ADAPTIVE TRACKING CONTROL OF SPROTT-H SYSTEM

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Abstract

This paper investigates the tracking problem of Sprott-H chaotic system which is three-dimensional chaotic systems discovered by Sprott (1994). New nonlinear control laws are derived for the tracking problem of uncertain Sprott-H chaotic system with unknown parameters. The adaptive generalized backstepping method is applied to control of uncertain Sprott-H chaotic system. Numerical simulations are presented to demonstrate the effectiveness of the control schemes.

KEYWORDS


1. INTRODUCTION

In recent years, various controllers have been proposed to achieve the control of chaotic systems [3-9]. The adaptive synchronization of an uncertain modified hyperchaotic Lü system was investigated in [10]. In [11], the output regulation problem for the Sprott-G chaotic system (1994) has been studied in detail. The stabilization and synchronization of the hyperchaotic Cai system with unknown system parameters was applied by adaptive control theory. The feedback controllers for control of the simplified Lorenz system was investigated in [13], [14] derive state feedback controllers for the output regulation problem of the Sprott-H chaotic system (1994). In [15], active controller has been designed to solve the output regulation problem for the Sprott-K chaotic system (1994) and a complete solution for the tracking of constant reference signals (set-point signals). [16] applied adaptive control theory for the control and synchronization of the Sprott H chaotic system (Sprott, 1994) with unknown system parameters.

The rest of the paper is organized as follows: In section 2, Sprott-H system is presented. In section 3, the generalized backstepping method is studied. In section 4, the generalized backstepping controller is designed for racking any desired inputs. In section 5, Represents simulation results. Finally, in section 6, Provides conclusion of this work.
2. SYSTEM DESCRIPTION

The three-dimensional Sprott-H chaotic system [17] described by the dynamics

\[
\begin{align*}
\dot{x} &= -ay + z^2 \\
\dot{y} &= x + by \\
\dot{z} &= x - z
\end{align*}
\]

(1)

Where \(a, b\) are positive constants and \(x, y, z\) are variables of the system, when \(a = 1, b = 0.5\), the system (1) is chaotic. See Figure 1 and Figure 2.

![Figure 1. Time response of the system (1).](image1)

![Figure 2. Phase portraits of the hyperchaotic attractors (1).](image2)

3. GENERALIZED BACKSTEPPING METHOD

Generalized backstepping method [7-9] is applied to nonlinear systems as follow

\[
\begin{align*}
\dot{X} &= F(X) + G(X)\eta \\
\dot{\eta} &= f_0(X, \eta) + g_0(X, \eta)u
\end{align*}
\]

(2)

Where \(\eta \in \mathbb{R}\) and \(X = [x_1, x_2, \ldots, x_n] \in \mathbb{R}\). Suppose the function \(v(x)\) is the lyapunov function.
\[ V(X) = \frac{1}{2} \sum_{i=1}^{n} x_i^2 \] (3)

The control signal and the extended lyapunov function of system (2) are obtained by equations (4),(5).

\[ u = \frac{1}{g_o(X,\eta)} \left\{ \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial \varphi_i}{\partial x_j} [f_i(X) + g_i(X)\eta] \right\}, k_i > 0, i = 1,2,\ldots, n \] (4)

\[ V_i(X, \eta) = \frac{1}{2} \sum_{i=1}^{n} x_i^2 + \frac{1}{2} \sum_{i=1}^{n} [\eta - \varphi_i(X)]^2 \] (5)

### 4. Tracking of Chaotic System

First, we will add control laws \( u_1, u_2 \) to the equation (1). Let \( \ddot{z} \) be the deviation between the output \( z \) and the desired trajectory \( r(t) \).

\[
\begin{align*}
\dot{x} &= -ay + (\ddot{z} + r)^2 + u_1 \\
\dot{y} &= x + by + u_2 \\
\dot{\ddot{z}} &= x - \ddot{z} - r - \dot{r} \\
\end{align*}
\] (6)

Stabilization of the state: In order to use the theorem, it is sufficient to establish equation (7).

\[
\begin{align*}
\varphi_1(x, y, \ddot{z}) &= r + \dot{r} \\
\varphi_2(x, y, \ddot{z}) &= 0 \\
\end{align*}
\] (7)

According to the theorem, the control signals will be obtained from the equations (8).

\[
\begin{align*}
u_1 &= - (\ddot{z} - r) - k_1(x - \varphi_1) + \hat{a}y - z^2 \\
u_2 &= -(\hat{b} + k_2)y - x \\
\end{align*}
\] (8)

The parameters \( a, b \) are unknown and \( \hat{a}, \hat{b} \) are respectively estimated values of parameters \( a, b \) which are updated by following equation.

\[
\begin{align*}
\dot{\hat{a}} &= -\hat{a}xy \\
\dot{\hat{b}} &= \hat{b}y^2 \\
\end{align*}
\] (9)

And Lyapunov function as

\[ V(x, y, z, w) = \frac{1}{2} x^2 + \frac{1}{2} y^2 + \frac{1}{2} z^2 + \frac{1}{2} (x - \varphi_1)^2 + \frac{1}{2} (y - \varphi_2)^2 \] (10)

we select the gains of controllers (8) in the following form

\[ k_1 = 10, k_2 = 9 \] (11)

### 5. Numerical Simulation

This section presents numerical simulations Sprott-H chaotic system. The generalized backstepping method (GBM) is used as an approach to control chaos in Sprott-H chaotic system. The initial values of the Sprott-H chaotic system are \( x(0) = -1, y(0) = 5, z(0) = -2 \).

- Case 1 : Tracking \( r(t) = 1 \),
- Case 2: Tracking $r(t) = \sin t$,
- Case 3: Tracking $r(t) = 2 - 2e^{-t}$.

Figure 3 shows that the scalar output can track the Step Input with the control inputs (8). Figure 4 shows that the scalar output can track the desired trajectory $r(t) = \sin(t)$. Figure 5 shows that the scalar output can track the desired trajectory $r(t) = 2 - 2e^{-t}$.
6. CONCLUSIONS

In this paper, we applied adaptive generalized backstepping control for the control of the Sprott-H chaotic system with unknown system parameters. The tracking problem considered for the Sprott-H chaotic system was for the tracking of any desired input signals. Numerical simulations show that the proposed method work effectively.

REFERENCES