A CHARACTERIZATION OF THE EGALITARIAN CORRESPONDENCE IN THE CONTEXT OF BARGAINING PROBLEMS

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ABSTRACT

In this note we provide a characterization of the egalitarian correspondence in the context of bargaining problems. The characterization is based on bilateral consistency, contraction and expansion independence.

KEYWORDS

Egalitarian correspondence, Bargaining problems.

1. INTRODUCTION

The theory of axiomatic bargaining, which originated in a fundamental paper by Nash (1950), refers to a basic problem in which a group of individuals faces a set of choices over which their preferences differ. The treatment of such basic problem, as introduced by Nash, is fairly abstract; it involves no detailed description of the characteristics of the actual physical choices that are available and, instead, only uses the utilities attached by the individuals to these choices. It also endorses the so-called axiomatic approach, which consists in formulating desiderata, or axioms, on how the bargaining problem should be solved and in checking whether these axioms are compatible, i.e., whether there exist solutions satisfying them all. Several well-known solutions have been singled out, such as the so-called Nash solution, Kalai-Smorodinsky solution or the egalitarian solution.

Convexity assumption already appeared in Nash's original formulation. The standard justification for restricting attention to convex problems is an assumption that players' preferences can be represented by von Neumann-Morgenstern utility functions, and then admitting the use of lotteries. In other words, convexity of feasible sets of bargaining problems may be derived from expected utility bargaining situations or the possibility of randomization between alternatives. Or the bargaining problems may concern the division of a commodity bundle where convexity is a consequence of concavity of the utility functions (See Peters, 1992, subsection 1.3.2). In general, however, economic situations may lead to non-convex feasible set. Such as the division of a bundle of commodities between individuals with non-concave utility functions. Then, the question arises if and how the various solution concepts and their axiomatic characterizations extend to bargaining problems with possibly non-convex feasible sets. Several papers have adopted the axiomatic approach without the convexity assumption, see Anant et al. (1990), Denicolo and Mariotti (2000), Herrero (1989), Kaneko (1980), Peters and Vermeulen (2012), Xu and Yoshihara (2006, 2013).

Much of the literature assume multi-valued solutions (e.g., Herrero (1989), Kaneko (1980), and Mariotti (1998)) under the absence of the convexity condition. It is because solutions satisfying axioms of equity and efficiency on non-convex problems generally cannot be single-valued. Thus multi-valued solutions may be of interest. Besides, multi-valued solution concepts are also common practice in game theory; e.g. the core for sidepayment and non-sidepayment games, the set of Nash equilibria or refinements thereof for non-cooperative games. Hence under the condition of multi-valued solution concept, one would like to know how the various solution concepts and their axiomatic characterizations extend to possibly non-convex bargaining problems. This note is aimed at answering the question for the egalitarian solution.

In this note we consider multi-valued solution concepts on the model of bargaining problems without the convexity assumption. We deal with the egalitarian correspondence. It is a multi-valued solution concept which is an extension of the egalitarian solution (The egalitarian solution in bargaining theory studied by Kalai (1977) and Thomson (1983), among others). We establish an axiomatization of the egalitarian correspondence by Pareto Optimality, Symmetry, Contraction Independence, Expansion Independence and Bilateral Consistency. Contraction Independence and Expansion Independence refer to the way in which the rule reacts to changes in the bargaining set. Bilateral Consistency is a variable-population axiom reflecting a sort of stability criterion: if a two-agent sub-group of the original group secedes with the choices allocated to it, and its members want to re-evaluate the resulting (two-agent) bargaining problem, no meaningful change should occur.

2. Definitions and Notations

Let $U \subseteq \mathbb{N}$ be the *universe* of players with at least three players. A pair $B = (S, d)$ is called a *bargaining problem* if S, the feasible set, is a subset of $\mathbb{R}^N$, where N is a finite subset of U, and d, the *status quo* or *disagreement point*, is a point of S. In bargaining theory, it is always assumed that $|N| \geq 2$, where $|N|$ denotes the *cardinality* of the set N. For each feasible alternative $x = (x_i)_{i \in N} \in S$, x_i denotes the level of utility of player i in N. If $x \in \mathbb{R}^N$ and $P \subseteq N$, then x_P denotes the restriction of x to P. Denote the *origin* of $\mathbb{R}^N$ by 0_N. If $A, B \subseteq \mathbb{R}^N$ and $x \in \mathbb{R}^N$ then $x + A = A + x = \{x + a : a \in A\}$, $A - B = \{a - b : a \in A \ and \ b \in B\}$. Denote the set of boundary points of A by ∂A. A nonempty subset of $\mathbb{R}^N$, S, is *comprehensive* if $(S - IR^N_+) \subseteq S$, where $IR^N_+ = \{x \in IR^N_+ : x_i \geq 0 \ \ for \ all \ i \in N\}$. For convenience, we employ conventions that for $x, y \in \mathbb{R}^N$, $x \gg y$ implies $x_i > y_i$ for all $i \in N$, $x \geq y$ implies $x_i \geq y_i$ for all $i \in N$, and $x > y$ implies $x \geq y$ and $x \neq y$. A nonempty subset of $\mathbb{R}^N$, S, is *non-level* if $x, y \in \partial S$ and $x \geq y$, then $x = y$. For S, a subset of $\mathbb{R}^N$, the weak Pareto optimal subset of S is defined by

$$WPO(S) = \{x \in S : y \gg x \Rightarrow y \notin S\}$$

and *PO(S)* denotes the strong Pareto optimal subset of S, that is,

$$PO(S) = \{x \in S : (x + \mathbb{R}_+^{\ N}) \cap S = \{x\}\}$$

Note that *WPO(S) = PO(S)* if S is non-level.

Let $B = (S, d)$ be a bargaining problem where $S \subseteq \mathbb{R}^N$ and $N \subseteq U$. In this paper, we always assume that S is non-level, closed, comprehensive and bounded above by a hyperplane with a positive normal. To make notation simpler, it is assumed throughout that $d = 0_N$: this convention is justified by an implicit assumption of translation covariance.
Moreover, let $S \subseteq \mathbb{R}^N$, for convenience, we will use S instead of $(S, 0_N)$ to denote a bargaining problem. We also assume that S is nondegenerate, that is, there is $x \in S$ such that $x_i > 0$ for all i in N because it will offer each player some potential reward for reaching an agreement. Σ^N denotes the set of all bargaining problems with player set N, and $\Sigma = \bigcup_{N \subseteq U} \Sigma^N$. A correspondence f defined on Σ associates to every problem S in Σ a non-empty subset $f(S)$ of S.

Definition 1 *The egalitarian correspondence* $\boldsymbol{E}$ *is defined by setting, for all* $S \in \Sigma$, $\boldsymbol{E}(S)$ *to be the set of the maximal point of S of equal coordinates.*

Axiom 1 *Pareto Optimality (PO): For all* $S \in \Sigma,\ f(S) \subseteq PO(S)$.

Axiom 2 *Symmetry (SYM): For all* $S \in \Sigma^N$ *, if for all permutations* π *on N,* $\pi(S) = S$ *, then* $x_i = x_j$ *for all* $i, j \in N$ *, where* $x \in f(S)$.

Axiom 3 *Contraction Independence (CI): For all* $S, S' \in \Sigma$ *, if* $S' \subseteq S$ *then* $f(S) \bigcap S' \subseteq f(S')$.
This says that if an element in the correspondence of a given problem remains feasible for a new problem obtained from it by contraction, then it should also be in the correspondence of this new problem.

Axiom 4 *Expansion Independence (EI): For all* $S, S' \in \Sigma$ *, if* $S' \subseteq S$ *then* $f(S') \bigcap \partial S \subseteq f(S)$.
Axiom EI states that it is not worth reconsidering the agreements if the set of available opportunities expand without offering the unanimously preferred alternatives (see Thomson and Myerson, 1980).

In order to formulate the axioms of bilateral consistency and converse consistency, an additional piece of notation is needed. Given $P \subseteq Q \subseteq U$, a subset A of $\mathbb{R}^Q$, and a point x of A, $t_p^x(A)$ is the intersection of A with the hyperplane through x parallel to $\mathbb{R}^P$, seen as a subset of $\mathbb{R}^P$, that is, $t_p^x(A) = \{x' \in \mathbb{R}^P : (x', x_{Q \setminus P}) \in A\}$. And if A is a bargaining problem, then we call $t_p^x(A)$ the reduced bargaining problem with respect to x and P.

Axiom 5 *Bilateral Consistency (BCON): For all* $P \subseteq Q \subseteq U$ *, for all* $S \in \Sigma^P$ *, for all* $T \in \Sigma^Q$ *, if* $S = t_p^x(T) \in \Sigma^P$ *where* $x \in f(T)$ *, then* $x_P \in f(S)$ *with* $|P| = 2$.

Axiom 6 *Converse Consistency (CCON): For all* $Q \in U$ *, for all* $T \in \Sigma^Q$ *, and all* $x \in T$ *, if for all* $P \subseteq Q$ *such that* $|P| = 2$ *,* $S = t_p^x(T) \in \Sigma^P$ *and* $x_p \in f(S)$ *, then* $x \in f(T)$.

Axiom BCON can be interpreted as an axiom of "equilibrium" in the sense of a self-consistent allocation-expectations property. Namely, it sustains an allocation by providing an exact description of the expectations that players and coalitions have if they were to deviate and reject the allocation in question. Axiom CCON, on the other hand, states that the solution outcome can be decentralized by being imposed on smaller coalitions, where each of them holds the appropriate expectations, as described by the relevant reduced problem.

3. MAIN RESULTS

In this section, we provide an axiomatic characterization of the egalitarian correspondence. Our characterization result of the egalitarian correspondence highlights the two crucial roles that Contraction Independence and Expansion Independence play in two-person bargaining problems. Then with the help of Bilateral Consistency, we obtain the desired result. It should be noted that our characterization of the egalitarian correspondence does not use the Monotonicity type axiom for characterization of egalitarian solution. The main results of this paper are the following:

Theorem 1 A *correspondence on* Σ^N *with* $|N|=2$ *satisfies PO, SYM, EI and CI if and only if it is the egalitarian correspondence* **E**.

Proof. It is easily verified that **E** satisfies the four axioms. Conversely, let *f* be a correspondence on Σ^N satisfying the four axioms. Let $N=\{1,2\}$, $S\in\Sigma^N$ and **E**(*S*) = $\{(e,\ e)\}$. Define a symmetric problem S^0 by

$$S^0 = S \bigcap \pi(S) .$$

where π is the reverse permutation, i.e., π(1) = 2 and π(2) = 1.
By PO and SYM, we have $f(S^0)=\mathbf{E}(S)$. Since $S^0 \subseteq S$, we obtain that $(e,e)\in f(S)$ by EI of *f*. Hence, $\mathbf{E}(S)\subseteq f(S)$.
On the other hand, let $p\in f(S)$. Define two bargaining problems $\overline{S}$ and S_0 by

$$\overline{S}=\{y\in I\!R^N:\ \ y_1+y_2\le p_1+p_2\}\ \ and\ \ S_0=S\bigcap \overline{S}$$

It is obvious that $f(\overline{S})=E(\overline{S})=\{(\frac{p_1+p_2}{2},\frac{p_1+p_2}{2})\}$ by PO and SYM. And $S_0\subseteq S$ and $S_0\subseteq\overline{S}$. Applying CI to S and S_0 , $p\in f(S)\bigcap S_0\subseteq f(S_0)$. Applying EI to S_0 and $\overline{S}$, $p\in f(S_0)\bigcap\partial\overline{S}\subseteq f(\overline{S})$. Since $f(\overline{S})=\{(\frac{p_1+p_2}{2},\frac{p_1+p_2}{2})\}$, this implies that $p_1=p_2$.

That is, $p\in E(S)$. Hence, $f(S)\subseteq\mathbf{E}(S)$. Thus, $f(S)=\mathbf{E}(S)$. ■

The following Theorem 2 could be obtained as a straightforward consequence of Theorem 1 and the so-called Elevator Lemma introduced by Thomson. For the sake of completeness we provide the proof.

Theorem 2 *A correspondence on* Σ *satisfies PO, SYM, EI, CI, and BCON if and only if it is the egalitarian correspondence* **E**.

Proof. It is easy to verify that **E** satisfies BCON. Note that **E** also satisfies CCON. Conversely, let *f* be a correspondence on Σ satisfying the five axioms. By Theorem 1, it only remains to consider the case of $|N|\ge 3$. Let $S\in\Sigma^N$. For any $x\in f(S)$, by BCON of *f* and Theorem 1, we have that $x_P\in f(t_P^x(S))=\mathbf{E}(t_P^x(S))$ for all $P\subseteq N$ with $|P|=2$. Then by CCON of **E**, $x\in\mathbf{E}(S)$. Hence $f(S)\subseteq\mathbf{E}(S)$. Since $|\mathbf{E}(S)|=1$ and $f(S)\neq\phi$, these imply $f(S)=\mathbf{E}(S)$. ■

The following theorem states that BCON can not be replaced by CCON in Theorem 2.

Theorem 3 *If a correspondence f on Σ satisfies PO, SYM, EI, CI, and CCON then it contains the egalitarian correspondence* **E**, *that is,* $\mathbf{E}(S) \subseteq f(S)$ *for all* $S \in \Sigma$. *Furthermore, there is no unique correspondence on* Σ *satisfying PO, SYM, EI, CI, and CCON.*

Proof. Let f be a correspondence on Σ satisfying the five axioms. By Theorem 1, it only remains to consider the case of $|N| \geq 3$. Let $S \in \Sigma^N$. For any $x \in \mathbf{E}(S)$, by BCON of **E** and Theorem 1, we have that $x_P \in f(t_P^x(S)) = \mathbf{E}(S)$ for all $P \subseteq N$ with $|P| = 2$. Then by CCON of f, $x \in f(S)$. Hence $\mathbf{E}(S) \subseteq f(S)$. ■

To verify the non-uniqueness, we construct a correspondence σ on Σ satisfying the five axioms but it is not **E**. Let $S \in \Sigma^N$ and $i, j \in N$. i and j are equivalent, written $i \sim j$, if $\max\{y : (y, x_{N\backslash\{i\}}) \in S\} = \max\{y : (y, x_{N\backslash\{i\}}) \in S\}$ for every $x \in PO(S)$. Let $\sigma(S) = \{x \in PO(S) : x_i = x_j \text{ if } i \sim j\}$. It is straightforward that $\sigma = \mathbf{E}$ for the class of two-person bargaining problems and σ satisfies PO, SYM, CI, EI and CCON. But $\sigma \neq \mathbf{E}$ This completes the proof.

4. INDEPENDENCE OF THE AXIOMS

The following examples show that the independence of axioms in Theorem 2.

Example 1 *For every bargaining problem* $S \in \Sigma^N$, *define* $\sigma^1(S) = \{0_N\}$. *Then* σ^1 *satisfies all axioms except PO.*

Example 2 *For every bargaining problem* $S \in \Sigma$, *define* $\sigma^2(S) = PO(S)$. *Then* σ^2 *satisfies all axioms except SYM.*

Example 3 *Let* $\sigma^3 = \sigma$, *where* σ *is defined as that in Theorem 3. Because* $\sigma^3 \neq \mathbf{E}$, σ^3 *does not satisfy BCON. Hence* σ^3 *satisfies all axioms except BCON.*

Example 4 *For every bargaining problem* $S \in \Sigma^N$, *recall that the Nash(1950) correspondence N is the set of maximizers of the product* $\prod_{i \in N} x_i$ *over S. We define a correspondence* σ^4 *by* $\sigma^4(S) = \mathbf{E}(S)$ *if S is symmetric (i,e,* $\pi(S) = S$ *for all permutations π on N); otherwise,* $\sigma^4 = N(S)$. *Then* σ^4 *satisfies all axioms except EI.*

Example 5 *Let* $N = \{1,2\}$. *Define a bargaining problem* $S' \in \Sigma^N$ *by* $S' = \{x : x_1 + 2x_2 \leq 2\}$. *Let* σ^5 *be a correspondence on* Σ *by*

$$\sigma^5(S) = \begin{cases} E(S) \cup \{(0,1)\}, \ \textit{if} \ \ S = S' \\ E(S) \cup \{(0,1)\}, \ \textit{if} \ \ S \supseteq S' \ \textit{and} \ \ (0,1) \in PO(S) \\ E(S), \ \textit{otherwise} \end{cases}$$

where $S \in \Sigma^N$; *and* $\sigma^5(S) = \mathbf{E}(S)$ *if* $S \notin \Sigma^N$

Clearly, the correspondence σ^5 *satisfies PO, EI and BCON. To verify it satisfies SYM, let S be a two-person bargaining problem with* $S \supseteq S'$, $(0,1) \in PO(S)$. *Then S must be not symmetric (since* $(0,1) \notin PO(S)$*). Combining this with both* **E** *satisfies SYM and* S' *is not symmetric, we have that* σ^5 *satisfies SYM. Hence* σ^5 *satisfies all axioms except CI.*

5. SINGLE-VALUED SOLUTION CONCEPT UNDER ABSENCE OF THE CONVEXITY CONDITION

The egalitarian solution in bargaining theory studied by Kalai (1977) and Thomson (1983) in the convex bargaining problems under the condition of single-valued solution concept. Removing the convexity requirement, Conley and Wilkie (1991) show that Kalai's (1977) characterization of the egalitarian solution is easily adapted to the non-convex case under the condition of single-valued solution concept. In page 52 of Thomson and Lensberg (1989), we also see that Thomson's (1983) characterization of the egalitarian solution remains true to the non-convex case under the condition of single-valued solution concept. However, as we stated in Introduction, much of the literature assume multi-valued solutions under the absence of the convexity condition. Hence an alternative approach, allowing a solution to be a correspondence (multi-valued solution), is used in this note. That is, we consider multi-valued solution concepts on the model of bargaining problems under absence of the convexity condition.

On the other hand, Theorems 1-3 are also easily adapted to the single- valued case as follows:
First we present some definitions and axioms under the condition of single-valued solution concept. A solution g defined on Σ associates to every problem *S* in Σ a point $g(S)$ of *S*.

Definition 2 *The egalitarian solution $\mathcal{E}$ is defined by setting for all $S \in \Sigma^N$, $\mathcal{E}(S)$ to be the maximal point of S of equal coordinates.*

Axiom 7 *Pareto optimality (Po): For all $S \in \Sigma$, $g(S) \in PO(S)$.*

Axiom 8 *Symmetry (Sym): For all $S \in \Sigma^N$, if for all permutations π on N, $\pi(S) = S$, then $x_i = x_j$ for all $i, j \in N$, where $x = g(S)$.*

Axiom 9 *Expansion independence (Pi): For all $S, S' \in \Sigma$, if $S' \subseteq S$ and $g(S') \in \partial S$ then $g(S') = g(S)$.*

Axiom 10 *Bilateral consistency (Bcon): For all $P \subseteq Q \subseteq U$, for all $S \in \Sigma^P$, for all $T \in \Sigma^Q$, if $S = t_P^x(T) \in \Sigma^P$ where $x = g(T)$, then $x_P = g(S)$ with $|P| = 2$.*

Axiom 11 *Converse consistency (Ccon): For all $Q \in U$, for all $T \in \Sigma^Q$, and all $x \in T$, if for all $P \subseteq Q$ such that $|P| = 2$, $S = t_P^x(T) \in \Sigma^P$ and $x_P = g(S)$, then $x = g(T)$.*

Then Theorems 1-3 are revised to be Theorems 4-6, respectively. The reader can easily verify these theorems, we omit these proofs.

Theorem 4 *A solution on Σ^N with $|N| = 2$ satisfies Po, Sym and Ei if and only if it is the egalitarian solution $\mathcal{E}$.*

Theorem 5 *A solution on Σ satisfies Po, Sym, Ei and Bcon if and only if it is the egalitarian solution $\mathcal{E}$.*

Theorem 6 *A solution on Σ satisfies Po, Sym, Ei and Ccon if and only if it is the egalitarian solution $\mathcal{E}$.*

6. NON-LEVELNESS

In this note the characterizations are obtained on non-level bargaining problems. The non-level condition says that the undominated boundary of a bargaining problem contains no segment parallel to a coordinate subspace. This condition means that strong and weak Pareto optimality coincide. It is a familiar regularity condition in game theory. Also, it has played a crucial role in several contributions to the theory of games in characteristic function form (see Aumann, 1985; Hart, 1985, 1989; Hwang and Sudh¨olter, 2001; Peleg, 1985; Tadenuma, 1992).

This condition also has important implications. In particular, there are a number of axioms that some solutions satisfy on the set of non-level problems but not on the set of all bargaining problems. We use it here in order to guarantee that for all $P \subseteq Q \subseteq U$, for all $S \subseteq \mathbb{R}^P$, for all $T \subseteq \mathbb{R}^Q$, if $x \in PO(T)$, then $x_P \in PO(S)$ where $S = t_P^x(T)$. That is, if x is in the strong Pareto optimal subset of T, then x_P is in the strong Pareto optimal subset of corresponding reduced bargaining problem S.

Since a bargaining problem can be approximated by a sequence of elements in the set of non-level bargaining problems, the subdomain will turn out to be quite useful. Of course, non-levelness also has conceptual significance since it guarantees that "utility transfers" are always possible along the north-east boundary of the problem.

7. Final Remark

As we stated in Section 5, the axiomatic characterizations of the egalitarian solution are easily adapted to the non-convex case under the condition of single-valued solution concept. Similarly, most results in the literature concerning the Kalai-Smorodinsky solution are also easily adapted to the non-convex case under the condition of single-valued solution concept (see Thomson and Lensberg, 1989, page 39). Here an alternative approach, allowing a solution to be a correspondence (multi-valued solution), is used in this note. It may be also interesting to deal with the "Kalai-Smorodinsky correspondence". Note that the "Kalai-Smorodinsky correspondence" does not satisfy EI, CI, and BCON. We consider such an extension as a challenging and interesting enterprise, and we plan to propose one extension in a subsequent work.

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