BLOCKWISE SOLID BURST ERROR CORRECTING CODES

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ABSTRACT

This paper presents a lower and upper bound for linear codes which are capable of correcting errors in the form of solid burst of different lengths within different sub blocks. An illustration of such kind of codes has also been provided.

KEYWORDS

Parity check matrix, Syndromes, Error correction, Solid burst.

1. INTRODUCTION

In most memory and storage system, the information is stored in various parts of the code length, known as sub-blocks. When error occurs in such a system, it does in a few places of the same sub-block. The errors which are likely to occur in one sub-block need not necessarily be of the type of errors in the other sub-blocks. Keeping this in view, there is a need to study block-wise error correcting (BEC) codes and this paper is in this direction. Dass and Tyagi [4] studied linear codes that are capable of correcting block wise burst error. These studies were further extended to the block-wise correction of errors which were in the form of repeated burst [5]. The development of codes correcting errors within a sub-block improves the efficiency of the communication channel as it reduces the number of parity-check digits required.

The nature of errors differs from channel to channel depending upon the behaviour of channels or the kind of errors which occur during the process of transmission. With the possibility of occurrence of solid burst error in various channels (viz. semiconductor memory data [6], supercomputer storage system [1]), one of the important areas in coding theory is to study solid burst error. A solid burst may be defined as follows:

Definition: *A solid burst of length s is a vector with non zero entries in some s consecutive positions and zero elsewhere.*

Schillinger [9] developed codes that correct solid burst errors. Shiva and Cheng [11] produced a paper for multiple solid burst error correcting codes in binary case with a very simple decoding scheme. Sharma and Dass [10] also studied solid burst error correcting perfect codes.

In a paper, Das [3] studies codes that detect and locate solid burst errors of length s or less within sub-blocks of same length. The idea of error locating codes is extendable to the codes that are able to correct errors occurring within sub-blocks of any length. In this correspondence, this paper obtains bounds on parity check digit that are required for correcting solid burst errors of length s_i or less lying within sub-block of length n_i in which the code length n is divided.

In what follows, the code length is taken to be n over GF(q), consisting of b mutually exclusive sub-blocks of lengths n_i such that $n = n_1+n_2+. . .+n_b$. The distance between two vectors shall be considered in the Hamming sense.

Note that an (n, k) linear code capable of correcting an error requires the syndromes of any two vectors to be different irrespective of whether they belong to the same sub-block or different sub-blocks. So, in order to correct solid burst errors of length s_i or less lying within a sub-block of length n_i, the following conditions need to be satisfied:

(a) The syndrome resulting from the occurrence of an solid burst error of length s_i or less must be non-zero and distinct from the syndromes resulting from any other solid burst errors of length s_i or less within the *same* sub-block of length n_i .
(b) The syndrome resulting from the occurrence of solid burst error of length s_i or less lying within a sub-block of length n_i must be distinct from the syndrome resulting likewise from any solid burst error of length s_j or less within *any other* sub block of length n_j, $i \neq j$.

Two results are derived in this paper. The first result gives a lower bound on the number of check digits required for the existence of a linear code over GF(q) capable of correcting errors that are solid burst of length s_i or less lying within a sub-block of length n_i. In the second result, an upper bound on the number of check digits which ensures the existence of such a code is derived. An illustration of such a code over GF(2) is also given.

2. A LOWER BOUND

The following theorem gives a lower bound on the necessary number of parity check digits required for a code that corrects solid burst of length s_i or less lying within a single sub-block of length n_i (i=1,2, . . . , b). The proof is based on the technique used in theorem 4.13, Peterson and Weldon [7].

Theorem 1 An (n, k) linear code over GF(q) is subdivided into b mutually exclusive sub-blocks of length n_i such that $n = n_1+n_2+. . .+n_b$. The number of parity check digits required for the code that corrects solid burst of length s_i or less lying within a single sub-block of length n_i is at least

$$loq_q\left\{1+\sum_{i=1}^{b}\sum_{\alpha=1}^{s_i}(n_i-\alpha+1)(q-1)^{\alpha}\right\}. \qquad (1)$$

Proof. Let there be an (n, k) linear code vector over GF(q) that corrects all solid burst of length s_i or less within a single corrupted sub-block of length n_i. The maximum number of distinct syndromes available using n-k check bits is q^{n-k}. The proof proceeds by first counting the number of syndromes that are required to be distinct by condition (a) and (b) and then setting this number less than or equal to q^{n-k}. First we consider a sub-block, say i-th sub-block of length n_i. Since the code is capable of correcting all errors which are solid burst of length s_i or less within the sub-block of length n_i, any syndrome produced by an solid burst of length s_i or less in the given sub-block must be different from any such syndrome likewise resulting from solid burst of length s_i or less in the same sub-block by condition (a).

Also by condition (b), syndromes produced by solid burst of length s_j or less in different sub-blocks must be distinct. Thus the syndromes produced by solid burst of length s_i or less, whether in the same sub-block or in different sub-blocks should be distinct.

Since the number of solid burst of length s_i or less within the sub-block of length n_i, excluding the vector of all zeros, is (refer Das[3])

$$\sum_{\alpha=1}^{s_i}(n_i-\alpha+1)(q-1)^{\alpha}\,. \tag{2}$$

So we must have at least $1+\sum_{i=1}^{b}\sum_{\alpha=1}^{s_i}(n_i-\alpha+1)(q-1)^{\alpha}$ distinct syndromes, including the all zero syndrome.

Therefore we must have

$$q^{n-k} \geq 1+\sum_{i=1}^{b}\sum_{\alpha=1}^{s_i}(n_i-\alpha+1)(q-1)^{\alpha} \tag{3}$$

or,

$$\text{n-k} \geq \; loq_q\left\{1+\sum_{i=1}^{b}\sum_{\alpha=1}^{s_i}(n_i-\alpha+1)(q-1)^{\alpha}\right\}.$$

Discussion 1.1. Taking $n_1 = n_2 = \ldots = n_b = N$ (say) and $s_1 = s_2 = \ldots = s_b = s$ (say), then the expression (1) reduces to

$$loq_q\left\{1+b\sum_{\alpha=1}^{s}(N-\alpha+1)(q-1)^{\alpha}\right\}.$$

This gives a bound when the lengths of the sub blocks and the solid bursts are same.

Discussion 1.2. Taking $b = 1$, $n_1 = N$ and $s_1 = s$, then the expression (1) reduces to

$$loq_q\left\{1+\sum_{\alpha=1}^{s}(N-\alpha+1)(q-1)^{\alpha}\right\},$$

which coincides with the result of the theorem 3, obtained by Das [3].

3. AN UPPER BOUND

In the following theorem, another bound on the number of parity check digits required for the existence of a code considered in **Theorem 1**.

The proof involves suitable modifications of the technique used in deriving Varshamov-Gilbert-Sacks bound by constructing a parity check matrix for such a code (refer Sacks [8], also Theorem 4.7, Peterson and Weldon [7]). This technique not only ensures the existence of such a code but also gives a method for the construction of such a code.

Theorem 2 An (n, k) linear code over GF(q) is subdivided into b mutually exclusive sub-blocks of length n_i such that $n = n_1+n_2+\ldots+n_b$. Such a code capable of correcting an solid burst of length s_i or less ($n_i > 2s_i$), occurring within a single sub-block of length n_i can always be constructed provided that

$$q^{n-k} \geq 1+\sum_{i=1}^{s_b}\sum_{l=1}^{s_b}(n_b-i-l+1)(q-1)^{i+l-1}+\sum_{i=1}^{s_b-1}(q-1)^i\times\sum_{i=1}^{b-1}\sum_{\alpha=1}^{s_i}(n_i-\alpha+1)(q-1)^{\alpha}\ . \qquad (4)$$

Proof. In order to prove the existence of such a code we construct an (n-k)×n parity check matrix H for such a code by a synthesis procedure as follows:

After adding $\sum_{i=1}^{b-1} n_i$ columns appropriately corresponding to the first (b-1) sub-blocks, suppose that we have added the first j-1 columns $h_1, h_2, \ldots, h_{j-1}$ of the b-th sub-block. We now lay down the conditions to add the j-th column h_j as follows:

According to the condition (a), for the correction of solid burst error of length s_b or less within the sub-block of length n_b, the syndrome of any solid burst errors of length s_b or less within the sub-block must be different from the syndrome resulting from any other solid burst error of length s_b or less within the *same* sub-block.

So h_j can be added provided it is not a linear sum of immediately preceding s' (s'<s_b) columns $h_{j-1}, h_{j-2}, \ldots, h_{j-s'}$, together with any other linear sum of s_b or less consecutive columns among the first j-1-s' columns of the b-th sub-block. i.e.,

$$\begin{aligned} h_j \neq\ &(u_{j-1}h_{j-1}+u_{j-2}h_{j-2}+\ldots+u_{j-s'+1}h_{j-s'+1}+u_{j-s'}h_{j-s'}) \\ &+(v_ih_i+v_{i+1}h_{i+1}+\ldots+v_{i+s''-1}h_{i+s''-1}), \end{aligned} \qquad (5)$$

where s'' are less or equal to s_b, j ≥ s', the columns h_i in the second bracket are any s_b or less consecutive columns among the first (j-1-s') columns in the b-th sub-block and all the coefficients u_i and v_i are non-zero.

Thus, the coefficients u_i form a solid burst of length s' and the coefficients v_i form a solid burst of length s_b or less in a (j-1-s')-tuple.

The number of possible linear combinations on R.H.S. of (5), including the zero vector, is (refer Das [3])

$$1+\sum_{i=1}^{s_b}\sum_{l=1}^{s_b}(j-i-l+1)(q-1)^{i+l-1}\ . \qquad (6)$$

Now according to condition (b), the syndrome of any solid burst error of length s_b or less within the sub-block of length n_b must be different from the syndrome resulting from any solid burst error of length s_i or less within any *other* sub-block of length n_i, i≠b.
In view of this, h_j can be added provided that

$$\begin{aligned} h_j \neq\ &(u_{j-1}h_{j-1}+u_{j-2}h_{j-2}+\ldots+u_{j-s'+1}h_{j-s'+1}+u_{j-s'}h_{j-s'}) \\ &+(v_ih_i^m+v_{i+1}h_{i+1}^m+\ldots+v_{i+s''-1}h_{i+s''-1}^m), \end{aligned} \qquad (7)$$

where $s' < s_b$, $s'' \le s_m$, j ≥ s', h_i^m 's in the second bracket are the columns corresponding to *m*-th sub-block among the previous b-1 sub-blocks and all the coefficients u_i and v_i are non-zero.

The number of ways in which the coefficients u_i can be selected is

$$\sum_{i=1}^{s_b-1}(q-1)^i . \qquad (8)$$

To enumerate the coefficients v_i is equivalent to enumerate the number of solid burst errors of length s_i or less in a vector of length n_i. This number of solid burst errors of length s_i or less within a sub-block of length n_i, excluding the vector of all zeros, is given by (2).

Since there are b-1 previously chosen sub-blocks of length n_i (i=1, 2, . . , b-1) in which solid burst of length s_i or less are to be corrected, therefore number of such linear combinations becomes

$$\sum_{i=1}^{b-1} Expr.(2) . \qquad (9)$$

Thus for the block-wise correction of solid burst errors of length s_b or less, the number of linear combinations to which h_j can not be equal to is the sum of linear combinations computed in (6) and (9).

At worst all these combinations might yield distinct sum.

Therefore h_j can be added to the b-th sub-block of H provided that

$$q^{n-k} > \text{expr. (6)} + \text{expr. (9)}$$

or,

$$q^{n-k} \quad \ge \quad 1+\sum_{i=1}^{s_b}\sum_{l=1}^{s_b}(j-i-l+1)(q-1)^{i+l-1}+\sum_{i=1}^{s_b-1}(q-1)^i \times \sum_{i=1}^{b-1}\sum_{\alpha=1}^{s_i}(n_i-\alpha+1)(q-1)^{\alpha} .$$

Replacing j by n_b, where n_b is the length of the b-th sub-block, gives the inequality (4) stated in the theorem.

Discussion 2.1. Taking $n_1 = n_2 = \ldots = n_b = N$ (say) and $s_1 = s_2 = \ldots = s_b = s$ (say), then the expression (4) reduces to

$$q^{n-k} \ge 1+\sum_{i=1}^{s}\sum_{l=1}^{s}(N-i-l+1)(q-1)^{i+l-1}+(b-1)\sum_{\alpha=1}^{s}\sum_{l=1}^{s-1}(N-\alpha+1)(q-1)^{\alpha+l} .$$

This gives an upper bound when the lengths of the sub blocks and the solid bursts are same.

Discussion 2.2. Taking b = 1, $n_1 = N$ and $s_1 = s$, then the expression (4) reduces to

$$q^{n-k} \ge 1+\sum_{i=1}^{s}\sum_{l=1}^{s}(N-i-l+1)(q-1)^{i+l-1} ,$$

which coincides with the result of the theorem 4, Das [3].

4. AN ILLUSTRATION

Consider a (14, 8) binary code with the 6 ×14 matrix H which has been constructed by the synthesis procedure given in the proof of theorem 2 by taking b=2, $s_1=s_2=3$, $n_1=7$, $n_2=7$, q=2.

$$H=\begin{bmatrix} 1000001 & 1000111 \\ 0100001 & 0100011 \\ 0010000 & 1010001 \\ 0001001 & 0101100 \\ 0000100 & 0010010 \\ 0000010 & 0001001 \end{bmatrix}$$

The null space of this matrix will give a code that corrects solid burst errors of length 3 or less within the two sub-blocks of length 3 each. It may be easily verified from error pattern-syndromes **table 1** that:

(i) Syndromes of all solid burst errors of length 3 or less within any one sub-block are all non-zero.
(ii) The syndrome of a solid burst error of length 3 or less within any sub-block is different from the syndrome of a solid burst error of length 3 or less within the *same* sub-block.
(iii) The syndrome of a solid burst error of length 3 or less within any sub-block is different from the syndrome of a solid burst error of length 3 or less within the *other* sub-block.

Table 1

Error patterns	Syndromes	Error patterns	Syndromes
1st sub-block		2nd sub-block	
1000000 0000000	100000	0000000 1000000	101000
0100000 0000000	010000	0000000 0100000	010100
0010000 0000000	001000	0000000 0010000	001010
0001000 0000000	000100	0000000 0001000	000101
0000100 0000000	000010	0000000 0000100	100100
0000010 0000000	000001	0000000 0000010	110010
0000001 0000000	110100	0000000 0000001	111001
1100000 0000000	110000	0000000 1100000	111100
0110000 0000000	011000	0000000 0110000	011110
0011000 0000000	001100	0000000 0011000	001111
0001100 0000000	000110	0000000 0001100	100001
0000110 0000000	000011	0000000 0000110	010110
0000011 0000000	110101	0000000 0000011	001011
1110000 0000000	111000	0000000 1110000	110110
0111000 0000000	011100	0000000 0111000	011011
0011100 0000000	001110	0000000 0011100	101011
0001110 0000000	000111	0000000 0001110	010011
0000111 0000000	110111	0000000 0000111	101111

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