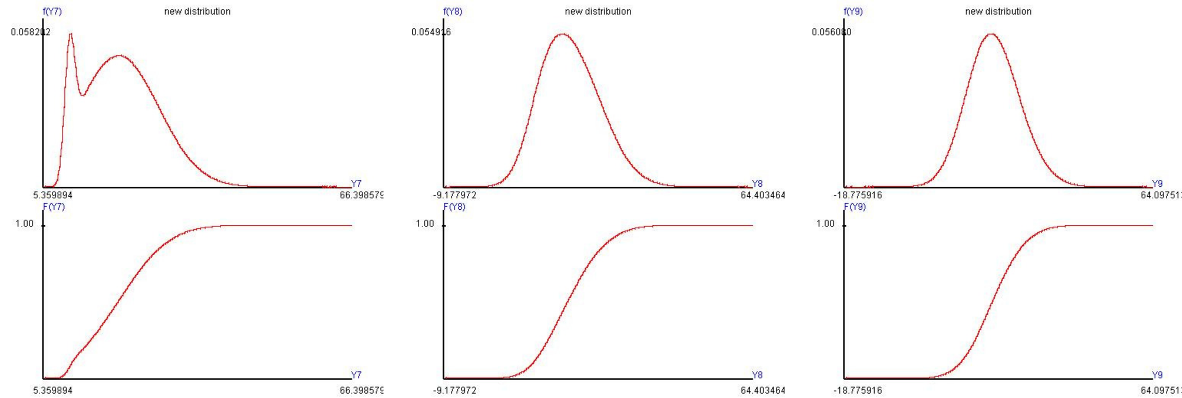


Case 2-6



	Case 2-4	Case 2-5	Case 2-6
Mathematical Mean:	20.04798	20.25124	20.69980
Variance :	23.91910	22.42319	23.14579
S.D. :	4.89072	4.73531	4.81101
Skewed Coef. :	0.13016	0.13269	0.08741
Kurtosis Coef. :	2.75554	2.95146	3.05525

Case 2-7	Case 2-8	Case 2-9



	Y7	Y8	Y9
Mathematical Mean:	20.41645	20.70049	21.17019
Variance :	52.90890	50.87011	50.92751
S.D. :	7.27385	7.13233	7.13635
Skewed Coef. :	0.42017	0.33238	0.17471
Kurtosis Coef. :	2.67076	2.94172	3.03002

According to Case 2-1, 2-2 and 2-3 with different Var(X1) and Case 2-1, 2-4 and 2-7 with different Var(X2), the increase of |Var(X1) – Var(X2)| also distorts the graphs of Y that cannot be shown by coefficients. However, the effect of changed Var(X1) on Y is not the same as the effect of changed Var(X2). Larger Var(X1) induces in more centralized and positive skewed shapes of Y, but Var(X2) induces in distorted shapes of Y. For example, the means of Y are similar between Case 2-3 and Case 2-7 but the variance of X2 dominates the variances of Y in Case 2, therefore, Var(Y7) >Var(Y3). On the other hand, when the NE payoffs are the same, Var(X2) is relatively more important and the players may devote to reduce Var(X2) without consideration of Var(X1) which changes the centralization of Y reverently. According to Case 2-4 to 2-6 with Var(X2)=25 and Case 2-7 to 2-9 with Var(X2)=64, the basic assumption of Var(X2) increases variances of Y which are still less than MAX(Var(X1), Var(X2)). More specifically, the large means help the decision-making process press the variances of NE payoffs down, such as Case 2-3.

Comparing Case 1 with Case 2, we can see that whatever means and variances of strategic payoffs are, the means of Y are DS payoffs plus risk premium and the variances of Y are less than the maximum of Var(X1) and Var(X2), moreover, in Case 2 Var(X2) dominates on the shapes of Y. Another difference is that Y in Case 2 are not easily distorted by the change of variances of strategic payoffs. On the other hand, large means of X1 and X2 can reduce the distortion effect on Y when the variances of strategic payoffs are changed. Thus, the means and variances of strategic payoffs interact and decide Y. One example is that risky and highly pricing stock can brings high risk and returns, but also high price stock needs more investing funds to create demand in order to push up the price. If the price of stock is low in Case 1, then investors easily push the stock price higher and involve the risk from the model setting with initial variances of strategic payoffs. Finally, the large means of strategic payoffs can assist decision-making process in more efficiently restraining the increasing rate of variance in Case 2 than in Case 1.

4. CONCLUSION

The paper examines the NE payoff distributions of strategic payoffs which are i.i.d. Normal distribution with different means and variances. The simulated game model can yield the variance of strategic payoffs has different impact on the NE payoff distribution, depending on the magnitude of means of strategic payoffs. We would like to highlight the role of parameters setting. It is intuitive that even with variance values of strategic payoffs the means of the NE payoff distributions become the DS means plus risk premium, whose relationship is shown in CAPM. Nevertheless, when the variances of strategic payoffs are the same, we do not obtain Normal distribution of NE payoff, except for strategic payoffs have large enough means than the value of variance. In that sense, a testable implication of the model is that the NE payoff distributions evolve more distorted shapes as the variances between strategic payoffs varies, starting from Case 1-1 when the variances of strategic payoffs are 1, then moving to distorted NE payoff distributions when the variances of strategic payoffs become larger. Eventually, for the distance between variances of strategic payoffs is 63, the NE payoff distribution is dominated by large variance of strategic payoff, nevertheless, starting form Case 2-1, then for the distance between variances of strategic payoffs is larger than 0, the NE payoff distribution is dominated by the DS variances.

In this paper, we have made the assumption that strategic payoffs with different means and variances in the game that can be implied on the choosing stocks, deciding investment plan. If the variances of strategic payoffs are large enough, the NE payoff distributions should obtain a mean included the DS means and risk premium, as the variances of the NE payoff distributions are lower than DS variances. But even if the variances of strategic payoffs are as small as 1, the NE payoff distributions may still have positive risk premium and smaller variances. In this case, it faces a trade-off between means and variances of the NE payoff distributions due to comparing with the DS. It may very well be the case, depending on the variance magnitude of strategic payoff sand how large the means of strategic payoffs happen in the decision-making process, that one effect dominates the other and NE payoff distributions have more reduced variances and higher extra risk premium. However, if means of strategic payoffs happen large, relative to Case 2-1, then large means of strategic payoffs suppress the distorted NE payoff distributions that induce from the variances of strategic payoffs.

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