APPLICATION OF Q-MEASURE IN A REAL TIME Fuzzy System for Managing Financial Assets

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Abstract

One of the major problems that a financial manager faces is the enormous amount of financial data. There is a variety of software systems used to support the process of investment decision making. In this paper, a software system for financial asset management is presented. The system is based on fuzzy logic, operates in real time and differs from existing systems for portfolio management in five key aspects. The system is tested on real data from Bulgarian Stock Exchange.

Keywords

Fuzzy System, Financial Data, Asset Management, Portfolio Management, Q-measure

1. Introduction

Financial investors use mainly two methodologies for analysing financial data in order to increase their wealth: fundamental analysis and technical analysis. While the former is based on exploring the countless macroeconomic factors of economic environment, the latter is about keeping track of price changes and finding patterns in their time series. Both aim at predicting the future behaviour of asset prices and making investment decisions depending on these predictions. However, the argument between fundamental and technical analysers is not an issue in this work.

In this paper, a software system for managing data and investments (both individual and portfolio) is presented. The system is based on tools that fuzzy logic provides and is an application of the Q-measure of an asset. [11]

The presented system differs from the existing systems for portfolio management in five key aspects:

1) There are no requirements for probability distributions of returns.
2) Geometric mean of daily returns is used as a measure of changes in prices.
3) A solution of the cardinality problem in portfolio theory is proposed.
4) Financial data is processed in real time.
5) The system is applicable to individual asset management as well as to portfolio construction.
The system consists of three modules (Fig.1):

- Module 1 – Data Managing Module (DMM)
- Module 2 – Q-Measure Fuzzy Logic Module (QFLM)
- Module 3 – Portfolio Construction Module (PCM).

DMM is an application for collecting, storing and managing financial data in real time. In addition, in this module calculations of precise measurements of important asset characteristics (return, risk and q-ratio) are implemented.

QFLM consists of applications based on fuzzy logic. Input data for this module are the crisp numerical values of asset characteristics from DMM. These crisp values are fuzzified and after applying the aggregation rules a fuzzy variable *Q-measure* for each of the assets is obtained. The output of this module is a defuzzified crisp value of *Q-measure*.

In PCM several portfolios are constructed. The investors’ utility preferences are the key factor for choosing the optimal portfolio.

2. DATA MANAGING MODULE (DMM)

2.1. Return, Risk and q-ratio

Let $P_1, P_2, ..., P_T$ be the sequence of daily close prices $P_t$ of an asset, where $t = 1, 2, ..., T$.

The use of geometric mean of returns aims at analysing the changes in the invested capital in its dynamics and is calculated as follows:

$$r_g = \sqrt[T-1]{\prod_{t=2}^{T} r_t},$$

where $r_t = \frac{P_t}{P_{t-1}}$ is the daily return for day and $t = 2, ..., T$. [1], [3]
If the standard deviation of the returns is to be used as a measure of the risk, it is appropriate to take the logarithms of returns and then the next formulae are derived:

$$\ln(r_t) = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln(P_t) - \ln(P_{t-1})$$

is the daily log return for $t = 2, ..., T$;

$$\bar{r}_g = \ln(r_g) = \ln\left(\prod_{t=2}^{T} P_t\right) = \frac{1}{T-1} \sum_{t=2}^{T} \ln r_t$$

is the arithmetic mean of log returns. [1], [2]

In time series of asset prices, there are days with no trading activity, e.g. there is missing data. One way to deal with this problem is to copy the last price corresponding number of times, e.g.

$$P_{t-1}, P_{t-1}, ..., P_{t-1}, P_t.$$ 

This operation does not change the arithmetic mean of log returns due to the properties of logarithms. In case of daily observations, the geometric mean of returns is calculated as follows:

$$\frac{k}{\sqrt[k]{\prod_{t=2}^{T} r_t}} = \frac{P_T}{P_1},$$

where $k = \sum_{i=2}^{T} \Delta_i$ and $\Delta_i$ is the number of days between the non-missing observations at days $t-1$ and $t$ decreased by 1.

If log returns are used, the above considerations should be made very carefully because of the different number of days between the observations. Thus, if the return for the period $\Delta_i$ is

$$r_i^* = \frac{r_t - 1}{\Delta_i} + 1,$$

then the log return at the moment $t$ is $\ln(r_i^*)$ and so the arithmetic mean of log returns is calculated:

$$\bar{r}^* = \ln\left(\prod_{t=2}^{T} r_t^*\right) = \frac{1}{T-1} \sum_{t=2}^{T} \ln r_t^*. [11]$$
Then the total return is \( TR = e^{\tilde{r}} \), the total norm of return is \( TNR = e^{\tilde{r}} - 1 \) and the annual norm of return is \( ANR = TNR \frac{D}{k+1} \), where \( D \) is the number of days in the financial year.

Finally, the annual return is \( AR = ANR - 1 \).

In the proposed system, annual return is used as a measure of asset return as it is an adequate estimator for the change of the investment.

The commonly used idea for risk in investment theory is the variability of returns. The variability is calculated with different statistical tools, based on probability distributions but most often by the variance of the returns. [1], [6], [7], [8], [9]

If log returns are used then the estimator of variance as arithmetic mean of log returns is calculated as follows:

\[
 s^2 = \frac{1}{T-2} \sum_{t=2}^{T} (\ln(r_t) - \bar{r}_T)^2 ,
\]

and the \( q \)-ratio equals the quotient of return and risk:

\[
 q = \frac{AR}{s} . [11]
\]

Some investors have specific expectations about both return and risk of their investments. This is why there are two additional inputs in the system- a lower threshold of return \( r_0 \) and risk \( \sigma_0 \), and these are predefined by the investor.

2.2. Description of DMM

DMM consists of a three-layer application (Fig 2.) The function of this module is to collect raw data (asset prices) in real time, to store and process this data.

![Figure 2. Scheme of the Data managing module](image)
The tests of the system are conducted on data from Bulgarian Stock Exchange. The changes in Bulgarian asset prices are published on the webpage of BSE (bse-sofia.bg). The information of interest (date, BSE code, open, close, high and low prices) is in the html code of this page. The application is started automatically by Windows Task Scheduler.

The data access layer contains methods for managing the database. The **Requester** realizes the request to the webpage of BSE. The **Parser** selects the data that is needed in a suitable form with respect to the next steps. For this reason, the html code of the downloaded page by the **Requester** is parsed with regular expressions. The **Filler** is the part of the application that deals with the missing data. There are two types of missing data – missing name and missing price. The name of an asset is missing in the database if it is a new asset listed on the exchange or there has been no trade with this asset for a period of time being longer than the time the system has been functioning. The price could be missing in the days with no deals with the given asset or on the holidays. The **Calculator** uses the information obtained by **Filler** and formulae from 2.1 to calculate the return, risk and \( q \)-ratio for each asset.

### 3. Q-MEASURE FUZZY LOGIC MODULE (QFLM)

Input data for QFLM is derived from the database after the module for data collection and storage has finished its work. The output is a crisp value for the Q-measure of each asset. The scheme of this module is presented on Fig. 3.

![Figure 3. Scheme of QFLM](image)

The input data consists of crisp values for return, risk, \( q \)-ratio, ID and name of each of the assets. These crisp values are fuzzified according to the predefined linguistic variables. Each linguistic variable is a triple consisting of a name, a domain and a set of fuzzy terms. The fuzzy terms are defined with their name and membership function. Three types of membership functions are used in this system: Gaussian, Bell and Sigmoid function. For each input variable a degree of membership to the corresponding term is calculated. Then the fuzzy rules are fired. The rules are of IF-THEN type and model the decision making process. The fuzzy output variable **Q-measure**
consists of fuzzy terms that describe the quality of an asset. The purpose of the defuzzification is to obtain a crisp output value.

3.1. Linguistic Variables

The fuzzy system comprises of three input and one output variables. Input variables are the characteristics of an asset: return $K_1 = \{\text{return}\}$, risk $K_2 = \{\text{Risk}\}$ and $K_3 = \{q\text{-ratio}\}$. The output variable is the Q-measure of an asset. These four fuzzy variables have different number of fuzzy terms with Gaussian, Bell or Sigmoid membership functions which are shown on Fig. 4, 5, 6.

![Figure 4. Input linguistic variable $K_1 = \{\text{return}\}$](image1)

![Figure 5. Input linguistic variable $K_2 = \{\text{Risk}\}$](image2)
3.2. Fuzzy Rules and Defuzzification

The goal of decision making is to find an optimal solution for a situation where a number of possible solutions exists. Bellman and Zadeh proposed a fuzzy model for decision-making in which objectives and goals are described as fuzzy sets and the solution is a suitable aggregation of these sets. [12]

Let \( G_i \) be a fuzzy set of objectives and let \( C_j \) be a fuzzy set of goals. Then, the fuzzy decision \( D \) under the given objectives \( G_i \) and goals \( C_j \), is the fuzzy set

\[
D(p) = \min \left[ \inf_{r \in N_r} G_i(p), \inf_{j \in N_m} C_j(p) \right],
\]

for each \( p \in P \), where \( P \) is the universe of possibilities. [10], [12]

If \( u_i \) and \( v_j \) are non-negative weights such that

\[
\sum_{i=1}^{n} u_i + \sum_{j=1}^{m} v_j = 1,
\]

then the fuzzy decision is obtained as
\[
D(p) = \sum_{i=1}^{n} u_i G_i(p) + \sum_{j=1}^{m} v_j C_j(p).
\]

In the proposed system the rules have the form:

**IF** (return \( K_1 \) is high) **AND** (risk \( K_2 \) is low) **AND** (\( q \)-ratio \( K_3 \) is big),

**THEN** (\( Q \)-measure is good)

There are 24 rules with respective weights that are implemented in the system. [12]

Although these rules adequately describe the most important possible situations that might arise in the process of investment decision-making, the list of fuzzy rules can be extended without changing the system’s architecture.

As a defuzzification method, the method of centre of gravity has been chosen:

\[
d_{CoG}(D) = \frac{\int_{-\infty}^{+\infty} x D(x) dx}{\int_{-\infty}^{+\infty} D(x) dx},
\]

and thus a crisp value for the asset quality is obtained as an output of QFLM.

Estimated value for each asset is then recorded in the database and used for managing individual assets or for constructing investment portfolios.

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**4. INDIVIDUAL ASSET MANAGEMENT**

The crisp values of \( Q \)-measure for each asset, obtained in QFLM, can be used to support investors in the process of decision making.

The \( Q \)-measure of an asset reflects three variables: return, risk and their ratio. In economics, it is believed that return could always be traded for risk, i.e. higher returns always bring more risk and vice versa. Empirical results confirm this belief. [4], [5]

However, for the non-speculative investor it is essential to what extent these two characteristics (return and risk) are stable over time. The proposed model for the assessment of individual assets uses one additional characteristic: the \( q \)-ratio which is a quotient of return and risk and reflects the extent to which the taken risk is justified by adequate returns. The conducted empirical tests for all assets listed on BSE, for different periods of time show that \( Q \)-measure is a proper indicator of the quality of the asset over time. Some of the results are presented in 6.
If the $Q$-measure is less than 0.4 (whatever return and risk) a dramatic decrease of price occurs in up to about 3 months. At the same horizon and a $Q$-measure between 0.4 and 0.6 the price of the asset does not change significantly and even if it increases, the transaction costs will exceed the potential benefits. When the $Q$-measure is greater than 0.6 the asset price increases steadily and such an asset is considered suitable for purchase.

5. PORTFOLIO CONSTRUCTION MODULE (PCM)

![Diagram of PCM]

Figure 8. Scheme of PCM

The scheme of PCM is shown on Fig. 8. Input data for PCM are received from the database after QFLM has finished its work. First, in Asset Selection all the assets are sorted in descending order by their $Q$-measure. Then several assets are selected with respect to a criterion set by the investor, e.g. maximum number of assets or other. Then all possible combinations of these assets are generated and they form corresponding portfolios. If a substantial amount of capital is unused then a procedure for allocation takes place. This procedure modifies the portfolio to the optimal reduction of unused capital. Constructed portfolios are stored in a database and the investor can make a decision.

Let $A = \{A_1; A_2; A_3; \ldots; A_n\}$ be a set of financial assets. The goal is to construct investment portfolios (subsets of $A$) according to the investor’s preferences. Here the case of max number of assets will be described.

If the investor wants no more than $m$ assets in his portfolio, then the first $m$ assets from the sort list are extracted. Then these assets are combined in all possible ways to sets with 1, 2, ..., $m$
elements. The number of these sets is \(2^m - 1\), because the empty set is not taken into consideration. Now for each of these sets a portfolio is constructed. The share of asset \(A_j\) is denoted by \(x_j\) and equals

\[
x_j = \frac{Q_j}{\sum_{j=1}^{n} Q_j},
\]

where \(n\) is the number of financial assets in the portfolio, \(n = 1, 2, \ldots, m\), and \(Q_j\) is the \(Q\)-measure of \(A_j\). The number of shares \(n_j\), which are to be purchased from the asset \(A_j\) is calculated as:

\[
n_j = \left\lfloor \frac{x_j S}{P_j} \right\rfloor,
\]

where \(x_j\) is the quota the asset \(A_j\), \(P_j\) the price of one share of \(A_j\) and \(S\) is the capital (total sum to be invested). Due to the cardinality problem, at this stage \(S\) is not fully used. Thus, the used capital is:

\[
S_u = \sum_{j=1}^{N} n_j P_j
\]

and the remaining capital is \(S - S_u\). In case \(S - S_u\) exceeds a given percentage of the initial capital an allocation procedure takes place. In the first step of this procedure, the remaining capital is compared to the price of the asset with the highest \(Q\)-measure. If \(S - S_u\) is bigger than that price then additional number of shares of this asset is added to the portfolio. Otherwise, the comparison is repeated for the next item in the sort list. This process is repeated until the remaining capital is small enough and no more shares can be bought. At each step, the number of additional shares \(n_{j_a}\) of an asset is

\[
n_{j_a} = \left\lfloor \frac{x_j (S - S_u)}{P_j} \right\rfloor.
\]

Finally, portfolio return \(R_p\), risk \(\sigma_p\) and \(q\)-ratio \(q_p\) are calculated for all \(2^m - 1\) portfolios:

\[
R_p = \sum_{j=1}^{N} x_j r_j, \quad \sigma_p = \sum_{j=1}^{N} x_j s_j, \quad q_p = \frac{R_p}{\sigma_p}.
\]

Now each portfolio is put through QFLM in order to obtain its \(Q\)-measure. The portfolios with their characteristics are stored in the data base.

6. RESULTS

In this section some of the test results are demonstrated.
6.1. Individual Asset Management

As discussed in section 4, the results of QFLM can be used for managing an individual asset. Four assets with different *Q-measures* are selected.

6.1.1. 4EC

On 15.02.2012 the asset 4EC has *Q-measure* of 0.81 and as can be seen on Fig. 9 its price steadily increases in the next 3-4 months.

![Figure 9. One-year change in price of 4EC](image)

6.1.2. E4A

On 15.02.2012 the asset E4A has *Q-measure* of 0.18. Fig. 10 shows that its price decreases in the next 3-4 months.

![Figure 10. One-year change in price of E4A](image)
6.1.3. 55B

On 15.02.2012 the asset 55B has Q-measure of 0.39 which indicates not good quality. However, as it can be seen on Fig. 11 in the next 3-4 weeks its price rises significantly and so does its Q-measure: 0.52, 0.71 and 0.82. Therefore, the system cannot predict for a long period of time, but as far as it is started daily it can quickly enough detect change in asset price behavior.

Figure 11. One-year change in price of 55B

6.1.4. 6F3

On 15.02.2012 the asset 6F3 has Q-measure of 0.24 which indicates bad quality. As it can be seen on Fig. 12 in the next 3-4 months its price fluctuates, so the Q-measure is a trusted indicator in this case again.

Figure 12. One-year change in price of 6F3

6.2. Portfolios

The following results are obtained after tests conducted on 20.06.2012 on Bulgarian Stock Exchange. The initial capital is fixed.
6.2.1. Two methods of portfolio construction

The first portfolio consists of 25 assets with quotas depending on their $Q$-measure. The BSE code, price, number of shares, and characteristics are presented on Table 1. Initial capital is BGN 100 000.

This portfolio has return $R_p = 1.4652573$, risk $\sigma_p = 0.0185322$ and $Q$-measure=0.7583479. The unused capital is BGN 232.27.

<table>
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<th>Quota</th>
<th>Risk</th>
<th>Annual norm of return</th>
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If an investment portfolio consists of the same 25 assets with equal quotas of 0.04 then the return is $R_p = 1.2231736$ (lower), the risk is $\sigma_p = 0.0199578$ (slightly higher) and the $Q$-measure is $0.4444046$ (lower). The unused capital is BGN 120.65.

The only advantage of the portfolio with equal quotas is the smaller amount of unused capital. Now, let us take only the first ten assets and construct a portfolio (Table 2) with quotas depending on their $Q$-measure. Now the return is $R_p = 1.832328$, the risk is $\sigma_p = 0.017806$ and the unused capital is BGN 37.89.

Table 2. Investment portfolio of first 10 assets with quotas depending on their $Q$-measure

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<th>BSE code</th>
<th>Price</th>
<th>Number of shares</th>
</tr>
</thead>
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<td>45</td>
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</table>

The results show that an investment portfolio with quotas, depending on the $Q$-measure, shows higher $Q$-measure despite the fact that some of the assets in the portfolio have low $Q$-measure. For a portfolio with equal quotas the $Q$-measure is smaller as it consists of significantly more shares of assets with low $Q$-measure. However, the best choice is to pick the best assets.

### 6.2.2. Allocation procedure

To study the relationship between the size of the investment capital and the need to implement additional allocation procedure, portfolios of seven assets have been constructed. These portfolios consist of the same assets, yet with different amounts of investment capital. It is important to note that an additional allocation procedure does not apply if unused capital is less than 0.5% of the initial capital. This percentage is used to avoid transaction costs, whenever unused capital is relatively small.

Initial capital **BGN 1,000**. Initial portfolio (Table 3): return $R_p = 1.78318324$, risk $\sigma_p = 0.01519687$, $Q$-measure 0.81230537 and unused capital BGN 66.58.
Table 3. Initial portfolio with 7 assets, \( S_0 = \text{BGN 66.58} \)

<table>
<thead>
<tr>
<th>BSE code</th>
<th>Price</th>
<th>Number of shares</th>
<th>Quota</th>
<th>Risk</th>
<th>Annual norm of return</th>
<th>Q-measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>3JU</td>
<td>45</td>
<td>3</td>
<td>0.14538565</td>
<td>0.0241193</td>
<td>1.99408284</td>
<td>0.82919283</td>
</tr>
<tr>
<td>5ORG</td>
<td>90</td>
<td>1</td>
<td>0.14245006</td>
<td>0.00247399</td>
<td>1</td>
<td>0.81245</td>
</tr>
<tr>
<td>6A6</td>
<td>1.74</td>
<td>81</td>
<td>0.14244872</td>
<td>0.01237877</td>
<td>1.85524372</td>
<td>0.81244236</td>
</tr>
<tr>
<td>BLKC</td>
<td>0.471</td>
<td>302</td>
<td>0.14244326</td>
<td>0.02221776</td>
<td>2.52884615</td>
<td>0.8124119</td>
</tr>
<tr>
<td>4EC</td>
<td>1.91</td>
<td>74</td>
<td>0.14243653</td>
<td>0.02754392</td>
<td>2.86830926</td>
<td>0.81237285</td>
</tr>
<tr>
<td>SO5</td>
<td>1.846</td>
<td>77</td>
<td>0.14243505</td>
<td>0.00874433</td>
<td>1.18275862</td>
<td>0.81236439</td>
</tr>
<tr>
<td>5BD</td>
<td>0.754</td>
<td>188</td>
<td>0.14240073</td>
<td>0.00871467</td>
<td>1.04851752</td>
<td>0.81216861</td>
</tr>
</tbody>
</table>

Originally, the portfolio is constructed with a high percentage of unused funds (over 0.5%). To reduce this rate, additional allocation procedure is applied.

Portfolio after the first allocation procedure (Table 4): return \( R_p = 1.79868443 \), risk \( \sigma_p = 0.01556071 \), Q-measure 0.81228124 and unused capital BGN 15.97.

Table 4. Portfolio with 7 assets after the first allocation, \( S_a = \text{BGN 15.97} \)

<table>
<thead>
<tr>
<th>BSE code</th>
<th>Price</th>
<th>Number of shares</th>
<th>Quota</th>
<th>Risk</th>
<th>Annual norm of return</th>
<th>Q-measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>3JU</td>
<td>45</td>
<td>3</td>
<td>0.135</td>
<td>0.0241193</td>
<td>1.99408284</td>
<td>0.82919283</td>
</tr>
<tr>
<td>5ORG</td>
<td>90</td>
<td>1</td>
<td>0.09</td>
<td>0.00247399</td>
<td>1</td>
<td>0.81245</td>
</tr>
<tr>
<td>6A6</td>
<td>1.74</td>
<td>87</td>
<td>0.15138</td>
<td>0.01237877</td>
<td>1.85524372</td>
<td>0.81244236</td>
</tr>
<tr>
<td>BLKC</td>
<td>0.471</td>
<td>325</td>
<td>0.153075</td>
<td>0.02221776</td>
<td>2.52884615</td>
<td>0.8124119</td>
</tr>
<tr>
<td>4EC</td>
<td>1.91</td>
<td>79</td>
<td>0.15089</td>
<td>0.02754392</td>
<td>2.86830926</td>
<td>0.81237285</td>
</tr>
<tr>
<td>SO5</td>
<td>1.846</td>
<td>82</td>
<td>0.151372</td>
<td>0.00874433</td>
<td>1.18275862</td>
<td>0.81236439</td>
</tr>
<tr>
<td>5BD</td>
<td>0.754</td>
<td>202</td>
<td>0.152308</td>
<td>0.00871467</td>
<td>1.04851752</td>
<td>0.81216861</td>
</tr>
</tbody>
</table>

It can be noted that the risk of this portfolio is less than the risk of the initial portfolio, the return is higher and the Q-measure is slightly smaller. The unused capital is reduced to BGN 15.97, but an additional allocation procedure is still possible.

Portfolio after the second application of the additional allocation (Table 5): return \( R_p = 1.81988325 \), risk \( \sigma_p = 0.01574007 \), Q-measure 0.81228817 and unused capital BGN 4.64.
Table 5. Portfolio with 7 assets after the second allocation, $S_t = BGN 4.64$

<table>
<thead>
<tr>
<th>BSE code</th>
<th>Price</th>
<th>Number of shares</th>
<th>Quota</th>
<th>Risk</th>
<th>Annual norm of return</th>
<th>Q-measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>3JU</td>
<td>45</td>
<td>3</td>
<td>0.135</td>
<td>0.0241193</td>
<td>1.99408284</td>
<td>0.82919283</td>
</tr>
<tr>
<td>5ORG</td>
<td>90</td>
<td>1</td>
<td>0.09</td>
<td>0.00247399</td>
<td>1</td>
<td>0.81245</td>
</tr>
<tr>
<td>6A6</td>
<td>1.74</td>
<td>88</td>
<td>0.15312</td>
<td>0.01237877</td>
<td>1.85524372</td>
<td>0.81244236</td>
</tr>
<tr>
<td>BLKC</td>
<td>0.471</td>
<td>331</td>
<td>0.155901</td>
<td>0.02221776</td>
<td>2.52884615</td>
<td>0.81241119</td>
</tr>
<tr>
<td>4EC</td>
<td>1.91</td>
<td>80</td>
<td>0.1528</td>
<td>0.02754392</td>
<td>2.86830926</td>
<td>0.81237285</td>
</tr>
<tr>
<td>SO5</td>
<td>1.846</td>
<td>83</td>
<td>0.153218</td>
<td>0.00874433</td>
<td>1.18275862</td>
<td>0.81236439</td>
</tr>
<tr>
<td>5BD</td>
<td>0.754</td>
<td>206</td>
<td>0.155324</td>
<td>0.00871467</td>
<td>1.04851752</td>
<td>0.81216861</td>
</tr>
</tbody>
</table>

After constructing portfolios with various investment capital a conclusion can be derived. It can be observed that unused capital is significantly larger if the ratio of capital/maximum price is small. The proportion of unused capital decreases with the increase of investment capital or, in other words, the more initial capital one has, the less part of it remains unused (Table 6).

Table 6. Correlation between the amount of investment capital and the unused capital

<table>
<thead>
<tr>
<th>Capital/max price per share</th>
<th>Unused capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000/90 = 11.(1)</td>
<td>6.658 %</td>
</tr>
<tr>
<td>10000/90 = 111.(1)</td>
<td>0.9261 %</td>
</tr>
<tr>
<td>100000/90 = 1111.(1)</td>
<td>0.1597 %</td>
</tr>
</tbody>
</table>

6.2.3. Construction of all possible portfolios with predefined maximum number of assets

The proposed system provides the investor with the opportunity to select portfolios from all possible portfolios with predefined maximum number financial assets. These portfolios are constructed by the combination constructor. If the best five assets are selected then 31 investment portfolios are constructed. Each of them is represented as a point on the graph shown in Fig. 13, investment risk being plotted on the $x$ axis, and the return being plotted on the $y$ axis.

Figure 13. Portfolios with different numbers of assets
All portfolios lie along an almost straight line and this is not incidentally. A basic idea in economics is that return is proportional to risk. Investors can choose any portfolio depending on their preference. For example, if they prefer an asset portfolio with only one asset, they have five different choices. Or if they prefer a portfolio with fixed return they have several choices again.

7. CONCLUSIONS AND FUTURE WORK

In this paper a real time software system for managing individual assets and portfolio investments is proposed. This system is based on the Q-measure of an asset. The Q-measure incorporates return, risk and their ratio, and being modelled with fuzzy logic tools it intuitively reflects the process of investment decisions in economic environment with enormous amount of data, which is often uncompleted and imprecise. The system is tested with real data from BSE.

Major difference from existing similar systems is that there are no requirements for probability distributions of returns. In addition, the system provides a procedure for portfolio allocation that aims at maximal possible use of investment capital. Although it is not based on an optimization algorithm it solves the cardinality problem in portfolio management. Finally, one advantage of the system is that the investor can chose between several portfolios. The system does not determine the ‘best’ portfolio but allows the investors to base their decision based on financial market information provided by the model and on their personal preferences.

The authors plan to develop the system in two directions: to incorporate a neural network for autonomous adjusting the parameters of the fuzzy terms, and second, to include a module for parsing other stock exchanges.

REFERENCES

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