ACTIVE CONTROLLER DESIGN FOR THE HYBRID SYNCHRONIZATION OF HYPERCHAOTIC XU AND HYPERCHAOTIC LI SYSTEMS

Sundarapandian Vaidyanathan[1]

[1]R & D Centre, Vel Tech University
Avadi, Chennai-600 062, Tamil Nadu, INDIA
sundarvtu@gmail.com

ABSTRACT

The synchronization of chaotic systems treats a pair of chaotic systems, which are usually called as master and slave systems. In the chaos synchronization problem, the goal of the design is to synchronize the states of master and slave systems asymptotically. In the hybrid synchronization design of master and slave systems, one part of the systems, viz. their odd states, are completely synchronized (CS), while the other part, viz. their even states, are completely anti-synchronized (AS) so that CS and AS co-exist in the process of synchronization. This research work deals with the hybrid synchronization of hyperchaotic Xi systems (2009) and hyperchaotic Li systems (2005). The main results of this hybrid research work are established with Lyapunov stability theory. MATLAB simulations of the hybrid synchronization results are shown for the hyperchaotic Xu and Li systems.

KEYWORDS

Hybrid Synchronization, Active Control, Chaos, Hyperchaos, Hyperchaotic Systems.

1. INTRODUCTION

Hyperchaotic systems are nonlinear chaotic systems with two or more positive Lyapunov exponents. These systems have a lot of miscellaneous applications in Engineering. The hyperchaotic system was first discovered by the German scientist, O.E. Rössler ([1], 1979).

Hyperchaotic systems possess salient features like high level of security, high level of capacity and high level of efficiency. Hence, the hyperchaotic systems have important applications in areas like neural networks [2], oscillators [3], communication [4-5], encryption [6], synchronization [7], etc.

We note that many methods have been devised for the synchronization of chaotic systems. Some important methods can be cited as Ott-Grebogi-Yorke method [8], Pecora-Carroll method [9], backstepping method [10-12], sliding control method [13-15], active control method [16-18], adaptive control method [19-20], sampled-data control method [21], delayed feedback method [22], etc.

We describe the hybrid synchronization of a pair of chaotic systems called the master and slave systems as follows. Here, one part of the systems, viz. the odd states, are completely

synchronized (CS) and the other part of the systems, viz. the even states, are anti-synchronized so that both CS and AS can exist together in the hybrid synchronization of the two systems.

This paper focuses upon active controller design for the hybrid synchronization of hyperchaotic Xu systems ([23], 2009) and hyperchaotic Li systems ([24], 2005). The main results derived in this paper have been proved using stability theorems of Lyapunov stability theory [25].

2. ACTIVE CONTROL METHODOLOGY

The *master system* is described by the chaotic dynamics

$$\dot{x} = Ax + f(x) \tag{1}$$

where A is the $n \times n$ matrix of the system parameters and $f : R^n \to R^n$ is the nonlinear part. The *slave system* is described by the chaotic dynamics

$$\dot{y} = By + g(y) + u \tag{2}$$

where B is the $n \times n$ matrix of the system parameters, $g : R^n \to R^n$ is the nonlinear part and $u \in R^n$ is the active controller to be designed.
Let us define the *hybrid synchronization error* as

$$e_i = \begin{cases} y_i - x_i, & \text{if } i \text{ is odd} \\ y_i + x_i, & \text{if } i \text{ is even} \end{cases} \tag{3}$$

We can calculate the error dynamics as

$$\dot{e}_i = \begin{cases} \sum_{j=1}^{n} (b_{ij} y_j - a_{ij} x_j) + g_i(y) - f_i(x) + u_i & \text{if } i \text{ is odd} \\ \sum_{j=1}^{n} (b_{ij} y_j + a_{ij} x_j) + g_i(y) + f_i(x) + u_i & \text{if } i \text{ is even} \end{cases} \tag{4}$$

As mentioned earlier, the research goal is to find a controller u so that

$$\lim_{t \to \infty} \|e(t)\| = 0 \text{ for all values of } e(0) \in R^n \tag{5}$$

We apply the matrix Lyapunov method as our methodology.
Hence, we consider a candidate Lyapunov function

$$V(e) = e^T P e, \tag{6}$$

where P is a positive definite $n \times n$ matrix.
We remark that $V : R^n \to R$ is a positive definite function.
If we find a feedback controller u so that

$$\dot{V}(e) = -e^T Q e, \tag{7}$$

where Q is a positive definite matrix, then $\dot{V} : R^n \to R$ is a negative definite function.

Thus, the error dynamics (4) is globally exponentially stable by Lyapunov stability theory [25]. Hence, the states of the chaotic systems (1) and (2) will be globally and exponentially hybrid synchronized for all initial conditions $x(0), y(0) \in R^n$.

3. HYPERCHAOTIC SYSTEMS

The hyperchaotic Xu system ([23], 2009) has the 4-D dynamics

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_4 \\
\dot{x}_2 &= bx_1 + rx_1x_3 \\
\dot{x}_3 &= -cx_3 - x_1x_2 \\
\dot{x}_4 &= x_1x_3 - dx_2
\end{aligned} \tag{8}$$

where a, b, c, r, d are constant, positive parameters of the system.

The Xu system (8) exhibits a hyperchaotic attractor when the system parameters are taken as

$$a = 10, \quad b = 40, \quad c = 2.5, \quad r = 16, \quad d = 2 \tag{9}$$

The Lyapunov exponents of the 4-D system (8) for the values of parameters in (9) are

$$\lambda_1 = 1.0088, \quad \lambda_2 = 0.1063, \quad \lambda_3 = 0, \quad \lambda_4 = -13.6191 \tag{10}$$

Note that there are two positive Lyapunov exponents in (10). This shows that the 4-D Xu system (8) is hyperchaotic for the parametric values (9).

The strange attractor of the hyperchaotic Xu system is displayed in Figure 1.

The hyperchaotic Li system ([24], 2005) has the 4-D dynamics

$$\begin{aligned}
\dot{x}_1 &= \alpha(x_2 - x_1) + x_4 \\
\dot{x}_2 &= \delta x_1 - x_1x_3 + \gamma x_2 \\
\dot{x}_3 &= -\beta x_3 + x_1x_2 \\
\dot{x}_4 &= x_2x_3 + \varepsilon x_4
\end{aligned} \tag{11}$$

where $\alpha, \beta, \gamma, \delta, \varepsilon$ are constant, positive parameters of the system.

The Li system (11) exhibits a hyperchaotic attractor when the parameter values are taken as

$$\alpha = 35, \quad \beta = 3, \quad \gamma = 12, \quad \delta = 7, \quad \varepsilon = 0.58 \tag{12}$$

The Lyapunov exponents of the 4-D system (11) for the values of the parameters in (12) are

$$\lambda_1 = 0.5011, \quad \lambda_2 = 0.1858, \quad \lambda_3 = 0, \quad \lambda_4 = -26.1010 \tag{13}$$

Note that there are two positive Lyapunov exponents in (13). This shows that the 4-D Li system (11) is hyperchaotic for the parametric values (12).

The strange attractor of the hyperchaotic Li system is displayed in Figure 2.
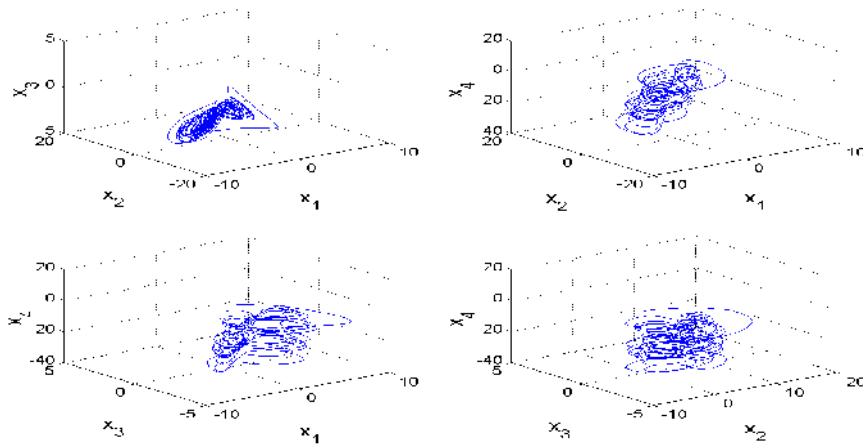
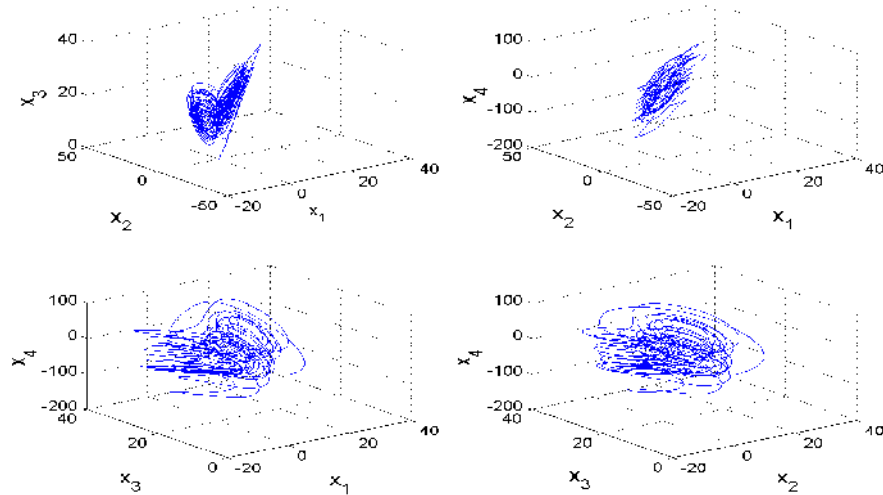Figure 1. The State Portrait of the Hyperchaotic Xu System

Figure 2. The State Portrait of the Hyperchaotic Li System

4. ACTIVE CONTROL DESIGN FOR THE HYBRID SYNCHRONIZATION OF HYPERCHAOTIC XU SYSTEMS

In this section, we design an active controller for the hybrid synchronization of two identical hyperchaotic Xu systems (2009) and establish our main result using Lyapunov stability theory.
The hyperchaotic Xu system is taken as the master system. The dynamics of the master system is given by

$$\dot{x}_1 = a(x_2 - x_1) + x_4$$
$$\dot{x}_2 = bx_1 + rx_1x_3$$
$$\dot{x}_3 = -cx_3 - x_1x_2$$
$$\dot{x}_4 = x_1x_3 - dx_2$$

(14)

where a, b, c, d are positive parameters of the system and $x \in R^4$ is the state of the system. The hyperchaotic Xu system is also taken as the slave system. The dynamics of the slave system is given by

$$\dot{y}_1 = a(y_2 - y_1) + y_4 + u_1$$
$$\dot{y}_2 = by_1 + ry_1y_3 + u_2$$
$$\dot{y}_3 = -cy_3 - y_1y_2 + u_3$$
$$\dot{y}_4 = y_1y_3 - dy_2 + u_4$$

(15)

where $y \in R^4$ is the state and u_1, u_2, u_3, u_4 are the active controllers to be designed.
We consider the hybrid synchronization problem for the systems (14) and (15).
Hence, the hybrid synchronization error e is defined as follows:

$$e_1 = y_1 - x_1$$
$$e_2 = y_2 + x_2$$
$$e_3 = y_3 - x_3$$
$$e_4 = y_4 + x_4$$

(16)

When we calculate the error dynamics, we obtain

$$\dot{e}_1 = a(e_2 - e_1) + e_4 - 2ax_2 - 2x_4 + u_1$$
$$\dot{e}_2 = be_1 + 2bx_1 + r(y_1y_3 + x_1x_3) + u_2$$
$$\dot{e}_3 = -ce_3 - y_1y_2 + x_1x_2 + u_3$$
$$\dot{e}_4 = -de_2 + y_1y_3 + x_1x_3 + u_4$$

(17)

Let the active controller for achieving hybrid synchronization be taken as

$$u_1 = -a(e_2 - e_1) - e_4 + 2ax_2 + x_4 - k_1e_1$$
$$u_2 = -be_1 - 2bx_1 - r(y_1y_3 + x_1x_3) - k_2e_2$$
$$u_3 = ce_3 + y_1y_2 - x_1x_2 - k_3e_3$$
$$u_4 = de_2 - y_1y_3 - x_1x_3 - k_4e_4$$

(18)

where k_i, $(i = 1, 2, 3, 4)$ are constants having positive values.
The error dynamics is simplified by the substitution of (18) into (17), and we obtain

$$
\begin{aligned}
\dot{e}_1 &= -k_1 e_1 \\
\dot{e}_2 &= -k_2 e_2 \\
\dot{e}_3 &= -k_3 e_3 \\
\dot{e}_4 &= -k_4 e_4
\end{aligned}
\tag{19}
$$

Theorem 4.1 The active control law defined by Eq. (18) achieves global and exponential hybrid synchronization of the identical hyperchaotic Xu systems (14) and (15) for all initial conditions $x(0), y(0) \in R^4$.

Proof. The proof is via Lyapunov stability theory [25] for global exponential stability.
As a candidate Lyapunov function , we consider the quadratic function defined by

$$
V(e) = \frac{1}{2} e^T e = \frac{1}{2} \left(e_1^2 + e_2^2 + e_3^2 + e_4^2 \right),
\tag{20}
$$

Note that V is a positive definite function on R^4.
Next, we differentiate (18) along the trajectories of (17).
A simple calculation gives

$$
\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2
\tag{21}
$$

Note that $\dot{V}$ is a negative definite function on R^4.
By Lyapunov stability theory [25], the error dynamics (19) is globally exponentially stable for all values of $e(0) \in R^4$.

This completes the proof. ■
Next, we illustrate our hybrid synchronization results with MATLAB simulations.
The classical fourth order Runge-Kutta method with time-step $h = 10^{-8}$ has been applied to solve the hyperchaotic Xu systems (14) and (15) with the active nonlinear controller (18).
The feedback gains in the active controller (18) are chosen as follows.

We take $k_i = 5, \ (i = 1, 2, 3, 4)$.

The parameters of the hyperchaotic Xu systems are taken as in the hyperchaotic case, *i.e.*

$$
a = 10, \quad b = 40, \quad c = 2.5, \quad r = 16, \quad d = 2
$$

For simulations, the initial conditions of the hyperchaotic Xu system (14) are chosen as

$$
x_1(0) = 21, \quad x_2(0) = 7, \quad x_3(0) = 22, \quad x_4(0) = -14
$$

Also, the initial conditions of the hyperchaotic Xu system (15) are chosen as

$$
y_1(0) = 12, \quad y_2(0) = -11, \quad y_3(0) = -20, \quad y_4(0) = 27
$$

Figure 3 depicts the hybrid synchronization of the identical hyperchaotic Xu systems.

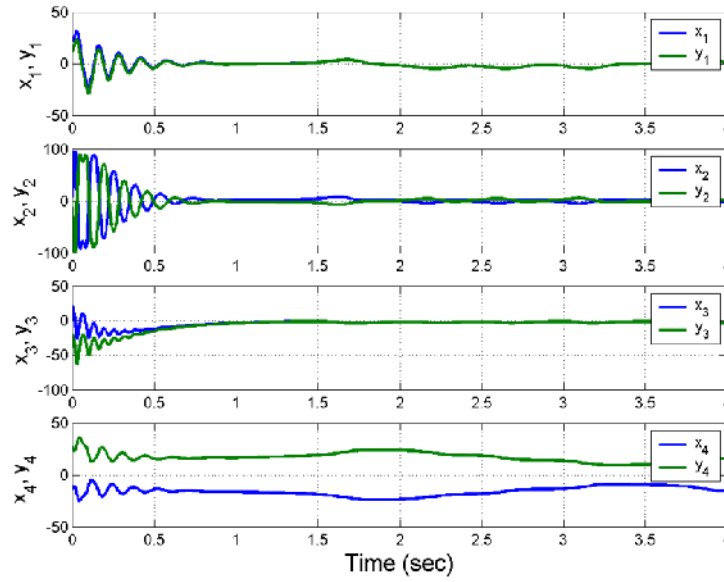Figure 4 depicts the time-history of the anti-synchronization errors e_1, e_2, e_3, e_4.

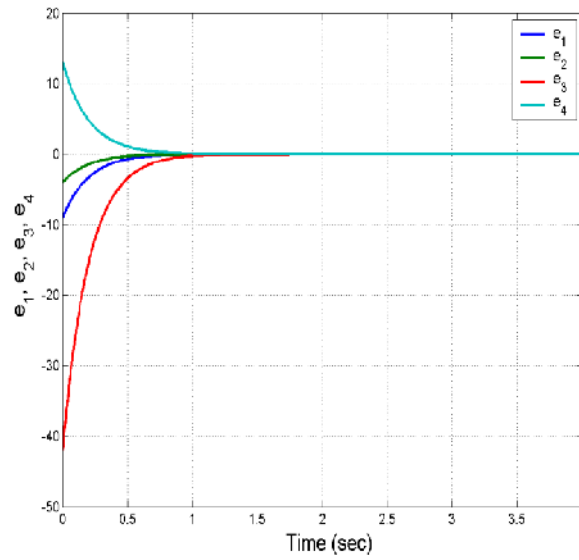Figure 3. Hybrid Synchronization of Identical Hyperchaotic Xu Systems

Figure 4. Time-History of the Hybrid Synchronization Errors e_1, e_2, e_3, e_4

5. ACTIVE CONTROLLER DESIGN FOR THE HYBRID SYNCHRONIZATION DESIGN OF HYPERCHAOTIC LI SYSTEMS

This section deals with the design of an active controller for the hybrid synchronization of two identical hyperchaotic Li systems (2005). In this section, we establish our main result using Lyapunov stability theory.

The hyperchaotic Li system is taken as the master system. Thus, the dynamics of the master system is given by

$$
\begin{aligned}
\dot{x}_1 &= \alpha(x_2 - x_1) + x_4 \\
\dot{x}_2 &= \delta x_1 - x_1 x_3 + \gamma x_2 \\
\dot{x}_3 &= -\beta x_3 + x_1 x_2 \\
\dot{x}_4 &= x_2 x_3 + \varepsilon x_4
\end{aligned}
\tag{22}
$$

where $\alpha, \beta, \gamma, \delta, \varepsilon$ are positive parameters of the system and $x \in R^4$ is the state of the system. The hyperchaotic Li system is taken as the slave system. The dynamics of the slave system is given by

$$
\begin{aligned}
\dot{y}_1 &= \alpha(y_2 - y_1) + y_4 + u_1 \\
\dot{y}_2 &= \delta y_1 - y_1 y_3 + \gamma y_2 + u_2 \\
\dot{y}_3 &= -\beta y_3 + y_1 y_2 + u_3 \\
\dot{y}_4 &= y_2 y_3 + \varepsilon y_4 + u_4
\end{aligned}
\tag{23}
$$

where $y \in R^4$ is the state and u_1, u_2, u_3, u_4 are the active controllers to be designed.

For the hybrid synchronization, the error e is defined as

$$
\begin{aligned}
e_1 &= y_1 - x_1 \\
e_2 &= y_2 + x_2 \\
e_3 &= y_3 - x_3 \\
e_4 &= y_4 + x_4
\end{aligned}
\tag{24}
$$

We obtain the error dynamics as

$$
\begin{aligned}
\dot{e}_1 &= \alpha(e_2 - e_1) + e_4 - 2\alpha x_2 - 2x_4 + u_1 \\
\dot{e}_2 &= \delta e_1 + 2\delta x_1 + \gamma e_2 - y_1 y_3 - x_1 x_3 + u_2 \\
\dot{e}_3 &= -\beta e_3 + y_1 y_2 - x_1 x_2 + u_3 \\
\dot{e}_4 &= \varepsilon e_4 + y_2 y_3 + x_2 x_3 + u_4
\end{aligned}
\tag{25}
$$

We choose the active controller for achieving hybrid synchronization as

$$u_1 = -\alpha(e_2 - e_1) - e_4 + 2\alpha x_2 + 2x_4 - k_1 e_1$$
$$u_2 = -\delta e_1 - 2\delta x_1 - \gamma e_2 + y_1 y_3 + x_1 x_3 - k_2 e_2$$
$$u_3 = \beta e_3 - y_1 y_2 + x_1 x_2 - k_3 e_3$$
$$u_4 = -\varepsilon e_4 - y_2 y_3 - x_2 x_3 - k_4 e_4$$

(26)

where k_i, $(i = 1, 2, 3, 4)$ are positive gains.

The error dynamics is simplified by the substitution of (26) into (25), and we obtain

$$\dot{e}_1 = -k_1 e_1$$
$$\dot{e}_2 = -k_2 e_2$$
$$\dot{e}_3 = -k_3 e_3$$
$$\dot{e}_4 = -k_4 e_4$$

(27)

Theorem 5.1 The active control law defined by Eq. (26) achieves global and exponential hybrid synchronization of the identical hyperchaotic Li systems (22) and (23) for all initial conditions $x(0), y(0) \in R^4$.

Proof. The proof is via Lyapunov stability theory [25] for global exponential stability.
As a candidate Lyapunov function, we take the quadratic function

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right),$$

(28)

Note that V is a positive definite function on R^4.
Next, we differentiate (26) along the trajectories of (25). A simple calculation gives

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2$$

(29)

Note that $\dot{V}$ is a negative definite function on R^4.
Hence, by Lyapunov stability theory, the error dynamics (27) is globally exponentially stable for all values of $e(0) \in R^4$. ∎

Next, we illustrate our hybrid synchronization results with MATLAB simulations.
The classical fourth-order Runge-Kutta method with time-step $h = 10^{-8}$ has been applied to solve the hyperchaotic Li systems (22) and (23) with the active controller defined by (26).
The feedback gains in the active controller (26) are taken as follows. Let $k_i = 5$, $(i = 1, 2, 3, 4)$.
We take the parameters of the hyperchaotic Li systems as

$$\alpha = 35, \quad \beta = 3, \quad \gamma = 12, \quad \delta = 7, \quad \varepsilon = 0.58$$

For simulations, the initial conditions of the hyperchaotic Li system (22) are chosen as

$$x_1(0) = 15, \quad x_2(0) = -26, \quad x_3(0) = -14, \quad x_4(0) = 5$$

Also, the initial conditions of the hyperchaotic Li system (23) are chosen as

$$y_1(0) = 27, \quad y_2(0) = 18, \quad y_3(0) = 10, \quad y_4(0) = -24$$

Figure 5 depicts the hybrid synchronization of the identical hyperchaotic Li systems.
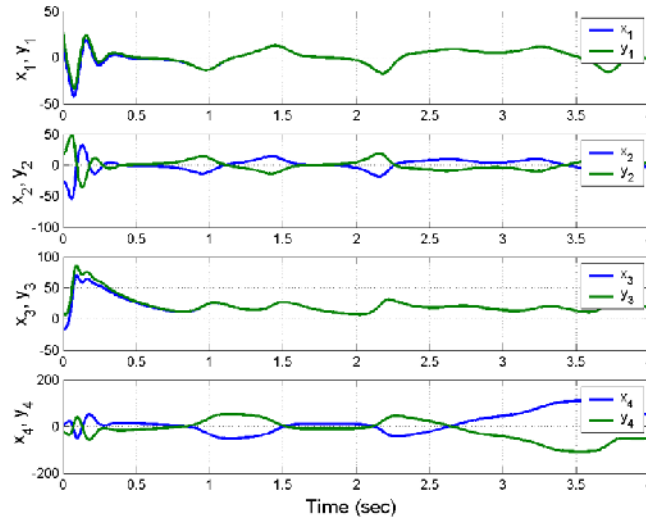Figure 6 depicts the time-history of the hybrid synchronization errors e_1, e_2, e_3, e_4.

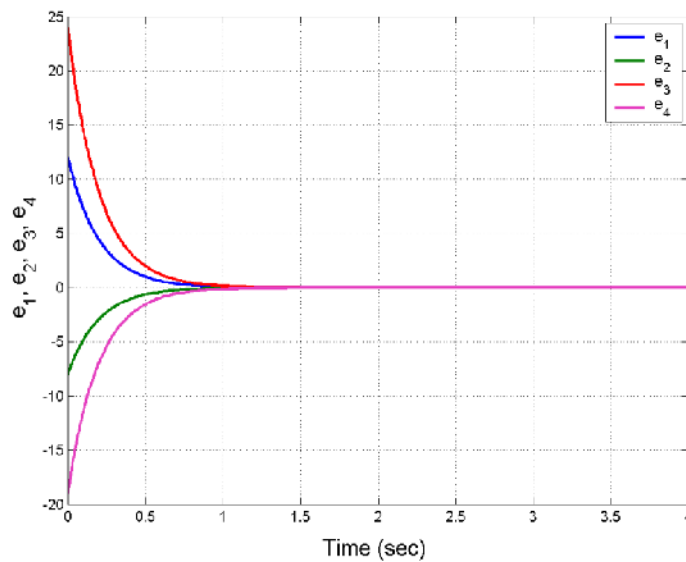Figure 5. Hybrid Synchronization of Identical Hyperchaotic Li Systems

Figure 6. Time-History of the Hybrid Synchronization Errors e_1, e_2, e_3, e_4

6. ACTIVE CONTROLLER DESIGN FOR THE HYBRID SYNCHRONIZATION OF HYPERCHAOTIC XU AND HYPERCHAOTIC LI SYSTEMS

In this section, we design an active controller for the hybrid synchronization of hyperchaotic Xu system (2009) and hyperchaotic Li system (2005) and establish our main result using Lyapunov stability theory.

The hyperchaotic Xu system is taken as the master system, whose dynamics is given by

$$
\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_4 \\
\dot{x}_2 &= bx_1 + rx_1x_3 \\
\dot{x}_3 &= -cx_3 - x_1x_2 \\
\dot{x}_4 &= x_1x_3 - dx_2
\end{aligned}
\tag{30}
$$

where a,b,c,d,r are positive parameters of the system and $x \in R^4$ is the state of the system.

The hyperchaotic Li system is taken as the slave system, whose dynamics is given by

$$
\begin{aligned}
\dot{y}_1 &= \alpha(y_2 - y_1) + y_4 + u_1 \\
\dot{y}_2 &= \delta y_1 - y_1y_3 + \gamma y_2 + u_2 \\
\dot{y}_3 &= -\beta y_3 + y_1y_2 + u_3 \\
\dot{y}_4 &= y_2y_3 + \varepsilon y_4 + u_4
\end{aligned}
\tag{31}
$$

where $\alpha,\beta,\gamma,\delta,\varepsilon$ are positive parameters of the system, $y \in R^4$ is the state and u_1,u_2,u_3,u_4 are the active controllers to be designed.

For the hybrid synchronization, the error e is defined as

$$
\begin{aligned}
e_1 &= y_1 - x_1 \\
e_2 &= y_2 + x_2 \\
e_3 &= y_3 - x_3 \\
e_4 &= y_4 + x_4
\end{aligned}
\tag{32}
$$

We obtain the error dynamics as

$$
\begin{aligned}
\dot{e}_1 &= \alpha(y_2 - y_1) - a(x_2 - x_1) + e_4 - 2x_4 + u_1 \\
\dot{e}_2 &= \delta y_1 + \gamma y_2 + bx_1 - y_1y_3 + rx_1x_3 + u_2 \\
\dot{e}_3 &= -\beta y_3 + cx_3 + y_1y_2 + x_1x_2 + u_3 \\
\dot{e}_4 &= \varepsilon y_4 - dx_2 + y_2y_3 + x_1x_3 + u_4
\end{aligned}
\tag{33}
$$

We choose the active controller for achieving hybrid synchronization as

$$
\begin{aligned}
u_1 &= -\alpha(y_2 - y_1) + a(x_2 - x_1) - e_4 + 2x_4 - k_1e_1 \\
u_2 &= -\delta y_1 - \gamma y_2 - bx_1 + y_1y_3 - rx_1x_3 - k_2e_2 \\
u_3 &= \beta y_3 - cx_3 - y_1y_2 - x_1x_2 - k_3e_3 \\
u_4 &= -\varepsilon y_4 + dx_2 - y_2y_3 - x_1x_3 - k_4e_4
\end{aligned}
\tag{34}
$$

where $k_i, \ (i = 1, 2, 3, 4)$ are positive gains.

The error dynamics is simplified by the substitution of (34) into (33), and we obtain

$$
\begin{aligned}
\dot{e}_1 &= -k_1 e_1 \\
\dot{e}_2 &= -k_2 e_2 \\
\dot{e}_3 &= -k_3 e_3 \\
\dot{e}_4 &= -k_4 e_4
\end{aligned}
\tag{35}
$$

Theorem 6.1 The active control law defined by Eq. (33) achieves global and exponential hybrid synchronization of the hyperchaotic Xu system (30) and hyperchaotic Li system (31) for all initial conditions $x(0), y(0) \in R^4$.

Proof. The proof is via Lyapunov stability theory [25] for global exponential stability.

As a candidate Lyapunov function, we consider the quadratic function

$$
V(e) = \frac{1}{2} e^T e = \frac{1}{2} \left(e_1^2 + e_2^2 + e_3^2 + e_4^2 \right),
$$

(36)

Note that V is a positive definite function on R^4.

Next, we differentiate (34) along the trajectories of (33). A simple calculation gives

$$
\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2
\tag{37}
$$

Note that $\dot{V}$ is a negative definite function on R^4.

Hence, by Lyapunov stability theory [25], the error dynamics (35) is globally exponentially stable for all values of $e(0) \in R^4$. ∎

Next, we illustrate our hybrid synchronization results with MATLAB simulations.

The feedback gains in the active controller (34) are taken as follows. Let $k_i = 5, \ (i = 1, 2, 3, 4)$.

The parameters of the hyperchaotic Xu and hyperchaotic Li systems are considered as in the hyperchaotic case. Thus, we take

$$
a = 10, \quad b = 40, \quad c = 2.5, \quad r = 16, \quad d = 2, \quad \alpha = 35, \quad \beta = 3, \quad \gamma = 12, \quad \delta = 7, \quad \varepsilon = 0.58
$$

For simulations, the initial conditions of the hyperchaotic Xu system (30) are chosen as

$$
x_1(0) = 27, \quad x_2(0) = 4, \quad x_3(0) = -13, \quad x_4(0) = 18
$$

Also, the initial conditions of the hyperchaotic Li system (31) are chosen as

$$
y_1(0) = 14, \quad y_2(0) = -28, \quad y_3(0) = 24, \quad y_4(0) = -5
$$

Figure 7 depicts the hybrid synchronization of the non-identical hyperchaotic Xu and hyperchaotic Li systems.

Figure 8 depicts the time-history of the hybrid synchronization errors e_1, e_2, e_3, e_4.
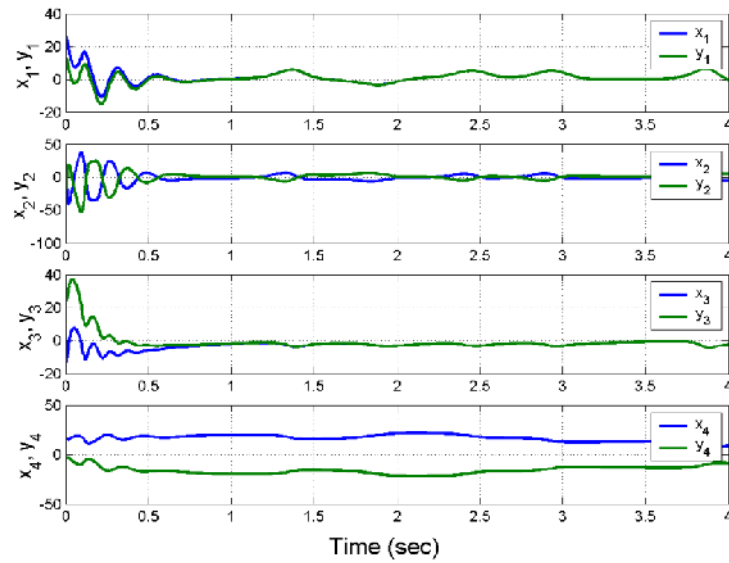

Figure 7. Hybrid Synchronization of Hyperchaotic Xu and hyperchaotic Li Systems
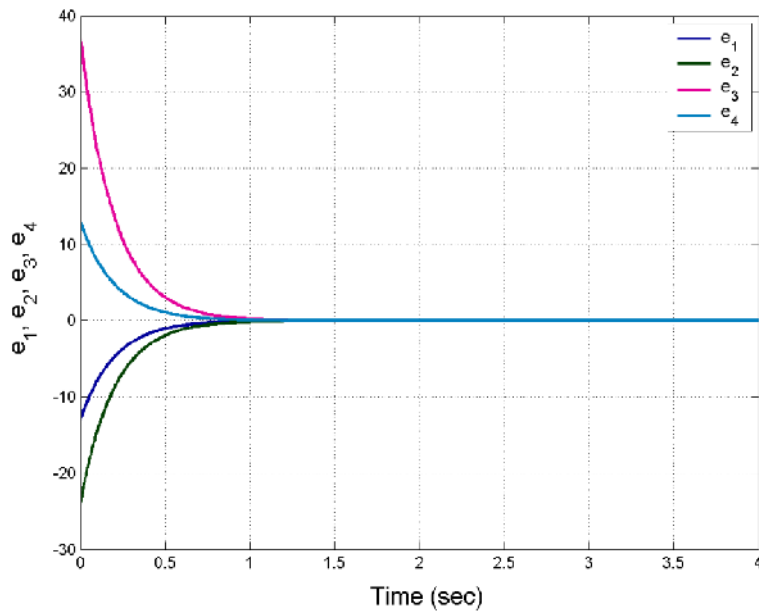

Figure 8. Time-History of the Hybrid Synchronization Errors e_1, e_2, e_3, e_4

7. CONCLUSIONS

This paper derived new results for the active controller design for the hybrid synchronization of hyperchaotic Xu systems (2009) and hyperchaotic Li systems (2005). Using Lyapunov control

theory, active control laws were derived for globally hybrid synchronizing the states of identical hyperchaotic Xu systems, identical hyperchaotic Li systems and non-identical hyperchaotic Xu and Li systems. MATLAB simulations were displayed in detail to demonstrate the hybrid synchronization results derived in this paper for hyperchaotic Xu and Li systems.

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AUTHOR

Dr. V. Sundarapandian earned his D.Sc. in Electrical and Systems Engineering from Washington University, St. Louis, USA in May 1996. He is Professor and Dean of the R & D Centre at Vel Tech Dr. RR & Dr. SR Technical University, Chennai, Tamil Nadu, India. So far, he has published over 310 research works in refereed international journals. He has also published over 200 research papers in National and International Conferences. He has delivered Key Note Addresses at many International Conferences with IEEE and Springer Proceedings. He is an India Chair of AIRCC. He is the Editor-in-Chief of the AIRCC Control Journals – *International Journal of Instrumentation and Control Systems*, *International Journal of Control Theory and Computer Modelling*, *International Journal of Information Technology, Control and Automation*, *International Journal of Chaos, Computing, Modelling and Simulation* & *International Journal of Information Technology, Modeling and Computing.* His research interests are Control Systems, Chaos Theory, Soft Computing, Operations Research, Mathematical Modelling and Scientific Computing. He has published four text-books and conducted many workshops on Scientific Computing, MATLAB and SCILAB.