A METAHEURISTIC OPTIMIZATION ALGORITHM FOR THE SYNCHRONIZATION OF CHAOTIC MOBILE ROBOTS

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ABSTRACT
We provide a scheme for the synchronization of two chaotic mobile robots when a mismatch between the parameter values of the systems to be synchronized is present. We have shown how meta-heuristic optimization can be used to adapt the parameters in two coupled systems such that the two systems are synchronized, although their behavior is chaotic and they have started with different initial conditions and parameter settings. The controlled system synchronizes its dynamics with the control signal in the periodic as well as chaotic regimes. The method can be seen also as another way of controlling the chaotic behavior of a coupled system. In the case of coupled chaotic systems, under the interaction between them, their chaotic dynamics can be cooperatively self-organized. A synergistic approach to meta-heuristic optimization search algorithm is developed. To avoid being trapped into local optimum and to enrich the searching behavior, chaotic dynamics is incorporated into the proposed search algorithm. A chaotic Levy flight is firstly incorporated in the proposed search algorithm for efficiently generating new solutions. And secondly, chaotic sequence and a psychology factor of emotion are introduced for move acceptance in the search algorithm. We illustrate the application of the algorithm by estimating the complete parameter vector of a chaotic mobile robot.

KEYWORDS
optimization search algorithm, chaotic dynamics, psychology factor of emotion, chaotic sequence

1. INTRODUCTION
The first chaotic mobile robot that can navigate following a chaotic pattern was proposed by Nakamura and Sekikuchi [1], where the Anorld's equation was used to generate the desired motions. Further investigations on chaotic trajectories of the same type of the robot using other equations were carried out in [2-12]. To scan the whole connected workspace, the main goal for a chaotic mobile robot is to increase and take advantage of the coverage areas resulting from its travelling paths. For many applications of mobile robots such as cleaning, patrolling, and grass-cutting tasks, large coverage areas are desirable for the mobile robots designed for implementation of navigating behaviors. As in single robot coverage, the objective is to completely cover the whole work area with shortest time. In the context of multiple robots, the key problem to be solved is to reduce the repetition times so that they simultaneously cover different parts of the environment. The use of collaborative robots is often suggested to have advantages over single robot systems. It is generally believed that proper organization of cooperating mobile robots provides significant benefits over single robot approaches for various missions. Cooperating robots have the capability to accomplish the same coverage work faster than a single robot. Synchronization of chaos is a cooperative behavior of coupled nonlinear
systems with chaotic uncoupled behavior. This behavior appears in many physical and biological processes. In most of the analysis done on two coupled chaotic systems, the two systems are assumed to be identical. In practical implementations this will not be the case. Chaos synchronization may seem unlikely due to the extreme sensitivity of chaos to initial conditions as well as small random disturbances and parameter variation. However, it has been realized that even chaotic systems may be coupled in a way such that their chaotic oscillations are synchronized. Mutual synchronization can be considered as a form of cooperative self-organization of interacting systems. In contrast to the case of coupled periodic systems, even in the case of coupled chaotic systems, under the interaction between them their chaotic dynamics can be cooperatively self-organized. When this phenomenon occurs there is complete or almost complete coincidence of regular or chaotic realizations generated by identical or almost identical systems with different initial conditions. We consider the case of synchronized chaos where two coupled systems evolve identically in time. In this paper, we investigate the synchronization of two coupled chaotic mobile robots which are not identical. We illustrate how adaptive controllers can be used to adjust the parameters of the systems such that the two chaotic mobile robots will synchronize. We proposed a simple yet effective meta-heuristic optimization algorithm for the synchronization of chaotic mobile robots.

2. Meta-heuristic Cuckoo Search Algorithm

Cuckoo search for the optimization problem is a meta-heuristic search algorithm which has been proposed by Yang and Deb [13, 14]. This search algorithm is inspired by the reproduction strategy of cuckoos. The cuckoo bird lays her egg in the nest of other host birds, which may be of different species. The host bird may discover that the egg is not her own and either throw this alien egg away or simply abandons the nest all together and then build a new nest elsewhere to increase the hatching probability of her own egg. The cuckoo breeding analogy is represented by a set of host nests. Each nest carries an egg (a solution). A new nest is generated by performing a random walk from some current nest. The new nest is then evaluated and compared to a current nest chosen at random. The new solution is formed by modifying only one solution with a random move (i.e., the solution is improved by generating a new solution via a random move from an existing solution). If the new solution is found to be better than another randomly chosen existing solution then the old solution is replaced with the new one. Furthermore, for each generation of evolution a new solution is generated with a certain probability and the lowest fit solution is replaced by this solution (i.e., the worst nests are removed with some probability and replaced with random nests.).

The foraging trajectory of an animal is essentially a random walk in that the next step is based on the current location and the probability of moving to the next location. One type of random walk is Lévy flights [15, 16] in which the step lengths are distributed according to a heavy-tailed probability distribution. Many studies have shown that flight behaviour of many animals has demonstrated the typical characteristics of Lévy flights. The cuckoo search implements Levy flight type of search behaviour by employing heavy tailed probability distribution. Due to the heavy tailed nature of the Lévy distribution, motion based on Lévy flights is able to search large areas very quickly. When exploring the area around a given solution, the search will mostly stay local, but will move a great distance occasionally, helping faster explore the space. Figure 1 shows the pseudo code for the basic steps involved in the cuckoo search proposed by Yang and Deb [14]. The cuckoo search use Levy flights for both local and global searching. The scale of this random search is controlled by multiplying the generated Levy flight by a step size \( \alpha \). Yang and Deb [14] found that the convergence rate for this search algorithm was not affected strongly by the value \( P_a \) and they suggested setting \( P_a = 0.25 \).
Cuckoo Search Algorithm

begin
Objective function \( f(x), x = (x_1, \ldots, x_d)^T; \)
Initial a population of \( n \) host nests \( x_i (i = 1, \ldots, n) \);
while \((t < \text{Maximum Generation}) \text{ or (stop criterion)}\);
Get a cuckoo (say \( i \)) randomly
and generate a new solution by Levy flights;
Evaluate its quality/fitness; \( F_i \)
Choose a nest among \( n \) (say \( j \)) randomly;
if \((F_i > F_j)\),
Replace \( j \) by the new solution;
end
Abandon a fraction \( (p_a) \) of worse nests
[build new ones at new locations by Levy flights];
Keep the best solutions (nests with quality solutions);
Rank the solutions and find the current best;
end while
Post process results and visualization;
end

Figure 1. Pseudo code for the Cuckoo Search optimization search algorithm

3. MOVE-GENERATION WITH CHAOTIC LEVY FLIGHTS

The trajectory of chaotic dynamics can travel ergodically over the whole search space. Generally, the chaotic variable has special characters including pseudo-randomness, irregularity and ergodicity. In general, the parameter \( \alpha \) in Levy flight for the cuckoo search algorithm described in the previous section is the key factor to affect the convergence of the search algorithm. However, it cannot ensure the ergodicity of optimization entirely in phase space in that they are absolutely random in the cuckoo search algorithm. The ergodicity property asserts that a system having a number of possible states will visit each one with equal frequency over a finite time [17]. Here we incorporate the chaotic Levy flight for the improved cuckoo search. We introduced a new neighbor selection by focusing on the concept of chaos. With the characteristic of nonlinear systems, chaos has a bounded unstable dynamic behavior that includes infinite unstable periodic motions and exhibits sensitive dependence on initial conditions. It appears to be stochastic, but it occurs in a deterministic nonlinear system under deterministic conditions. The well-known logistic map which exhibits the sensitive dependence on initial conditions is employed in this paper to generate the chaotic sequence \( c_s \) for the parameter in Levy flight:

\[
c_s(t+1) = 4.0 \times c_s(t) \times (1 - c_s(t)), \quad 0 \leq c_s(0) \leq 1.
\] (1)

By Levy flights with the infinite variance of Levy distribution, it permits occasionally large steps of the previous solution. Large steps are needed to avoid being trapped into local optimum. Thus, a synergy of a chaotic sequence and Levy flights may result in better solutions. For the proposed algorithm, a new method to search a solution is introduced. The new solution generation method is shown in the following equation:
A chaotic sequence and Levy distribution are used to generate $c_s$ and $\text{Levy} (\lambda)$ respectively. The product $\otimes$ means entry-wise multiplications. The Levy flights essentially provide a random walk, while their random steps are drawn from a Levy distribution for large steps

$$\text{Levy} \sim u^{-\lambda}, \quad (1 \leq \lambda \leq 3)$$

which has an infinite mean and an infinite variance. Specifically, the distribution used is a power law of the form in equation (3), and therefore has an infinite variance. Because the chaotic sequence can generate several neighborhoods of suboptimal solutions to maintain the variability in the solutions, it can prevent the search process from becoming premature [18]. That is, the algorithm probably converges to a space in the search space with denser good solutions.

4. Move-Acceptance by a Psychology Factor of Emotion and Chaotic Sequence

Among most computational intelligence algorithms for optimization problem including metaheuristic search algorithms, the solution is drawn like a moth to a flame and cannot keep away. To avoid being trapped into local optimum and to enrich the searching behavior, chaotic sequence and a psychology factor of emotion are proposed for move acceptance decision in the improved cuckoo search algorithm. In the context of psychology, emotion is considered a response to stimulus that involves characteristic physiological changes such as rise in body temperature and increase in pulse rate. By Weber-Fechner Law, the relationship between perception and stimulus is logarithmic [19]. This relationship means that if the perception is altered in an arithmetic progression the corresponding stimulus varies as a geometric progression. In other words, if the weight is 1 kg, an increase of a few grams does not be noticed. And when the mass is increased by a certain factor, an increase in weight will be perceived. Further, if the mass is doubled, the threshold will be also doubled. This relationship can be described by a differential equation as:

$$dp = k \frac{dS}{S}$$

where $dp$ is the differential change in perception, $S$ is the stimulus at the instant and $dS$ is the differential increase in the stimulus. A constant factor $k$ is to be determined experimentally. By integrating the above equation,

$$p = k \ln S + C$$

with $C$ is the constant of integration, $\ln$ is the natural logarithm. In order to determine $C$, let $p = 0$, that is, no perception, then

$$C = -k \ln S_0$$

where $S_0$ is the Absolute Stimulus Threshold. The equation then becomes:

$$p = -k \ln \frac{S}{S_0}$$
For the proposed emotional chaotic cuckoo search algorithm, we define only two emotions cuckoos could have, positive and negative, and correspond to two reactions to perception respectively as follow:

\[
\text{IF (} c_s < e_j \text{) THEN positive}
\]
\[
\text{ELSE negative}
\]

where \( c_s \) is the chaotic sequence number. The emotion of cuckoos can determine by \( e_s = p \). The perception of cuckoo can be described by following:

\[
e_s = -k \ln \left( \frac{S(F(x) - F(x_j))}{S_0} \right)
\]

(8)

Here \( S \) means stimulus function and \( S_0 \) means stimulus threshold. \( F(*) \) is a fitness function. A candidate move is generated by chaotic Levy flight, and the system must decide whether to accept that move, based upon the chaotic sequence number and the emotion factor. This process of move generation with chaotic Levy flight and move acceptance is repeated. This mechanism enables a system to transcend fitness barriers, thereby moving from one valley in the fitness landscape to another. The decision to accept new solutions is based on the acceptance criterion. We apply the psychology factor of emotion, which is given by (8):

\[
P(\text{accept}) = \min(1, e_s)
\]

(9)

This criterion produce real numbers in \([0,1]\) interval as the acceptance probability, which are used in the algorithm in the decision making process. Random numbers are replaced by chaotic sequence. The proposed algorithm compares the value of \( P(\text{accept}) \) with a value from a chaotic sequence. The proposed emotional chaotic cuckoo search algorithm is shown in Figure 2. The chaotic sequence used in this part produces not just a gradual divergence of the sequences of values, but also an exponential divergence, bringing the complexity and unpredictability features of chaotic theory into the proposed algorithm. Hence, the probability of evading local minima increases dramatically. The distinctive characteristics of the emotional chaotic cuckoo search algorithm are listed below to recapitulate the main proposal:

- Improved quality of the neighborhood search, and neighbor selection, using a chaotic sequence and a Levy random number generator.
- Increased probability of escaping from local minima by using the new acceptance criterion and the new method of space search.

**Emotional Chaotic Cuckoo Search Algorithm**

begin

Objective function \( f(x) \), \( x = (x_1, \cdots, x_d)^T \);
Initial a population of \( n \) host nests \( x_i \) (\( i = 1, \cdots, n \));

while \( (t < \text{Maximum Generation}) \) or (stop criterion);
Get a cuckoo (say \( i \)) randomly
and generate a new solution by Chaotic Levy flights;
Evaluate its quality/fitness; \( F_i \);
Choose a nest among \( n \) (say \( j \)) randomly;

\]
if \((F_i > F_j)\),
Replace \(j\) by the new solution;
else if \((c_s < e_r)\),
Replace \(j\) by the new solution;
end
Abandon a fraction \((p_a)\) of worse nests
[and build new ones at new locations via Chaotic Levy flights];
Keep the best solutions (nests with quality solutions);
Rank the solutions and find the current best;
end while
Post process results and visualization;
end

Figure 2. Pseudo code of the proposed Emotional Chaotic Cuckoo Search Algorithm

5. Synchronization of Chaotic Mobile Robots

The chaotic dynamics of the mobile robot is achieved by incorporating nonlinear equations into
the robot kinematic equations, such as Arnold, Lornez, and the Chua’s circuit equations, that are
well known equations with chaotic dynamics.

5.1. Incorporating Arnold Equation into Robot Kinematic Equation

The Arnold's equation can be described in the form of

\[
\begin{pmatrix}
    \dot{x}_1 \\
    \dot{x}_2 \\
    \dot{x}_3
\end{pmatrix} = \begin{pmatrix}
    A_1 \sin x_3 + A_2 \cos x_2 \\
    A_1 \sin x_3 + A_2 \cos x_2 \\
    A_1 \sin x_3 + A_2 \cos x_2
\end{pmatrix}
\]

(10)

where \(A_1, A_2, A_3\) are constants. The Arnold equation behaves periodic orbit when one of
the constants, for example \(A_3\), is small or 0, and behaves chaotic orbit when \(A_3\) is large.
Incorporating
the Arnold equation into the controller of the mobile robot by adopting the methodology in [1], we
can describe the state equation of two-wheel mobile robots as

\[
\begin{pmatrix}
    \dot{x} \\
    \dot{y} \\
    \dot{\theta}
\end{pmatrix} = \begin{pmatrix}
    \cos \theta & 0 & v \\
    \sin \theta & 0 & 0 \\
    0 & 1 & \sigma
\end{pmatrix}
\]

(11)

where \(v\) [m/s] is the velocity of the robot and \(\sigma\) [rad/s] is the angular velocity input of the system.
Therefore, the state equation of the mobile robot becomes

\[
\begin{pmatrix}
    \dot{x}_1 \\
    \dot{x}_2 \\
    \dot{x}_3 \\
    \dot{x} \\
    \dot{y}
\end{pmatrix} = \begin{pmatrix}
    A_1 \sin x_3 + A_2 \cos x_2 \\
    A_1 \sin x_3 + A_2 \cos x_2 \\
    A_1 \sin x_3 + A_2 \cos x_2 \\
    v \cos x_1 \\
    v \sin x_1
\end{pmatrix}
\]

(12)
Equation (12) includes the Arnold equation. The Arnold equation has chaotic dynamics or not, depending on the initial states. With some values of initial states for the Arnold equation, the trajectory behave chaotically. It is guaranteed that a chaotic dynamics of the Arnold equation is not attracted to a limit cycle or a quasi-periodic orbit. As seen in equation (12), the whole states evolve in a 5-dimensional space which includes a 3-dimensional subspace of the Arnold equation. The state evolution in the 2-dimensional complementary space is highly coupled with that in the 3-dimensional subspace. This coupled system can be interpreted physically by the fact that the mobile robot moves with a constant velocity and is steered by the third variable of the Arnold equation. Therefore, the trajectory in x-y space of the mobile robot should also behave chaotic likely. The integrated system with appropriate initial conditions and adjusting parameters guaranteed that the trajectories of the Arnold’s equation behave chaotically. The resultant trajectory of the mobile robot with $v=1$, $A_1=0.25$, $A_2=0.25$, $A_3=0.5$ and initial conditions $x_1=4$, $x_2=3.5$, $x_3=0$, $x=5$, $y=5$ is shown in Figure 3. The initial conditions were selected from a domain where the Poincare section do not construct a closed trajectory. The trajectory of the robot is highly sensitive dependence on the initial conditions and unpredictable.

![Figure 3. Trajectory of the chaotic mobile robot controlled by Arnold equation in x−y−θ.](image)

### 5.2. Synchronization Through Parameter Adaptation

Consider two chaotic systems with evolution equations $\dot{X}_1 = f(X_1, \mu_1)$, $\dot{X}_2 = f(X_2, \mu_2)$, where $\mu_1$, $\mu_2$ are parameters of the systems. Complete synchronization between the primary and secondary can be realized by matching the parameters of the response to that of the drive through a loop of adaptive control. This is implemented by augmenting the evolution equation for the dynamical system by an additional equation for the evolution of the parameter(s) as described below.

\[
\begin{align*}
\dot{X}_1 &= f(X_1, \mu_1) \\
\dot{X}_2 &= f(X_2, \mu_2) + F(X_1, X_2) \\
\dot{\mu}_2 &= G(\bullet)
\end{align*}
\]

(13)

Here, $F(X_1, X_2)$ denotes coupling between the primary system ($X_1$) and the secondary system ($X_2$). The function $G$ acts on the meta-heuristic optimization as presented in the previous section. The scheme is adaptive since in the above procedure the parameters which determine the nature of the dynamics self-adjust or adapt themselves to yield the desired dynamics. Using such an adaptive control function the primary system and the secondary system eventually synchronized, although their behavior is chaotic and they have started with different initial conditions and parameter settings. Our aim is to devise an algorithm to adaptively adjust the parameters in the secondary system, $\mu_2$, until the system variables, $Y$, and the parameters themselves converge to
their counterparts in the primary system, i.e., both $X_2 \rightarrow X_1$ and $\mu_2 \rightarrow \mu_1$. In this way, synchronization between both systems is achieved and the parameters of the primary system are identified. Let $X=[x_1, x_2, x_3, x, y]^T$ be the state vector of the chaotic mobile robot, $\dot{X}$ is the derivative of the state vector $X$. Based on the measurable state vector $X$, for individual $i$, we define the following fitness function

$$
\text{Fitness} = \sum_{t=0}^{k} \left( (x_i(t) - x_{0i}(t))^2 + (x_2(t) - x_{2i}(t))^2 + (x_3(t) - x_{3i}(t))^2 \\
+ (x(t) - x_{i}(t))^2 + (y(t) - y_{i}(t))^2 \right)
$$

where $t = 0, \ldots, k$. Therefore, the problem of synchronization is transformed to that of using the meta-heuristic optimization to search for the suitable value of the parameter $\mu_2$ such that the fitness function is globally minimized.

6. SIMULATION RESULTS

In order to evaluate the proposed chaos synchronization strategy, the integrated systems of the Arnold equation with the mobile robot equation are employed in the simulation study. Let us consider a primary system $\dot{X}_1 = f(X_1, \mu_1)$, where $X_1 = [x_1, x_2, x_3, x, y]^T$ is the dynamic system state and $\mu_1 = [A_1, A_2, A_3]^T$ contains the parameters. The system is in the chaotic state when $A_1 = 0.25$, $A_2 = 0.25$, $A_3 = 0.5$. The secondary systems is $\dot{X}_2 = f(X_2, \mu_2)$, where $X_2$ are the state variables and $\mu_2$ are the parameters that can be adjusted in order to estimate $\mu_1$ and attain synchronization. In order to observe nonlinear dynamical searching process of the emotional chaotic cuckoo search algorithm as a whole, we plot search values of all the individuals for parameters $A_1, A_2, A_3$ in Fig. 4-6. Fig. 7 shows the identification of the parameters $A_1, A_2, A_3$ in chaotic mobile robot for the best fitness evolution. Fig. 8 shows the evolution of fitness function. We can see that the trajectories of the identification of the parameters converge at the real values of the parameters, indicating that the model of our proposed emotional chaotic cuckoo search algorithm can be used as an effective optimization model. Preliminary simulation results show that the proposed method can provide greater efficiency and satisfactory accuracy.
Figure 5. Identification of the parameter $A_2$ in chaotic mobile robot

Figure 6. Identification of the parameter $A_i$ in chaotic mobile robot

Figure 7. Identification of the parameters $A_i$ ('------'), $A_2$ ('......'), $A_3$ ('.-.-.-') in chaotic mobile robot for the best fitness evolution
7. CONCLUSIONS

We have proposed a new approach to optimization search algorithm in meta-heuristics for the synchronization of chaotic mobile robots. The proposed method includes both effects of chaotic dynamics and psychology factor of emotion for optimization search. In this paper, we have shown how meta-heuristic optimization can be used to adapt the parameters in two coupled systems such that the two systems are synchronized. The parameters of the secondary system are adaptively optimized by the proposed emotional chaotic cuckoo search algorithm to make it follow the dynamics of the primary system. Simulation results have been presented to show the effectiveness and feasibility of this approach.

REFERENCES


Authors

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