HYBRID SLIDING SYNCHRONIZER DESIGN OF IDENTICAL HYPERCHAOTIC XU SYSTEMS

Sundarapandian Vaidyanathan[1]

[1]Research and Development Centre, Vel Tech Dr. RR & Dr. SR Technical University
Avadi, Chennai-600 062, Tamil Nadu, INDIA
sundarvtu@gmail.com

ABSTRACT

In this paper, new results have been obtained via sliding mode control for the hybrid chaos synchronization of identical hyperchaotic Xu systems (Xu, Cai and Zheng, 2009). In hybrid synchronization of master and slave systems, the odd states are completely synchronized, while the even states are anti-synchronized. The stability results derived in this paper for the hybrid synchronization of identical hyperchaotic Xu systems are established using Lyapunov stability theory. MATLAB simulations have been shown for the numerical results to illustrate the hybrid synchronization schemes derived for the identical hyperchaotic Xu systems.

KEYWORDS

Sliding Mode Control, Hybrid Synchronization, Hyperchaotic Systems, Hyperchaotic Xu System.

1. INTRODUCTION

Chaotic systems are dynamical systems showing the butterfly effect [1], i.e. they are highly sensitive to initial conditions. By synchronization of chaotic systems, we describe the problem of considering a pair of chaotic systems called master and slave systems and using a controller that drives the slave system to track the trajectories of the master system asymptotically.
Hyperchaotic system is a special type of chaotic systems having more than one positive Lyapunov exponent. The hyperchaotic system was first discovered by O.E. Rössler (1979) and the hyperchaotic systems possess special characteristics such as high capacity, high security and high efficiency. Such special characteristics make the hyperchaotic systems suitable for a broad range of applications in nonlinear circuits, lasers, secure communications, neural networks, biological systems, etc. There is a great deal of interest in the research problems such as control and synchronization of hyperchaotic systems and also in the circuit design of hyperchaotic systems.

A seminal paper on chaos synchronization is due to Pecora and Carroll ([2], 1990). Chaos theory has been well studied in the last few decades and found applications in a variety of fields such as physics [3], chemistry [4], ecology [5], secure communications [6-8], etc.

In the last two decades, various schemes have been successfully applied for chaos synchronization such as PC method [2], OGY method [9], active control method [10-14], adaptive control method [15-20], time-delay feedback method [21], backstepping design method [22], sampled-data feedback method [23], etc.

In hybrid synchronization of master and slave systems, the odd states are completely synchronized (CS) and the even states are anti-synchronized (AS). The co-existence of CS and AS in the synchronization enhances the security of secure communication systems.

In this paper, we derive new results based on the sliding mode control [24-26] for the hybrid synchronization of identical hyperchaotic Xu systems ([27], Xu, Cai and Zheng, 2009).

This paper has been organized as follows. In Section 2, we describe the problem statement and our main results on hybrid synchronization using sliding mode control (SMC). As an application of the main results derived in Section 2, we discuss the hybrid synchronization of identical hyperchaotic Xu systems in Section 3. Finally, we summarize the main results obtained in Section 4.

2. PROBLEM STATEMENT AND MAIN RESULTS

Consider the master system given by the dynamics

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in R^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : R^n \to R^n$ is the nonlinear part of the system.

Consider the slave system given by the dynamics

$$\dot{y} = Ay + f(y) + u \tag{2}$$

where $y \in R^n$ is the state of the system and $u \in R^m$ is the controller to be designed.
We define the *hybrid chaos synchronization error* as

$$e_i = \begin{cases} y_i - x_i & \text{if } i \text{ is odd} \\ y_i + x_i & \text{if } i \text{ is even} \end{cases} \tag{3}$$

A direct calculation gives the error dynamics as

$$\dot{e} = Ae + \eta(x, y) + u \tag{4}$$

The design goal is to find a sliding controller u such that

$$\lim_{t \to \infty} \| e(t) \| = 0 \quad \text{for all } e(0) \in R^n. \tag{5}$$

To solve this problem, we first define the control u as

$$u = -\eta(x, y) + Bv \tag{6}$$

where B is a constant gain vector selected such that (A, B) is controllable.

Substituting (6) into (4), the error dynamics simplifies to

$$\dot{e} = Ae + Bv \tag{7}$$

which is a linear time-invariant control system with single input v.

Thus, the original anti-synchronization problem can be replaced by an equivalent problem of stabilizing the zero solution $e = 0$ of the system (7) by a suitable choice of the sliding mode control. In the sliding mode control, we define the variable

$$s(e) = Ce = c_1 e_1 + c_2 e_2 + \cdots + c_n e_n \tag{8}$$

where $C = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}$ is a constant vector to be determined.

In the sliding mode control, we constrain the motion of the system (7) to the sliding manifold defined by

$$S = \left\{ e \in R^n \mid s(e) = 0 \right\}$$

which is required to be invariant under the flow of the error dynamics (7).

When in sliding manifold S, the system (7) satisfies the following conditions:

$$s(e) = 0 \tag{9}$$

which is the defining equation for the manifold S and

$$\dot{s}(e) = 0 \tag{10}$$

which is the necessary condition for the state trajectory $e(t)$ of (7) to stay on the sliding manifold S.

Using (7) and (8), the equation (10) can be rewritten as

$$\dot{s}(e) = C\left[Ae + Bv \right] = 0 \tag{11}$$

Solving (11) for v, we obtain the equivalent control law

$$v_{\text{eq}}(t) = -(CB)^{-1} CA \; e(t) \tag{12}$$

where C is chosen such that $CB \neq 0$.

If we substitute the equivalent control law (12) into the error dynamics (7), we get the closed-loop dynamics given by the equation

$$\dot{e} = \left[I - B(CB)^{-1} C \right] Ae \tag{13}$$

In Eq. (13), we choose the row vector C so that the matrix $\left[I - B(CB)^{-1} C \right] A$ is Hurwitz, *i.e.* it has all eigenvalues with negative real parts.

With this choice of C, the controlled system (13) is globally asymptotically stable.

To design the sliding mode controller for (7), we apply the constant plus proportional rate reaching law

$$\dot{s} = -q\,\text{sgn}(s) - k\ s \tag{14}$$

where $\text{sgn}(\cdot)$ denotes the sign function and the gains $q > 0$, $k > 0$ are determined such that the sliding condition is satisfied and sliding motion will occur.

From equations (11) and (14), we can obtain the control $v(t)$ as

$$v(t) = -(CB)^{-1}\left[C(kI + A)e + q\,\text{sgn}(s)\right] \tag{15}$$

which yields

$$v(t) = \begin{cases} -(CB)^{-1}\left[C(kI + A)e + q\right], & \text{if } s(e) > 0 \\ -(CB)^{-1}\left[C(kI + A)e - q\right], & \text{if } s(e) < 0 \end{cases} \tag{16}$$

Theorem 2.1. *The master system (1) and the slave system (2) are globally and asymptotically hybrid synchronized for all initial conditions* $x(0), y(0) \in R^n$ *by the feedback control law*

$$u(t) = -\eta(x, y) + Bv(t) \tag{17}$$

where $v(t)$ *is defined by (15) and* B *is a column vector such that* (A, B) *is controllable.*

Proof. First, we note that substituting (17) and (15) into the error dynamics (4), we obtain the closed-loop error dynamics as

$$\dot{e} = Ae - B(CB)^{-1}\left[C(kI + A)e + q\,\text{sgn}(s)\right] \tag{18}$$

To prove that the error dynamics (18) is globally asymptotically stable, we consider the candidate Lyapunov function defined by the equation

$$V(e) = \frac{1}{2}s^2(e) \tag{19}$$

which is a positive definite function on R^n.

Differentiating V along the trajectories of (18) or the equivalent dynamics (14), we get

$$\dot{V}(e) = s(e)\dot{s}(e) = -ks^2 - q\,\text{sgn}(s)s \tag{20}$$

which is a negative definite function on R^n.

Hence, by Lyapunov stability theory [28], error dynamics (18) is globally asymptotically stable for all initial conditions $e(0) \in R^n$.

This completes the proof. ■

3. HYBRID SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC XU SYSTEMS VIA SLIDING MODE CONTROL

3.1 Theoretical Results

As an application of the sliding mode control results derived in Section 2, we design a sliding controller for the hybrid synchronization of hyperchaotic Xu systems ([27], Xu *et al.* 2009).
Thus, the master system is described by the Xu dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4 \\ \dot{x}_2 &= bx_1 + fx_1x_3 \\ \dot{x}_3 &= -cx_3 - \varepsilon x_1x_2 \\ \dot{x}_4 &= -dx_4 + x_1x_3 \end{aligned} \tag{21}$$

where x_1, x_2, x_3, x_4 are state variables and $a, b, c, d, \varepsilon, f$ are positive, constant parameters of the system.

The Xu system (21) is *hyperchaotic* when the parameters are chosen as

$$a = 10, \quad b = 40, \quad c = 2.5, \quad d = 2, \quad \varepsilon = 1 \ \text{ and } \ f = 16$$

Figure 1 illustrates the phase portrait of the hyperchaotic Xu system.

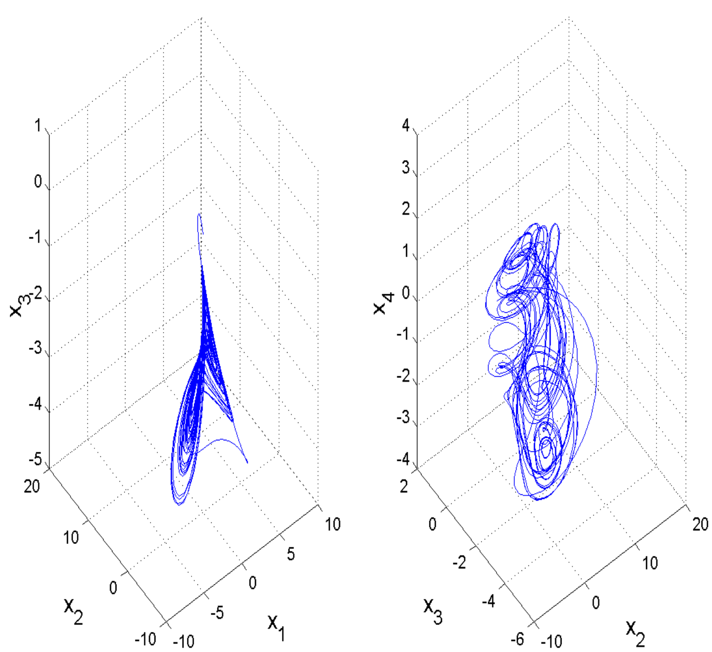

Figure 1. Phase Portrait of the Hyperchaotic Xu System

The slave system is described by the controlled hyperchaotic Xu dynamics

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + y_4 + u_1 \\
\dot{y}_2 &= by_1 + fy_1 y_3 + u_2 \\
\dot{y}_3 &= -cy_3 - \varepsilon y_1 y_2 + u_3 \\
\dot{y}_4 &= -dy_4 + y_1 y_3 + u_4
\end{aligned} \tag{22}$$

where y_1, y_2, y_3, y_4 are state variables and u_1, u_2, u_3, u_4 are the controllers to be designed.
Recall that the hybrid synchronization error is defined by

$$\begin{aligned}
e_1 &= y_1 - x_1 \\
e_2 &= y_2 + x_2 \\
e_3 &= y_3 - x_3 \\
e_4 &= y_4 + x_4
\end{aligned} \tag{23}$$

It is easy to see that the error dynamics is given by

$$\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) + e_4 - 2ax_2 - 2x_4 + u_1 \\
\dot{e}_2 &= be_1 + 2bx_1 + f(y_1 y_3 + x_1 x_3) + u_2 \\
\dot{e}_3 &= -ce_3 - \varepsilon y_1 y_2 + \varepsilon x_1 x_2 + u_3 \\
\dot{e}_4 &= -de_4 + y_1 y_3 + x_1 x_3 + u_4
\end{aligned} \tag{24}$$

The error dynamics (24) can be expressed in the matrix notation as

$$\dot{e} = Ae + \eta(x, y) + u \tag{25}$$

where

$$A = \begin{bmatrix} -a & a & 0 & 1 \\ b & 0 & 0 & 0 \\ 0 & 0 & -c & 0 \\ 0 & 0 & 0 & -d \end{bmatrix}, \quad \eta(x, y) = \begin{bmatrix} -2ax_2 - 2x_4 \\ 2bx_1 + f(y_1 y_3 + x_1 x_3) \\ -\varepsilon(y_1 y_2 - x_1 x_2) \\ y_1 y_3 + x_1 x_3 \end{bmatrix} \quad \text{and} \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{26}$$

As detailed in Section 2, we first set u as

$$u = -\eta(x, y) + Bv \tag{27}$$

where B is chosen such that (A, B) is controllable.

We take B as

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{28}$$

In the hyperchaotic case, the parameter values are taken as

$$a = 10, \quad b = 40, \quad c = 2.5, \quad d = 2, \quad \varepsilon = 1 \quad \text{and} \quad f = 16$$

The sliding mode variable is selected as

$$s = Ce = \begin{bmatrix} 9 & 1 & 1 & -9 \end{bmatrix} e = 9e_1 + e_2 + e_3 - 9e_4 \tag{29}$$

which makes the sliding mode state equation asymptotically stable.

A simple choice of the sliding mode gains is

$$k = 6 \text{ and } q = 0.2.$$

By applying Eq. (15), we can obtain $v(t)$ as

$$v(t) = -2e_1 - 48e_2 - 1.75e_3 + 13.5e_4 - 0.1\,\text{sgn}(s) \tag{30}$$

Thus, the required sliding mode controller is given by the equation

$$u = -\eta(x, y) + Bv \tag{31}$$

As an application of Theorem 2.1, we obtain the following result.

Theorem 3.1. *The identical hyperchaotic Xu systems (21) and (22) are hybrid synchronized for all initial conditions with the sliding mode controller* u *defined by (31).* ■

3.2 Numerical Results

In this section For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the hyperchaotic Xu systems (21) and (22) with the sliding mode controller u given by (31) using MATLAB.

In the hyperchaotic case, the parameter values are given by

$$a = 10, \quad b = 40, \quad c = 2.5, \quad d = 2, \quad \varepsilon = 1 \quad \text{and} \quad f = 16$$

The sliding mode gains are chosen as

$$k = 6 \text{ and } q = 0.2.$$

The initial values of the master system (21) are taken as

$$x_1(0) = 14, \; x_2(0) = 9, \; x_3(0) = 27, \quad x_4(0) = -10$$

The initial values of the slave system (22) are taken as

$$y_1(0) = 4, \; y_2(0) = 12, \; y_3(0) = -6, \; y_4(0) = 22$$

Figure 2 illustrates the hybrid synchronization of the hyperchaotic Xu systems (21) and (22).

Figure 3 illustrates the time-history of the synchronization errors e_1, e_2, e_3, e_4.

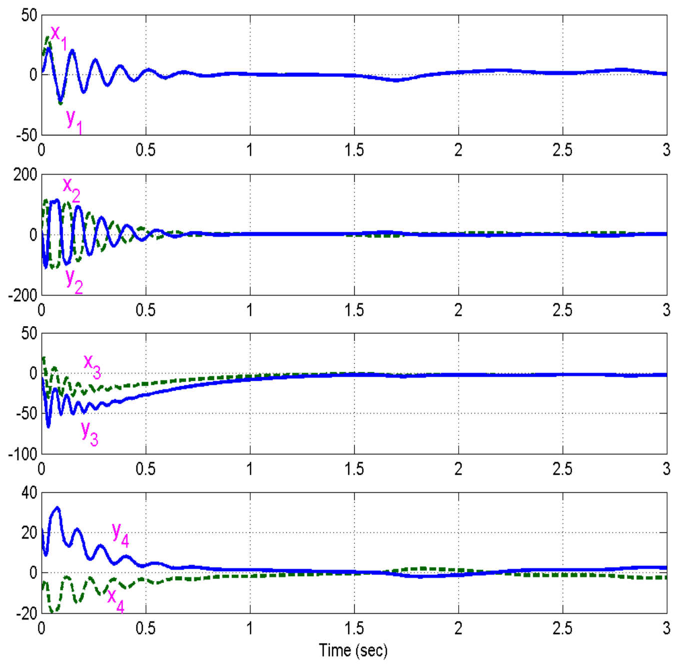

Figure 2. Hybrid Synchronization of Identical Hyperchaotic Xu Systems

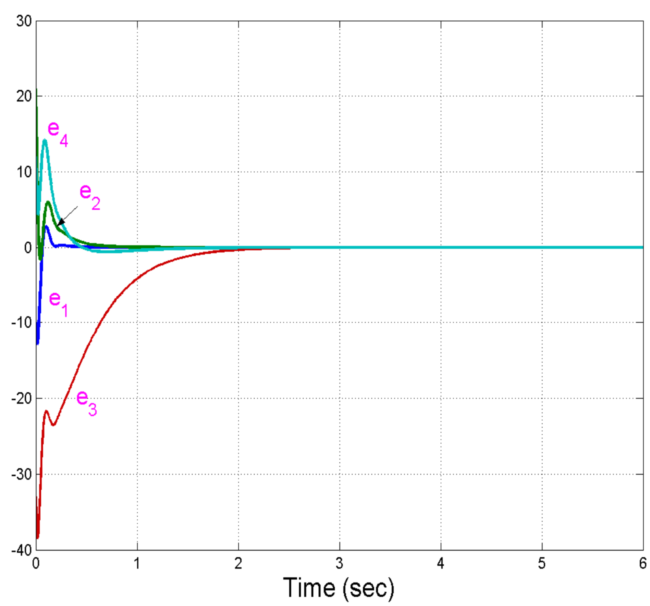

Figure 3. Time-History of the Hybrid Synchronization Error

4. CONCLUSIONS

In this paper, we have applied sliding mode control (SMC) to derive new results for the hybrid chaos synchronization of nonlinear chaotic systems. As an application of these main results, we have developed a sliding controller for the hybrid synchronization of hyperchaotic Xu systems (2009). Numerical simulations using MATLAB have been shown in detail to validate and illustrate the hybrid synchronization results derived for the hyperchaotic Xu systems.

REFERENCES

[1] Alligood, K.T., Sauer, T. & Yorke, J.A. (1997) *Chaos: An Introduction to Dynamical Systems*, Springer, New York.
[2] Pecora, L.M. & Carroll, T.L. (1990) "Synchronization in chaotic systems", *Phys. Rev. Lett*., Vol. 64, pp 821-824.
[3] Lakshmanan, M. & Murali, K. (1996) *Nonlinear Oscillators: Controlling and Synchronization*, World Scientific, Singapore.
[4] Han, S.K., Kerrer, C. & Kuramoto, Y. (1995) "Dephasing and burstling in coupled neural oscillators", *Phys. Rev. Lett.,* Vol. 75, pp 3190-3193.
[5] Blasius, B., Huppert, A. & Stone, L. (1999) "Complex dynamics and phase synchronization in spatially extended ecological system", *Nature*, Vol. 399, pp 354-359.
[6] Cuomo, K.M. & Oppenheim, A.V. (1993) "Circuit implementation of synchronized chaos with applications to communications," *Physical Review Letters*, Vol. 71, pp 65-68.
[7] Kocarev, L. & Parlitz, U. (1995) "General approach for chaotic synchronization with applications to communication," *Physical Review Letters*, Vol. 74, pp 5028-5030.
[8] Tao, Y. (1999) "Chaotic secure communication systems – history and new results," *Telecommun. Review*, Vol. 9, pp 597-634.
[9] Ott, E., Grebogi, C. & Yorke, J.A. (1990) "Controlling chaos", *Phys. Rev. Lett*., Vol. 64, pp 1196-1199.
[10] Ho, M.C. & Hung, Y.C. (2002) "Synchronization of two different chaotic systems using generalized active network," Physics Letters A, Vol. 301, pp 424-428.
[11] Huang, L., Feng, R. & Wang, M. (2005) "Synchronization of chaotic systems via nonlinear control," *Physical Letters A*, Vol. 320, pp 271-275.
[12] Chen, H.K. (2005) "Global chaos synchronization of new chaotic systems via nonlinear control," *Chaos, Solitons & Fractals*, Vol. 23, pp 1245-1251.
[13] Sundarapandian, V. (2011) "Global chaos synchronization of four-scroll and four-wing chaotic attractors by active nonlinear control," *International Journal on Computer Science and Engineering*, Vol. 3, No. 5, pp. 2145-2155.
[14] Sundarapandian, V. (2011) "Global chaos synchronization of Li and Liu-Chen-Liu chaotic systems by active nonlinear control," *International Journal of Advances in Science and Technology*, Vol. 3, No. 1, pp. 1-12.
[15] Lu, J., Wu, X., Han, X. & Lü, J. (2004) "Adaptive feedback synchronization of a unified chaotic system," *Physics Letters A*, Vol. 329, pp 327-333.
[16] Chen, S.H. & Lü, J. (2002) "Synchronization of an uncertain unified system via adaptive control," *Chaos, Solitons & Fractals*, Vol. 14, pp 643-647.
[17] Sundarapandian, V. (2011) "Adaptive control and synchronization of hyperchaotic Cai system", *International Journal of Control Theory and Computer Modelling*, Vol. 1, No. 1, pp 1-13.
[18] Sundarapandian, V. (2011) "Adaptive synchronization of hyperchaotic Lorenz and hyperchaotic Liu systems", *International Journal of Instrumentation and Control Systems*, Vol. 1, No. 1, pp 1-18.
[19] Sundarapandian, V. (2011) "Adaptive control and synchronization of Liu's four-wing chaotic system with cubic nonlinearity," International Journal of Computer Science, Engineering and Applications, Vol. 1, No. 4, pp 127-138.
[20] Sundarapandian, V. & Karthikeyan, R. (2011) "Global chaos synchronization of Pan and Lü chaotic systems via adaptive control," *International Journal of Information Technology, Convergence and Services,* Vol. 1, No. 5, pp. 49-66.
[21] Park, J.H. & Kwon, O.M. (2003) "A novel criterion for delayed feedback control of time-delay chaotic systems," *Chaos, Solitons & Fractals*, Vol. 17, pp 709-716.

[22] Wu, X. & Lü, J. (2003) "Parameter identification and backstepping control of uncertain Lü system," *Chaos, Solitons & Fractals*, Vol. 18, pp 721-729.
[23] Zhao, J. & J. Lu (2006) "Using sampled-data feedback control and linear feedback synchronization in a new hyperchaotic system," *Chaos, Solitons & Fractals*, Vol. 35, pp 376-382.
[24] Slotine, J.E. & Sastry, S.S. (1983) "Tracking control of nonlinear systems using sliding surface with application to robotic manipulators," *International Journal of Control*, Vol. 38, pp 465-492.
[25] Utkin, V.I. (1993) "Sliding mode control design principles and applications to electric drives," *IEEE Trans. Industrial Electronics*, Vol. 40, pp 23-36, 1993.
[26] Sundarapandian, V. (2011) "Global chaos synchronization of four-wing chaotic systems by sliding mode control", *International Journal of Control Theory and Computer Modelling*, Vol. 1, No. 1, pp 15-31.
[27] Xu, J., Cai, G. & Zheng, Song. (2009) "A novel hyperchaotic system and its control", *Journal of Uncertain Systems*, Vol. 3, No. 2, pp. 137-144.
[28] Hahn, W. (1967) *The Stability of Motion*, Springer, New York.

Author

Dr. V. Sundarapandian obtained his Doctor of Science degree in Electrical and Systems Engineering from Washington University, St. Louis, USA in May 1996. He is Professor and Dean of the Research and Development Centre at Vel Tech Dr. RR & Dr. SR Technical University, Chennai, Tamil Nadu, India. He has published over 285 refereed international journal publications. He has published over 175 papers in National and International Conferences. He is an India Chair of AIRCC and he is the Editor-in-Chief of the AIRCC Control Journals – IJICS, IJCTCM, IJITCA, IJCCMS and IJITMC. His research interests are Linear and Nonlinear Control Systems, Chaos Theory and Control, Soft Computing, Optimal Control, Operations Research, Mathematical Modelling and Scientific Computing. He has delivered several Key Note Lectures on Control Systems, Chaos Theory, Scientific Computing, Mathematical Modelling, MATLAB and SCILAB.