FEEDBACK ADAPTIVE CONTROL OF A CLASS OF NONLINEAR SYSTEMS USING FUZZY APPROXIMATORS

Mohamed Bahita[1] and Khaled Belarbi[2]

[1]Department of Automatic Control, Faculty of Hydrocarbons and Chemistry, Boumerdes University, Boumerdes 35000, Algeria
mbahita@yahoo.fr
[2] Department of Automatic Control, Faculty of engineering, Constantine University, 25000, Algeria
kbelarbi@yahoo.com

ABSTRACT

This paper describes the design of an adaptive direct control scheme for a class of nonlinear systems. The architecture is based on a fuzzy inference system (FIS) of Takagi Sugeno (TS) type to approximate a feedback linearization control law. The parameters of the consequent part of the fuzzy system are adapted and changed according to a law derived using Lyapunov stability theory. The asymptotic Lyapunov stability will be established with the tracking errors converging to a neighborhood of the origin. Finally, the adaptive direct fuzzy controller is applied in simulation to control three nonlinear systems.

KEYWORDS

Adaptive control, Feedback Linearization, Lyapunov Stability, TS Fuzzy Inference System, Nonlinear systems

1. INTRODUCTION

Conventional PID controllers are still the most widely adopted method in industry for various control applications due to their simple structure, ease of design, and low cost in implementation. For example, in one part of the work of Maidi et al [1], a design of the control (of PID type) of the internal fluid temperature at the outlet of a parallel-flow heat exchanger by manipulating the inlet external fluid temperature is proposed. However, PID controllers might not perform satisfactorily if the system to be controlled is of highly nonlinear and/or uncertain nature, such as aeronautics, robotics. On the other hand, conventional fuzzy control has long been known for its ability to handle nonlinearities and uncertainties through use of fuzzy set theory. It is thus believed that by combining these two techniques together a better control system can be achieved [2]. Fuzzy logic controller (FLC) has proven to be a successful control approach to many complex nonlinear systems [3]-[4]. In other words, fuzzy control is useful in situations where there is no acceptable mathematical model for the plant and where there are experienced human operators who can satisfactorily control the plant and provide qualitative control rules in terms of vague and fuzzy sentences. Based on the differences of fuzzy control rules and their generation methods, approaches to fuzzy logic control can be roughly classified into the following categories [4]: i) *Conventional fuzzy control*; ii) *fuzzy proportional-integral-derivative (PID) control*; iii) *neuro-fuzzy control*; iv) *fuzzy-sliding mode control*; v) *adaptive fuzzy control*; and vi) *Takagi–Sugeno (T–S) model-based fuzzy control*. However, it should be noted that the overlapping among these categories is inevitable. For example, conventional fuzzy control can be adaptive, fuzzy PID control can be tuned by neuro-fuzzy systems, or neuro-fuzzy control is adaptive in nature in many cases.

An adaptive controller is a controller that can modify its behaviour in response to changes in the dynamics of the process. An adaptive controller is formed by combining an on-line parameter estimator, which provides estimates of unknown parameters at each instant. In the direct adaptive control approach, the plant model is parameterized in terms of the controller parameters that are estimated directly without intermediate calculations involving plant parameter estimates. Whereas, when the plant parameters are estimated on-line and used to calculate the controller parameters, this approach is referred to as indirect adaptive control [5]-[6]. Adaptive fuzzy controller is a system that combines adaptive control theory and fuzzy systems. Based on this, a great number of works on adaptive fuzzy control have been proposed [6]-[9] where the general approach is usually based on the feedback linearization technique [10], and the used fuzzy inference system is introduced for approximating part or all the components of the control law. This last nonlinear control theory or feedback linearization technique is based on coordinate transformations by which a class of nonlinear systems can be transformed into linear systems through feedback. Hence the word feedback linearization denoting such methodology. With the advent of feedback linearization, both adaptive and fuzzy adaptive control found their way into nonlinear control. The combination of adaptive control and feedback linearization applied to fight control can be found in [11].

In most cases however [8]-[9], [12]-[14], a complementary term, called a supervisory controller, is added to the output of the fuzzy inference system as a part of the control law in order to guarantee the global stability using the Lyapunov theory. When the system is operating within the prescribed range, the supervisory controller is turned off. It is activated only if the system tends to go beyond the prescribed tolerance.

The main theme of the research presented in this work is the development of an alternative feedback direct fuzzy adaptive controller for a class of affine in control nonlinear systems. The architecture employs a fuzzy system of Takagi-Sugeno (T-S) type tow on line approximate the ideal control law which can not be computed, because of the lack of system dynamics knowledge as we will see later. To compare our work with related works, the authors is [8] used a fuzzy system of Mamdani type to approximate the control law instead of our (T-S) type and added the supervisory controller to the output of the fuzzy inference system as a part of the control law. In [14], the authors used a fuzzy inference system of T-S type to approximate the control law and added also the supervisory controller. Based on the initial proposal works [8], [14] we construct our feedback fuzzy direct adaptive control of Takagi-Sugeno (T-S) type for SISO nonlinear systems, and based on the universal function approximation property of fuzzy systems [4], [16], [17], we use only one part of control law without use of the supervisory controller (the second part) as done in [8] and [14]. As a comparison to [14], where the authors used six (6) fuzzy membership functions to define the state space, in our work we minimize the number of this membership functions to only two (2) for every input to the TS fuzzy controller, thus the number of rules is minimized where we have got good results as we will see in the simulation results section. The parameters of the consequent part of the fuzzy system are adapted and changed according to a law derived using Lyapunov stability theory where the asymptotic stability is established with the tracking errors converging to a neighborhood of the origin.

This work is organised as follows: in section 2, the problem formulation is introduced, in section 3, the stability analysis is developed and the adaptive laws are derived, in section 4, the adaptive direct fuzzy controller is applied in simulation to control three nonlinear systems, and section 5 concludes the paper.

2. PROBLEM FORMULATION

Consider a non linear system that can be transformed into the following form [10]:

$$x^{(n)} = f(x, \dot{x}, \ldots x^{(n-1)}) + b.u, \qquad y = x \tag{1}$$

Where $u \in R$ and $y \in R$ are the input and output of the system respectively, $f(x)$ is a non linear function and b is a positive constant (this is a usual assumption [7], [15]. We assume that the state vector $\underline{x} = (x_1, x_2, \ldots, x_n)^T = (x, \dot{x}, \ldots x^{(n-1)})^T \in R^n$ is available for measurement. The control objective is to force the output y to follow a given bounded reference signal $y_m(t)$, under the constraints that all signals involved must be bounded. More specifically, determine a feedback control estimation $u(\underline{x}, \underline{\theta})$ of u, and all this is based on a fuzzy inference system FIS of Takagi-Sugeno TS type. Determine also an adaptive law using Lyapunov theory for adjusting the parameters vectors $\underline{\theta}$ such that the closed-loop system must be globally stable in the sense that all variables must be uniformly bounded and the tracking error $e = y - y_m$ should be as small as possible. Define now the error vector as

$$\underline{e} = (e, \dot{e}, \ldots, e^{(n-1)})^T \in R^n \tag{2}$$

Step1: We choose u to cancel the nonlinearities in a nonlinear system so that the closed-loop dynamics is in a linear form, and guarantees tracking convergence based on a feedback linearization theory. If the function $f(\underline{x})$ is known and assuming b to be positive constant, then, from (1), the optimal control law is

$$u^{\bullet} = \frac{1}{b}.(v - f(\underline{x})) \tag{3}$$

Substituting (3) into (1), we can cancel the nonlinearities and obtain the simple input-state relation

$$x^{(n)} = v \tag{4}$$

Step2: We choose the artificial input v (an equivalent input) as a simple linear pole-placement controller $v = y_m^{(n)} - K^T.\underline{e}$ that provides guarantee about the stability of the overall system, with the vector

$$K = (k_0, k_1, \ldots, k_{n-1})^T \in R^n \tag{5}$$

chosen so that the polynomial :

$$s^n + k_{n-1}.s^{n-1} + \ldots\ldots + k_0 = 0 \tag{6}$$

has all its roots strictly in the left-half complex plane. Then the optimal control law is:

$$u^{\bullet} = \frac{1}{b}.(y_m^{(n)} - K^T.\underline{e} - f(\underline{x})) \tag{7}$$

based on $e = y - y_m$ then

$$e^{(n)} = y^{(n)} - y_m^{(n)} \tag{8}$$

Substituting (7) into (1), using (8) and based on $y = x$ (see(1)) we have :

$$e^{(n)} + k_{n-1}.e^{(n-1)} + ... + k_0.e = 0 \tag{9}$$

This implies that $\lim_{t\to\infty} e(t) = 0$ (exponentially stable dynamics), which is the main objective of control. Since $f(x)$ is unknown, the optimal control $u^{\bullet}$ of (7) can not be implemented. Our purpose is to design a TS system with output $u(\underline{x},\underline{\theta})$ to approximate this optimal control law as will be described in the following section.

3. THE TS FUZZY ADAPTIVE CONTROLLER

A TS fuzzy inference system with linear consequences is composed of rules of the form:

R^i : *if* z_1 *is* A_1^i *and* ...z_n *is* A_n^i *then* $u_i = a_0^i + a_1^i z_1 + ... + a_n^i z_n$,

where $z_1 \ldots z_n$ are functions of state variables. A_j^i are fuzzy sets. If we take $\theta_i^T = [a_0^i \; a_1^i ... a_n^i]$ as the vector of adjustable parameters of the consequence of rule R^i. The output of a TS fuzzy system can be put in the following form

$$u_c(\underline{x}) = \underline{\theta}^T.\underline{\xi}(\underline{x}) \tag{10}$$

where $\underline{\theta}^T = [\theta_1^T \; \theta_2^T ... \theta_n^T]$ contains all adjustable parameters and $\underline{\xi}(\underline{x})$ is a vector of fuzzy basis functions. It has been proven that (10) can approximate over a compact set Ω_Z, any smooth function up to a given degree of accuracy [4], [16], [17]. It can thus be used to approximate the ideal control law $u^{\bullet}$ as given in (7). In the following, we derive the adaptation law for the parameters of the fuzzy TS system using Lyapunov synthesis approach. As mentioned in section 2 (Problem formulation), since $f(\underline{x})$ is unknown, the optimal control $u^{\bullet}$ of (7) can not be implemented. Our purpose is then to design a fuzzy inference system FIS of TS type with output $u(\underline{x},\underline{\theta})$ to approximate this optimal control law. Thus, we replace the control input u in (1) by the FIS system with output $u(\underline{x},\underline{\theta})$, then (1) becomes:

$$x^{(n)} = f(\underline{x}) + b.u(\underline{x},\underline{\theta}) \tag{11}$$

Now adding and subtracting $b.u^{\bullet}$ to (11) we will have:

$$x^{(n)} = f(\underline{x}) + b.u(\underline{x},\underline{\theta}) + b.u^{\bullet} - b.u^{\bullet} \tag{12}$$

Substituting (7) into (12), we obtain:

$$x^{(n)} = f(\underline{x}) + b.u(\underline{x},\underline{\theta}) - b.u^{\bullet} + y_m^{(n)} - K^T.\underline{e} - f(\underline{x}) \tag{13}$$

thus:

$$x^{(n)} - y_m^{(n)} = -K^T.\underline{e} + b.(u(\underline{x},\underline{\theta}) - .u^{\bullet}) \tag{14}$$

Based on $y = x$ in (1) and using (2) and (8), equation (14) leads to the error system

$$\underline{\dot{e}} = A_c.\underline{e} + b_c[b.(u(\underline{x},\underline{\theta}) - .u^{\bullet})] \quad (15)$$

with

$$A_c = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ & & & & & \\ 0 & 0 & 0 & \dots & 0 & 1 \\ -k_0 & -k_1 & -k_2 & \dots & -k_{n-2} & -k_{n-1} \end{bmatrix}, \quad b_c = \begin{bmatrix} 0 \\ 0 \\ . \\ 0 \\ 1 \end{bmatrix} \quad (16)$$

Let's now study the stability of the system in order to develop an adaptive law to adjust the parameter vector $\underline{\theta}$ of the fuzzy controller. Define the optimal parameter vector $\underline{\theta}^{\bullet}$ as the parameter vector which corresponds to the best (optimal) approximator term $u(\underline{x},\underline{\theta}^{\bullet})$ of the optimal control signal $u^{\bullet}$ of (7). Based on this and by using an appropriate fuzzy approximator [4], [16], [17], we can write

$$u^{\bullet} \approx u(\underline{x},\underline{\theta}^{\bullet}). \quad (17)$$

Thus, the error equation (15) can be rewritten as

$$\underline{\dot{e}} = A_c.\underline{e} + b_c[b.(u(\underline{x},\underline{\theta}) - u(\underline{x},\underline{\theta}^{\bullet}))] \quad (18)$$

Based on (10) we have

$$u(\underline{x},\underline{\theta}) = \underline{\theta}^T.\underline{\xi}(\underline{x}), \text{ and } u(\underline{x},\underline{\theta}^{\bullet}) = \underline{\theta}^{\bullet T}.\underline{\xi}(\underline{x}) \quad (19)$$

let $\varphi = \underline{\theta} - \underline{\theta}^{\bullet}$ and using (19), thus (18) becomes

$$\underline{\dot{e}} = A_c.\underline{e} + b_c.b.\varphi^T.\underline{\xi}(\underline{x}) \quad (20)$$

Define the Lyapunov function candidate:

$$V = \frac{1}{2}\underline{e}^T.P.\underline{e} + \frac{b}{2.\gamma}\varphi^T\varphi \quad (21)$$

Where γ is a positive constant and P is a solution of the Lyapunov equation:

$$A_c^T.P + PA_c = -Q \text{ with } Q > 0. \quad (22)$$

Differentiate V with respect to time:

$$\dot{V} = \frac{1}{2}\underline{\dot{e}}^T.P.\underline{e} + \frac{1}{2}\underline{e}^T.P.\underline{\dot{e}} + \frac{b}{2\gamma}\dot{\varphi}^T\varphi + \frac{b}{2\gamma}\varphi^T\dot{\varphi} \quad (23)$$

using (20) and (22), we have :

$$\dot{V} = -\frac{1}{2}\underline{e}^T Q\underline{e} + \underline{e}^T Pb_c b.\varphi^T \underline{\xi}(\underline{x}) + \frac{b}{\gamma}\varphi^T \dot{\varphi} \tag{24}$$

Let P_n be the last column of *P*, and using (16) we obtain:

$$\underline{e}^T Pb_c = \underline{e}^T P_n \tag{25}$$

Substituting (25) into (24), we obtain

$$\dot{V} = -\frac{1}{2}\underline{e}^T Q\underline{e} + \frac{b}{\gamma}\varphi^T [\gamma.\underline{e}^T P_n \underline{\xi}(\underline{x}) + \dot{\varphi}] \tag{26}$$

If we choose the adaptive law:

$$\underline{\dot{\theta}} = -\gamma.\underline{e}^T P_n \underline{\xi}(\underline{x}) \tag{27}$$

This will result in

$$\frac{b}{\gamma}\varphi^T (\gamma.\underline{e}^T P_n \underline{\xi}(\underline{x}) + \dot{\varphi}) = 0 \tag{28}$$

We use the fact that $\dot{\varphi} = \underline{\dot{\theta}} - \underline{\dot{\theta}}^{\bullet} = \underline{\dot{\theta}}$, because the optimal parameter vector $\underline{\theta}^{\bullet}$ is constant and obviously its derivative is zero, i.e., $\underline{\dot{\theta}}^{\bullet} = 0$, then (26) becomes

$$\dot{V} = -\frac{1}{2}\underline{e}^T Q\underline{e} \tag{29}$$

From (22) we have $Q > 0$, it follows that:

$$\dot{V} = -\frac{1}{2}\underline{e}^T .Q.\underline{e} \le 0 \tag{30}$$

According to Lyapunov stability theory, i.e., $\dot{V}$ is negative definite (or semi-definite), we conclude that $\lim_{t\to\infty}\|\underline{e}(t)\| = 0$, which is the objective.

Remark 1:

In the above developments, stability results are provided using Lyapunov theory without use of the compensatory (supervisory) control term in addition to the control law as usually done in most cases [8], [14] as mentioned in the introduction.

Remark 2:

To understand how to apply our proposed feedback fuzzy (T-S) adaptive controller scheme in the simulation section, we summarize the leading steps in the following.

Step 1. Off line computations

- Specify the parameters: $k_0, ..., k_{n-1}$, such that all roots of $s^n + k_{n-1}.s^{n-1} + ... + k_0 = 0$ are in the open left-half plane.
- Specify a positive definite $n\,x\,n$ matrix Q.
- Solve the Lyapunov equation (22) to obtain a symmetric matrix $P > 0$.
- Specify the premise parameters of the (T-S) fuzzy controller, which are the centres, the widths and the shape of each basis function (membership functions) for each input to the (T-S) controller.
- give an initial value to any constant to be initialized, for example the parameters of the consequent part of the (T-S) controller and so on.

Step 2. On-line adaptation

- Apply the feedback control (10) to the plant (1), where u_c in (10) is the output of the (T-S) fuzzy system (controller).
- Use the adaptive law (27) to adjust the connections weights $\underline{\theta}$ of the (T-S) controller.

4. SIMULATION RESULTS

4.1. Example 1

In this example, we apply the direct adaptive fuzzy controller to regulate to the origin an unstable system where the dynamic equation as given in [8], [14] is as follows:

$$\dot{x}(t) = \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} + u(t) \qquad (31)$$

From (31), if the input $u(t) = 0$, we have: $\dot{x}(t) = \dfrac{1 - e^{-x(t)}}{1 + e^{-x(t)}} > 0$ for $x(t) > 0$, and $\dot{x}(t) = \dfrac{1 - e^{-x(t)}}{1 + e^{-x(t)}} < 0$ for $x(t) < 0$. Based on this, we can confirm that the plant (31) is unstable. $y = x$ is the output of the system in (31). The reference signal will be $y_m = 0$. According to the steps of the design procedure given in remark 2, the parameters are chosen as $\gamma = 0.8$, step size $dt = 0.2$, and $k = k_0 = 7.5$ in order to have all roots of $s + k_0 = 0$ in the left-half plane. We chose Q in (22) as $Q = 5 > 0$. Then by solving (22) we can obtain $P = 0.3333$. The (T-S) fuzzy controller has three inputs $\underline{z} = [z_1 \;\; z_2 \;\; z_3] = [x \quad x_m \quad (\dot{x}_m - k^T \underline{e})]$ with $\underline{e} = [x - x_m]$. All the three inputs to the (T-S) fuzzy controller are fuzzified with two fuzzy sets and similar Gaussian membership functions given by:

$$\mu_N(z_i) = \exp(-(z_i - c_N)^2 / 2.\sigma_N) \qquad (32)$$

$$\mu_P(z_i) = \exp(-(z_i - c_P)^2 / 2.\sigma_P) \qquad (33)$$

where z_i stands for the input number i. The widths of the membership functions are $\sigma_P = \sigma_N = 0.7$ for the first and the second input z_1, z_2 and are $\sigma_P = \sigma_N = 0.2$ for the third input z_3. The centres are set to $c_N = -1$, $c_P = 1$ for the three inputs z_1, z_2 and z_3.
This gives eight rules of the form:

R^i : *if* z_1 *is* A_1^i *and* z_2 *is* A_2^i *and* z_3 *is* A_3^i
then $u_i = a_1^i z_1 + a_2^i z_2 + a_3^i z_3$, with $i = 1$ to 8 (34)

We have 24 parameters to tune. All parameters (a_1^i, a_2^i and a_3^i) are initialised to zero. The initial conditions are $x(0) = 1$. Figure 1 shows the system state $x(t)$ and the desired position $x_m(t)$. We see from this Figure 1 that the proposed fuzzy direct adaptive control could lead rapidly the plant to the origin, i.e., $y_m(t) = x_m(t) = 0$. Figure 2 shows the corresponding control input $u(t)$. Clearly both the state and the control signal are bounded. Compared with the result in [8], [14], a good improvement on our system performance is observed, especially the response time (1.6 sec in our system and 8 sec in [8] and 11 sec in [14]).

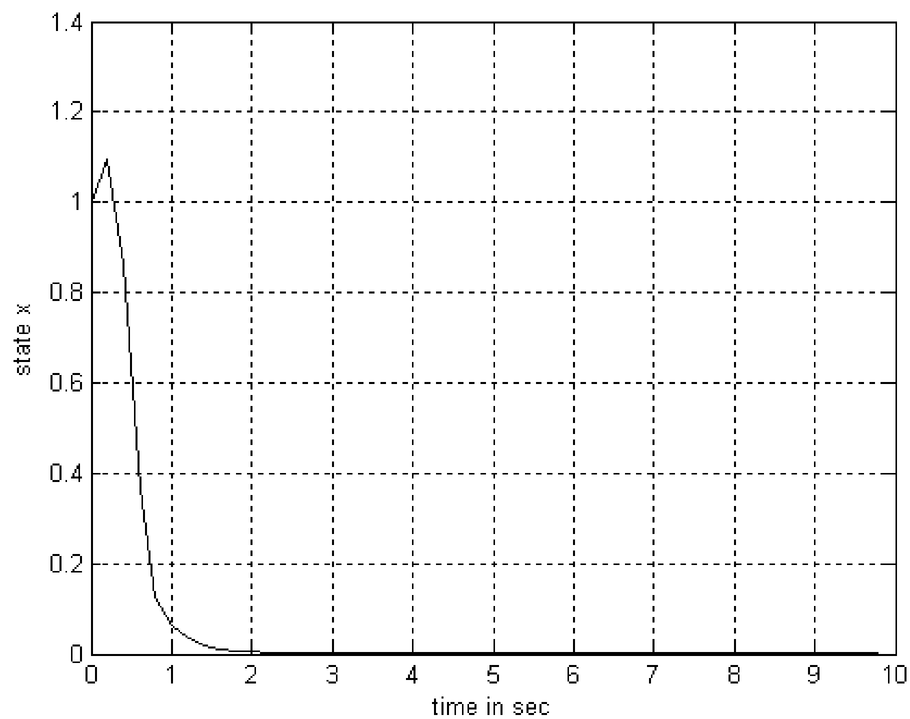


Figure 1. The system state

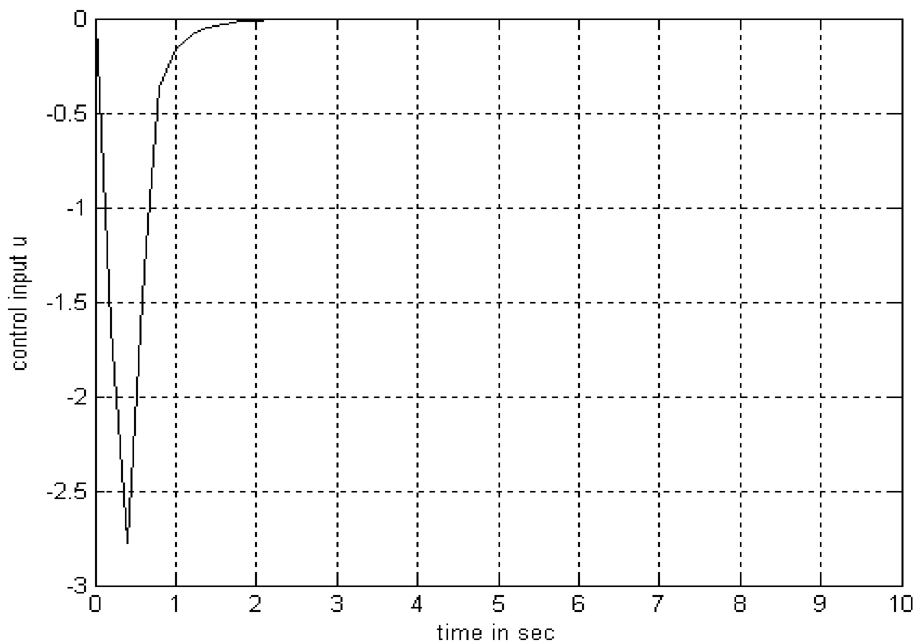


Figure 2. The control input.

4.2. Example 2

In this example, we consider a two dimensional nonlinear system controlled in [18]:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = 4.\left(\frac{\sin(4.\pi.x_1)}{\pi.x_1}\right).\left(\frac{\sin(\pi.x_2)}{\pi.x_2}\right)^2 + b.u(t). \qquad (35)$$

We apply the direct adaptive fuzzy controller to control the system state $x_1(t)$ to track a desired trajectory which is specified as the output of a second order with a bandwidth driven by a unity amplitude, 0.5 mean, square wave (see Figure 3). The parameters of the controller are chosen as $\gamma = 0.9$, step size $dt = 0.01$, Since the degree of the system is $n = 2$ the error polynomial is $s^2 + k_1.s + k_0 = 0$, we set $k_0 = 45$ and $k_1 = 5$, so that all their roots are in the open left-half plane. We choose $Q = diag(125, 125) > 0$, then by solving (22) we can obtain :

$$P = \begin{bmatrix} 581.9444 & 1.3889 \\ 1.3889 & 12.7778 \end{bmatrix} \qquad (36)$$

The structure of the TS fuzzy controller is exactly the same as in the previous example. The widths of the membership functions are $\sigma_P = \sigma_N = 1.5$ for the first, the second and the third input $z_1 = x_1$, $z_2 = x_{m1}$ and $z_3 = \dot{x}_{m1} - k^T \underline{e}$. The centres are set to $c_N = -1$, $c_P = 1$ for z_1, z_2, and $c_N = -0.5$, $c_P = 0.5$ for z_3. All initial conditions are set to zero. We assume that the control gain b is taken as a non unity gain ($b = 2$). From above, it is clear that b is bounded. Figure 3 shows that the system state $x_1(t)$ (in continuous) could track the desired trajectory $y_m(t) = x_m(t)$ (in dashed) perfectly. Figure 4 and Figure 5 show respectively the corresponding velocity of the system $x_2(t)$ (in continuous) with the desired velocity $\dot{y}_m(t)$ (in dashed) and the control input $u(t)$. From these Figures, we can confirm the smooth property of our fuzzy adaptive control system without the use of the supervisory term in the control law under the assumption that the control gain is bounded.

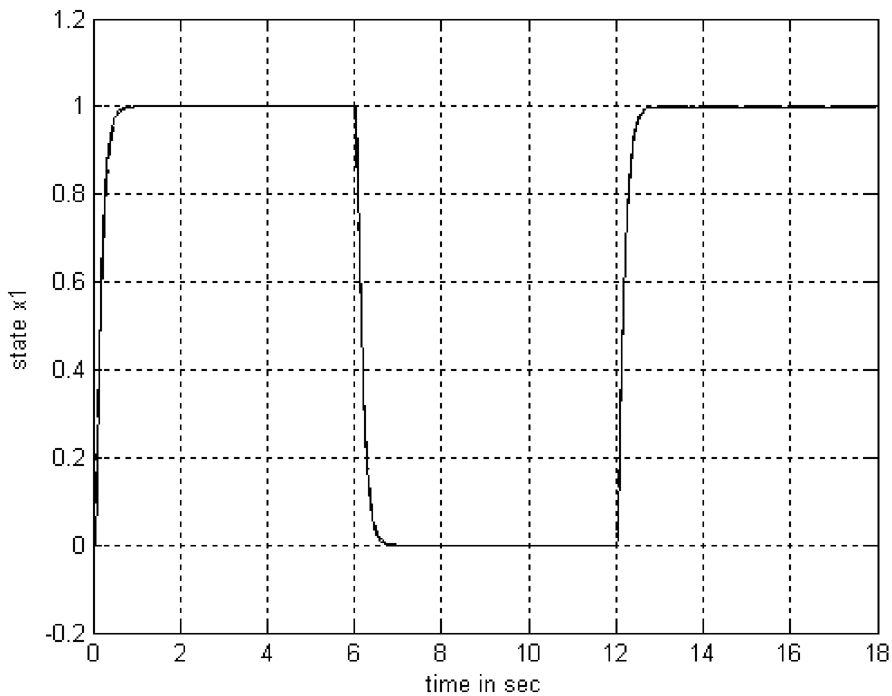


Figure 3. The system state (—) and the desired trajectory (---)

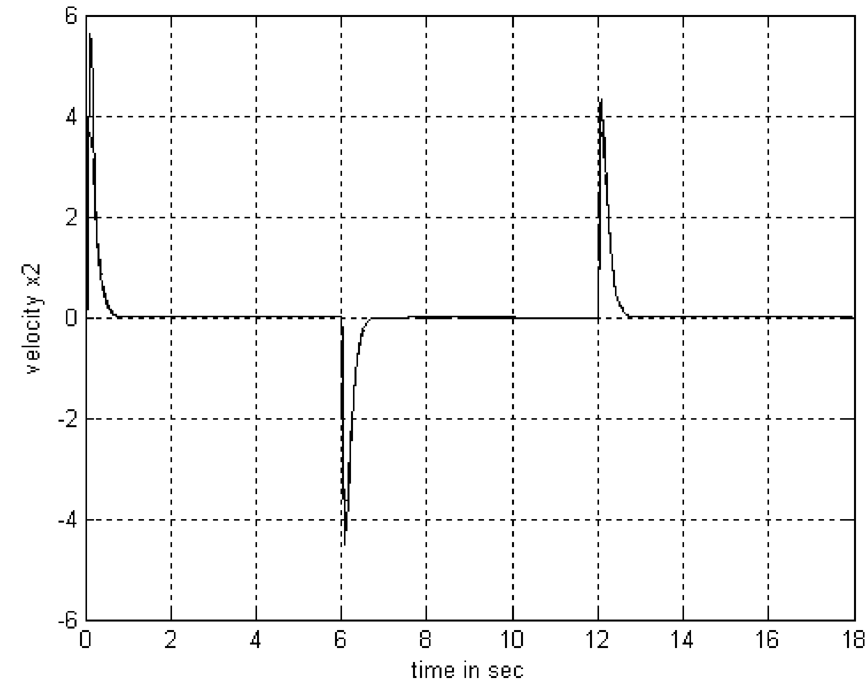


Figure 4. The velocity of the system (—) with the desired velocity (---)

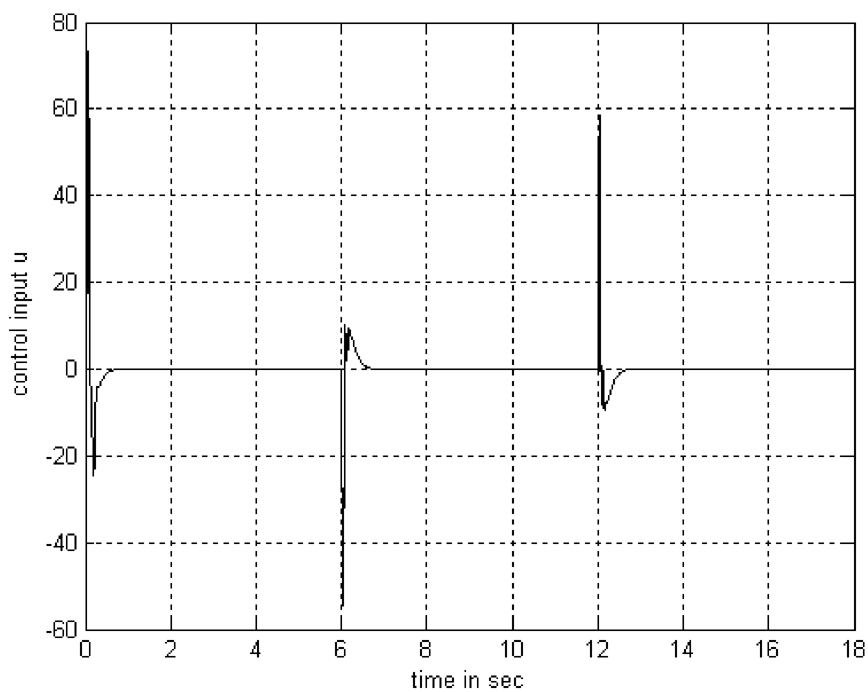


Figure 5. The corresponding control input

4.3. Example 3

In this example, we apply the performance of the proposed fuzzy adaptive system to control the level in a Three Tank System by simulation and compare its behaviour to a proportional integral PI controller by simulation. The Three-Tank System [2], [19], [20] is a benchmark process widely used for modelling and control strategies nonlinear systems. The nonlinear controlled system consists of three plexiglass cylinders T1, T2 and T3 with identical cross-sectional area A which are interconnected in series by two connecting pipes. The liquid leaving T2 is collected in a reservoir from which pumps 1 and 2 (driven by DC motors) supply tanks T1 and T2 with flow rates Q1 and Q2. All three tanks are equipped with piezo-resistive pressure transducer for measuring the level of the liquid (L_1, L_2 and L_3 in *cm*). The tanks are coupled by two connecting cylindrical pipes with a cross section S and an outflow coefficient $\mu_1 = \mu_3$. The nominal outflow is located at tank T2, it also has a circular cross section of S and an outflow coefficient μ_2. The connecting pipes and the tanks are additionally equipped with manually adjustable valves and outlets for the purpose of simulating clogs as well as leaks. In this example, we will consider the Three Tank System as a SISO system, i.e., we will be interested to control the level L_2 in tank T2 by the flow rate Q2. The dynamic equation describing the SISO Three Tank System [2], [19] is as follows:

$$A\frac{dL_2}{dt} = Q_2 - \mu_2 S\sqrt{2gL_2} \quad (37)$$

Where, $S = 0.5\,(cm^2)$, $\mu_2 = 0.4896$, $A = 154\,(cm^2)$, $g = 9.81*100\,(cm/s^2)$ is the universal gravitation and $y = x_1 = L_2$ is the level in tank T2. The reference signal will be $xm_1 = y_m = Lm_2$. The parameters are chosen as $\gamma = 0.008$, step size $dt = 1$, and $k = k_0 = 1.5$ in order to have all roots of $s + k_0 = 0$ in the left-half plane. We chose Q in (22) as $Q = 2 > 0$. Then by solving (22) we can obtain $P = 0.6667$. The TS fuzzy controller has three inputs $\underline{z} = [z_1 \;\; z_2 \;\; z_3] = [L_2 \;\; Lm_2 \;\; (\dot{L}m_2 - k^T \underline{e})]$ with $\underline{e} = [L_2 - Lm_2]$. All the three inputs to the TS fuzzy controller are fuzzified with two fuzzy sets and similar Gaussian membership functions given by:

$$\mu_N(z_i) = \exp(-(z_i - c_N)^2 / 2.\sigma_N) \quad (38)$$

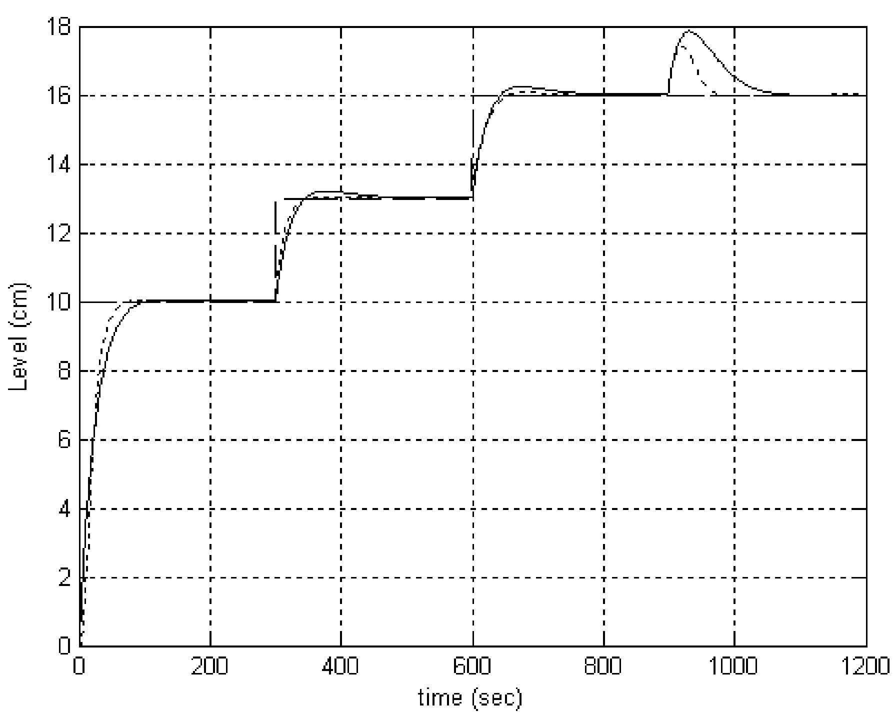
$$\mu_P(z_i) = \exp(-(z_i - c_P)^2 / 2.\sigma_P) \quad (39)$$

where z_i stands for the input number i. The widths of the membership functions are $\sigma_P = \sigma_N = 6$ for the first and the second input $z_1 = L_2$, $z_2 = Lm_2$ and are $\sigma_P = \sigma_N = 3.5$ for the third input $z_3 = \dot{L}m_2 - k^T \underline{e}$. The centres are set to $c_N = 10$, $c_P = 20$ for z_1 and z_2, and $c_N = -1$, $c_P = 3$ for z_3. This gives eight rules of the form:

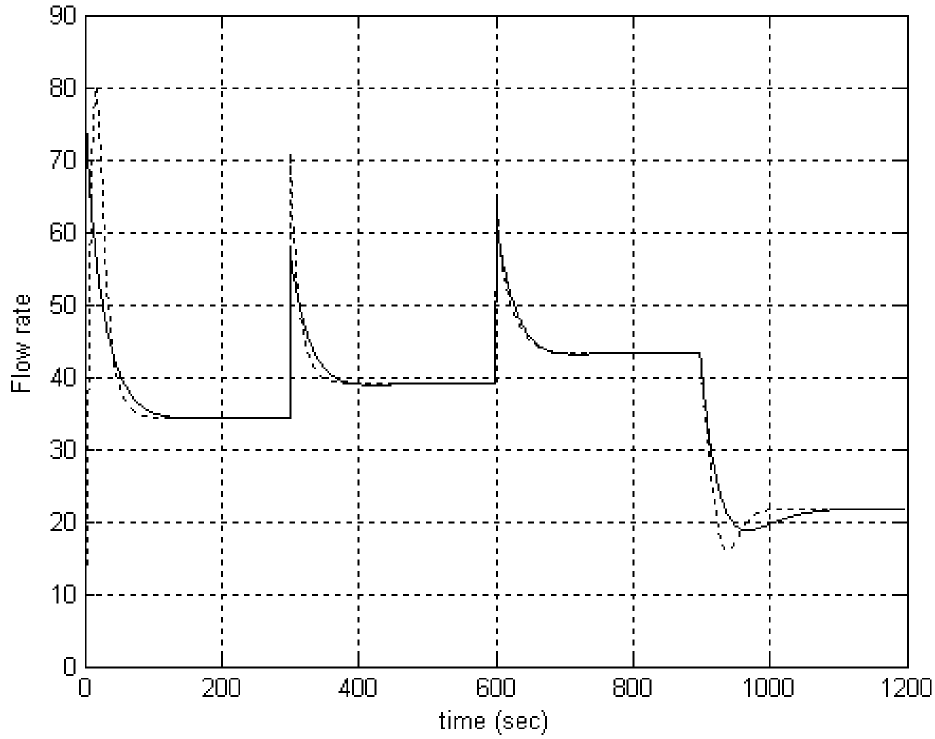
$$R^i: \; if\; z_1 \; is\; A_1^i \; and\; z_2\; is\; A_2^i \; and\; z_3\; is\; A_3^i$$
$$then\;\; u_i = a_1^i z_1 + a_2^i z_2 + a_3^i z_3, \text{ with } i = 1 \text{ to } 8 \quad (40)$$

We have 24 parameters to tune. All parameters (a_1^i, a_2^i and a_3^i) are initialised to zero. The initial conditions are $L_2(0) = 0$. The used PI parameters are $k_p = 7.5$ (proportional action) and $T_i = 6.5$ (integral action). Simulation results are shown in Figures 6 and 7, where the corresponding results to the TS controller are in dotted, while those corresponding to the PI controller are in continuous and the reference signal is in dashed. Figure 6 shows the evolution of the level L_2 in tank T2 driven by the TS controller and by the PI controller. Figure 7 shows the corresponding control inputs respectively for the TS and for the PI controller. In the first time interval ($0 \le t < 900\, s$) of Figure 6, we can see that the system output (level in tank T2) with the TS controller has got the reference rapidly (less response time than with the PI controller), and also has less overshoot at every reference variation than with the PI controller.

In the same time when controlling level L_2 in tank T2, we check the ability of our controller against perturbations. So, we create a clogging, i.e., we close the nominal outflow valve of tank T2 with degree of $50\,\%$ at time $t = 900\,s$. In other words, in tank T2, the cross section S of the nominal outflow valve will take the value $S = 0.5/2\,(cm^2)$ at $t = 900\,s$ instead of the nominal value $S = 0.5\,(cm^2)$. Simulation results are shown in the remaining (last) time interval ($900 \le t \le 1200\, s$) of the same previous Figures 6 and 7. Clearly, we can see that disturbances are suppressed rapidly with less amplitude for the TS controller than with the PI controller. i.e., the disturbances are suppressed at around $t = 970\,s$ with less amplitude for the TS fuzzy controller, and at around $t = 1040\,s$ for PI controller. Corresponding control inputs for the TS fuzzy controller and for the PI controller are also shown in the last time interval ($900 \le t \le 1200\, s$) of Figure 7. As a concluding remarks, from these figures, we can confirm that the proposed TS controller was able to stabilise the level of the liquid in tank T2 at each interval and also was able to eliminate disturbances introduced through the outflow pipe of tank T2 on a better way than with the PI controller, confirming also the robust smoothing property of the TS system without use of the supervisory term in the control law as discussed in the introduction.



Figures 6. The Level in tank T2 with the TS controller (…..) and with the PI controller (—)



Figures 7. The control signals of the TS controller (…..) and the PI controller (—)

5. CONCLUSIONS

In this paper, we developed a stable fuzzy direct adaptive control scheme for a class of unknown nonlinear systems based on feedback linearization theory. We used for this purpose an on-line Takagi-Sugeno (T-S) system to approximate the ideal control law. The consequent parameters of the used fuzzy controller are adapted and changed according to a law derived using Lyapunov stability theory. The proposed method could guarantee the stability of the resulting closed-loop system in the sense that all signals involved were uniformly bounded. All this was achieved without the use of the supervisory term in the control law. Finally, we used the direct adaptive fuzzy system to control an unstable nonlinear system (Example 1), a two dimensional nonlinear system (Example 2) and the level in a Three Thank System (Example 3). The results were encouraging with comparison to other related works confirming the smoothing capability of our feedback fuzzy adaptive control architecture.

REFERENCES

[1] A, Maidi., M. Diaf and J. P. Corriou, (2010) "Boundary control of a parallel-flow heat exchanger by input- output linearization", *journal of Process Control*, Vol. 20, pp1161-1174.

[2] M, Suresh, G, J, Srinivasan, and R, R, Hemamalini, (2009) "Integrated Fuzzy Logic Based Intelligent Control of Three Tank System", *Serbian Journal of Electrical Engineering*, Vol. 6, No. 1, pp1-14.

[3] Lee, C. C, (1990) “Fuzzy logic in control systems, Fuzzy logic controller, Parts I and II”, *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. 20, No. 2, pp404-435.

[4] Feng, G., senior member IEEE, (2006) “A Survey on analysis and Design on Model Based-Fuzzy Control Systems”, *IEEE Transactions on Fuzzy systems*, Vol. 14, No. 5, pp676-697.

[5] Astrom, K. J., and Wittenmark, B, (1995),“Adaptive Control”, *Addison Wesley*.

[6] Hojati, M., and Gazor, S, (2002), “Hybrid Adaptive Fuzzy Identification and Control of Nonlinear Systems”, *IEEE Transactions on Fuzzy Systems*, Vol. 10, No. 2, pp198-210.

[7] Vélez-Díaz, D., Tang., Y., (2004), “Adaptive Robust Fuzzy Control of Nonlinear Systems”, *Transactions on Systems, Man, and Cybernetics, part B*, Vol. 34, No. 3, pp1596-1601.

[8] Wang, L.-X., (1993), “Stable Adaptive Fuzzy Control of Nonlinear Systems”, *IEEE Transactions on Fuzzy Systems*, Vol. 1, No. 2, pp146-155.

[9] Qi, R., Brdys, M. A, (2009), “Indirect Adaptive Controller Based on a Self-tructuring Fuzzy System for Nonlinear Modeling and Control”, *International Journal of Applied Mathematics and Computer Sciences*, Vol. 19, No. 4, pp619-630.

[10] Slotine, J.J.E., and Weiping, L., (1991), “Applied Nonlinear control”, *Prentice Hall*.

[11] S.N. Singh and M. Steinberg., (1996), “Adaptive Control of Feedback Linearizable Nonlinear Systems with Application to Flight Control”, Journal of Guidance, Control, and Dynamics*, Man, and Cybernetics Part B*, Vol. 19, No. 4, pp871-877.

[12] Wang, L. X., (1996), “Stable adaptive fuzzy controllers with application to inverted pendulum tracking”, *Transactions on Systems, Man, and Cybernetics Part B*, Vol. 26, pp677-691.

[13] Wang, C. H., Liu, H., and Lin, T., (2002), “Direct Adaptive Fuzzy-Neural Control With State Observer and Supervisory Controller for Unknown Nonlinear Dynamical Systems”, *IEEE Transactions on Fuzzy Systems*, Vol. 10, No. 1, pp39-49.

[14] Tsay, D.-L., Chung, H. Y., and Lee, C. J., (1999), “The Adaptive Control of Nonlinear Systems Using the Sugeno–Type of Fuzzy Logic”, *IEEE Transactions on Fuzzy Systems*, Vol. 7, No. 2, pp225-229.

[15] Spooner., J.T., and Passino, K. M., (1996), “Stable Adaptive Control Using Fuzzy Systems and Neural Networks”, *IEEE Transactions on Fuzzy Systems*, Vol. 4, No. 3, pp339-359.

[16] Castro, J. L., (1995), “Fuzzy logic controllers are universal approximators”, *Transactions on Systems, Man, and Cybernetics*, Vol. 25, No. 4, pp629–635.

[17] Wang, L.-X., and Mendel, J. M., (1992), “Fuzzy basis functions, universal approximation, and orthogonal least-squares learning”, *IEEE Transactions on Neural Networks*, Vol. 3, No. 5, pp 807-814.

[18] R. M, Sanner, and J. J. E., Slotine, (1992), “Gaussian networks for direct adaptive control”, *IEEE Transactions on Neural Networks*, Vol. 3, pp837-863.

[19] GmbH, A., AMIRA DTS200, (2002), “Laboratory Setup Three-Tank-System”, AMIRA GmbH, Disburg, Germany.

[20] D. Theilliol, C. Join., and Y. Zhang, (2008), “Actuator Fault Tolerant Control Design Based on a Reconfigurable Reference Input”, *International Journal of Applied Mathematics and Computer Sciences*, Vol. 18, No. 4, pp553–560.

Authors

Mohamed BAHITA obtained his “Ingénieur“ and his Master Degree both in control Engineering from the University of Constantine, Algeria. He is working towards his PhD at the same University. He is currently a lecturer with the Faculty of Hydrocarbons and Chemistry (FHC), University of BOUMERDES, Algeria. His main interests are in artificial intelligence and adaptive control of nonlinear systems.

Khaled BELARBI obtained his “Ingénieur “ degree from “Ecole polytechnique, Algiers, Algeria, and MSc and PhD in control Engineering both from Control System Center UMIST, Manchester, UK . He is currently a professor with the Faculty of Engineering, University of Constantine, Algeria. His current interests are in predictive control and fuzzy and neural control.