# A Comprehensive Comparison Of the Original Forms of Biogeography-Based Optimization Algorithms 

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#### Abstract

Biogeography-based optimization (BBO) is a new population-based evolutionary algorithm and one of metaheuristic algorithms. This technique is based on an old mathematical study that explains the geographical distribution of biological organisms. The first original form of BBO was introduced in 2008 and known as a partial migration based BBO. After three months, BBO was re-introduced again with additional three other forms and known as single, simplified partial, and simplified single migration based BBOs. Then a lot of modifications and hybridizations were employed to boost-up the performance of BBO and solve its weak exploration. However, the literature lacks the explanations and the reasons on which the modifications of the BBO forms are based on. This paper tries to clarify this issue by making a comparison between the four original BBO algorithms through 23 benchmark functions with different dimensions and complexities. The final judgment is confirmed by evaluating the performance based on the effect of the problem's dimensions, the side constraints and the population size. The results show that both single and simplified single migration based BBOs are faster, but have less performance as compared to the others. The comparison between the partial and the simplified partial migration based BBOs shows that the preference depends on the population size, problem's complexity and dimensions, and the values of the upper and lower side constraints. The partial migration model wins when these factors, except the population size, are increased, and vice versa for the simplified partial migration model. The results can be used as a foundation and a first step of modification for enhancing any proposed modification on BBO including the existing modifications that are described in literature.


## KEYWORDS

Biogeography-Based Optimization, BBO, Simplified BBO, Meta-Heuristic Algorithms, Evolutionary Algorithms, Partial Migration, Single Migration

## 1. INTRODUCTION

The science ofbiology becomes one of the main resources of inspiration to develop the modern optimization techniques, such as ant colony optimization (ACO), bee colony optimization (BCO), wasp swarm optimization (WSO), bacterial foraging optimization (BFO), genetic algorithm (GA), evolutionary strategy (ES), differential evolution (DE), particle swarm optimization (PSO), etc. Biogeography-based optimization (BBO) is a new population-based evolutionary algorithm (EA) that was introduced by Dan. Simon in 2008 [1], and its performance was evaluated based on 14 benchmark functions, and then was tested to solve a real sensor selection problem for aircraft engine health estimation. BBO did well and proved that it is a very competitive method as compared to the other EAs. Since then, a lot of researches have been conducted, some of them to solve practical problems such as economic emission load dispatch [20], land cover feature extraction [21], and unit commitment [22]; while the others were focused to enhance its performance, by modifying its
algorithm or/and hybridizing it by adding sub-algorithms form other optimization techniques [23,24,25,26,27,29,33].

The objective of this paper is to outline a clear path for selecting the best algorithm among the four original forms, and thus, any present modification with wrong selected form can be reviewed again with this guidance to enhance its performance. In addition, it can be used as a foundation for any future modification.

This paper is organized as follows: Section II gives a quick introduction about the theory of island biogeography to be as a strong basis to understand the principles of the original BBOs which are described in Section III; after that, Section IV gives a comparison between the original forms of BBO. Section V is set for the conclusions and suggestions.

## 2. The Theory of IsLand Biogeography

Biogeography is a branch of biology, and it is a synthetic discipline, relying heavily on theory and data from ecology, population biology, system matics, evolutionary biology, and the earth sciences [4]. Biogeography seeks to describe, analyze and explain the geographic patterns and changing distributions of ecosystems and fossil species of plants (flora) and animals (fauna) through geological space and time $[5,6]$.

Island, in biogeography, is any area of suitable habitat (local environment occupied by an organism [7]) surrounded by an expense of unsuitable habitat and is endowed with exceptionally rich reservoirs of endemic, exclusive, strange and relict species [8]. Islands as ecological systems have such salient features as simple biotic, varying combinations of biotic and a biotic factors, and variability in isolation, shape, and size. Thus, island could be aquatic island, desert oasis, individual plants, caves, lakes or ponds, mountain-tops (sky-islands), microcosms or even patches of terrestrial ecosystems. [9,14]. With these characteristics, islands represent themselves as natural experiments, and got highly attentions by the nineteenth century naturalists of the first rank, such as Alfred R. Wallace in East Indies [10], Charles Darwin in Galapagos Islands [11] and Joseph D. Hooker in Southern Ocean [12].

Island biogeography is a special field within biogeography science. This field was initially started by the ecologists Robert H. MacArthur and Edward O. Wilson in 1960 to 1963 with their published paper [2], and continued their studies till 1967 when the final achievement were presented in [3]; and recently, this theory has been revisited and expanded more in [13].

Island biogeography theory fully integrates much of ecology, population biology, evolution, and paleontology, with important implications for conservation of species [13]. It was developed with mathematical models for attempting to translate the ecology and biogeography from the traditional view to analytical view, and answering why some islands are rich of species while the others are poor, by establishing and explaining the biotic (like predation, competition and interactions between species) and a biotic (like wind, water, sunlight, temperature, pressure and soil) factors that affect the species richness of natural communities in an island [15]. Thus, it gives the ability to predict the species counts that migrate between islands and then can find the optimum conservation areas $[4,5,6,8]$.

The equilibrium theory of island biogeography proposes that the number of inhabited species on an island is based on the dynamic equilibrium between new immigrated species onto an island and the extinct species out from that island [2,3,13].

Fig. 1 graphically represents the equilibrium model with exponential immigration (or speciation) rate $\lambda$ and emigration (or extinction) rate $\mu$, where they can also be plotted as a logistic, linear or any proper function $[4,16,17]$, while the equilibrium location will be shifted to the right or left side based on the type of rate functions, the island's area and/or the distance (isolation) between the source and recipient islands $[4,3,13]$.


Figure 1. Equilibrium model of a biota of a single island
$I$ and $E$ are the maximum possible immigration and emigration rates, respectively. $\hat{S}$ is the number of species at equilibrium, $\widehat{T}$ is the species turnover rate at equilibrium, and $S_{\max }$ is the maximum number of species on that island.
$I$ occurs when there is no colonization process, or in other word, the island is empty and offers maximum opportunity to the species on the other islands for immigrating to settle on it; and as the number of arrived species on that island increases, the opportunity for settlement will decrease and thus the immigration rate $\lambda$ will decrease too. Based on that, the species density increases, so the predation, competition and parasitism factors will increase; and as a result, the emigration rate $\mu$ will increase, and reaches its maximum value $E$ at the minimum value of $\lambda$ [18].

MacArthur and Wilson [2,3] simplified the exponential model to be as a linear function, where $I=E$ as shown in Fig. 2 with mathematical expressions in order to theoretically explain how the migration process on a single island happens.


Figure 2. Simplified equilibrium model of a biota of a single island

Now, let at time $t$, the recipient island has $S$ species with probability $P_{s}(t)$, and $\lambda_{s}$ and $\mu_{s}$ are respectively the immigration and emigration rates at the present of $S$ species on that island. Then the variation from $P_{S}(t)$ to $P_{S}(t+\Delta t)$ can be described as:

$$
\begin{equation*}
P_{s}(t+\Delta t)=P_{s}(t)\left(1-\lambda_{s} \Delta t-\mu_{s} \Delta t\right)+P_{s-1}(t) \lambda_{s-1} \Delta t+P_{s+1}(t) \mu_{s+1} \Delta t \tag{1}
\end{equation*}
$$

Also, $\hat{S}$ can be found by using different methods. From the basic of trigonometry:

$$
\begin{gather*}
\frac{\hat{T}}{\hat{S}}=\frac{E}{S_{\max }} \Rightarrow \hat{T}=\frac{E}{S_{\max }} \hat{S}  \tag{2}\\
\frac{\hat{T}}{\left(S_{\max }-\hat{S}\right)}=\frac{I}{S_{\max }} \tag{3}
\end{gather*}
$$

Substituting Eq. 2 in Eq. 3 for $\hat{T}$ :

$$
\begin{equation*}
\hat{S}=\left(\frac{I}{I+E}\right) S_{\max } \tag{4}
\end{equation*}
$$

Eq. 4 can also be obtained by equalizing $\lambda_{s}$ and $\mu_{s}$ rates at $\hat{S}$ as follows:

$$
\begin{gather*}
\mu_{s}=\frac{E}{S_{\max }} S  \tag{5}\\
\lambda_{s}=1-\mu_{s}=I\left(1-\frac{S}{S_{\max }}\right)  \tag{6}\\
\text { at } S=\hat{S}: \quad \lambda_{s}=\mu_{s} \Rightarrow I\left(1-\frac{\hat{S}}{S_{\max }}\right)=\frac{E}{S_{\max }} \hat{S} \tag{7}
\end{gather*}
$$

Solving Eq. 7 for $\hat{S}$ gives Eq. 4; where at the intersection point, the island's biota will be at a state of dynamic equilibrium, and thus $P_{s}(t+\Delta t)=P_{\max }[1,3,17]$.

From Eq. 1, to have $S$ at time $(t+\Delta t)$, one of the following three conditions should hold:

1. $S$ species at time $t$, and no immigration or emigration took place during the interval $\Delta t$;
2. ( $S-1$ ) species at time $t$, and one species immigrated;
3. $(S+1)$ species at time $t$, and one species emigrated.

To neglect the probability of more than one immigration or emigration, then $\Delta t$ has to be set with small value. As $\Delta t$ approaches 0 , the ratio $\left(\frac{\Delta P_{s}}{\Delta t}\right)$ approaches $\dot{P}_{s}(t)$ :

$$
\begin{equation*}
\frac{d P_{s}(t)}{d t} \cong \lim _{\Delta t \rightarrow 0} \frac{P_{s}(t+\Delta t)-P_{s}(t)}{\Delta t} \cong-\left(\lambda_{s}+\mu_{s}\right) P_{s}(t)+\lambda_{s-1} P_{s-1}(t)+\mu_{s+1} P_{s+1}(t) \tag{8}
\end{equation*}
$$

By considering the previous three conditions, Eq. 8 becomes:

$$
\dot{P}_{s}(t)=\left\{\begin{array}{cc}
-\left(\lambda_{s}+\mu_{s}\right) P_{s}+\mu_{s+1} P_{s+1}, & S=0  \tag{9}\\
-\left(\lambda_{s}+\mu_{s}\right) P_{s}+\lambda_{s-1} P_{s-1}+\mu_{s+1} P_{s+1}, & 1 \leq S \leq S_{\max }-1 \\
-\left(\lambda_{s}+\mu_{s}\right) P_{s}+\lambda_{s-1} P_{s-1}, & S=S_{\max }
\end{array}\right.
$$

The value of $\dot{P}_{S}(t)$ can also be determined by using a matrix technique [1].

Thus, using the known values of $P_{S}(t)$ and $\dot{P}_{S}(t)$, the value of $P_{S}(t+\Delta t)$ given in Eq. 1 can be approximated as:

$$
\begin{equation*}
P_{s}(t+\Delta t) \cong P_{s}(t)+\dot{P}_{s}(t) \Delta t \tag{10}
\end{equation*}
$$

Eq. 10 is the final form that has to be used in the program of BBO for calculating $P_{s}(t+\Delta t)$.
For finding $P_{S}(t)$, Dan Simon in [1] used two methods; either by solving Eq. 9 numerically, or by applying the following theorem:

Theorem 1: The steady-state value for the probability of the number of each species is given by:

$$
\begin{equation*}
P(\infty)=\frac{v}{\sum_{i=1}^{s_{\max }+1} v_{i}} \tag{11}
\end{equation*}
$$

where $v$ and $v_{i}$ are computed as:

$$
\begin{gather*}
v=\left[v_{1}, v_{2}, \ldots, v_{S_{\max }+1}\right]^{T}  \tag{12}\\
v_{i}=\frac{S_{\max }!}{\left(S_{\max }+1-i\right)!(i-1)!} \quad\left(i=1, \ldots, S_{\max }+1\right) \tag{13}
\end{gather*}
$$

## 3. Biogeography-Based Optimization (BBO)

The involvement of the science of biogeography into BBO is that the general problem solution means the natural distribution of species [1]. Each island represents one solution, where the good solution in biogeography means that the island has many species, and the density of these species depends on the availability of good features offered by that island (the good things of biotic "living: trees, shrubs, meadow, diversity of prey, etc" and a biotic "non-living: wind, temperature, humidity, water, area, etc" factors [19] - as described in section II), and vice versa for the poor solution islands [18]. Each feature is called suitability index variable (SIV), and represents the independent variable of such a problem in BBO [30].

Island suitability index (ISI) depends on the availability of those features on that island; and, in $\mathrm{BBO}, I S I$ is the dependent variable [30]. Thus, for problem with $n$-independent variables and $k$ islands or individuals, then it can be expressed as:

$$
\begin{equation*}
I S I_{i}=f\left(S I V_{1}, S I V_{2}, \ldots, S I V_{n}\right) \quad i=1,2, \ldots, k \tag{14}
\end{equation*}
$$

The algorithm of BBO consists of two main sub-algorithms, migration and mutation.
The original forms of BBO depend on the type of the migration process, which are partial migration based (PMB-BBO), single migration based (SMB-BBO), simplified partial migration based (SPMB-BBO), and simplified single migration based (SSMB-BBO) $[1,28]$.

### 3.1. Migration

Considering Eq. 14, the high $I S I$ for island $i$ represents a good solution, and also high $I S I$ means large number of available species on that island, which forces immigration rate $\lambda_{s}$ to be low and
emigration rate $\mu_{s}$ to be high; while low $I S I$ for island $i$ represents a poor solution, which means a shortage indication in the availability of species on that island, where at this condition $\lambda_{s}$ is high and $\mu_{s}$ is low.

Referring to Fig. 2, $S_{I}$ is located before $\hat{S}$, where $\lambda_{s}$ is high, $\mu_{s}$ is low and the solution $I S I_{I}$ is poor; while $S_{2}$ is located after $\hat{S}$, where $\lambda_{s}$ is low, $\mu_{s}$ is high and the solution $I S I_{2}$ is good. Thus, $\lambda_{s}$ and $\mu_{s}$ are indications of poor and good solutions, respectively.

In migration process, the high ISI islands share their features to modify the low ISI islands, where the islands of both sides are probabilistically selected. The high $I S I$ islands become the source of modification, while the low $I S I$ islands become the recipients to those emigrated species.

Although the species will emigrate from the rich islands to the poor islands, this phenomena does not mean that the species will completely disappear from its home islands. However, only a few representatives emigrate [1]. Thus, the recipient islands are enhanced, and at the same time the source islands are kept away from any shortage on its richness of species.

The migration process of the four original forms of BBO can be described as:-

### 3.1.1. PMB-BBO Model:

```
Let \(I S I_{i}\) denote the \(i\) th population member and contains \(n\) features
For each island \(I S I_{i}\) (where \(i=1,2,3, \ldots, k\) )
    For each SIV \(s\) (where \(s=1,2,3, \ldots, n\) )
        Use \(\lambda_{i}\) to probabilistically select the immigrating island \(I S I_{i}\)
            If rand \(<\lambda_{i}\)
                For \(j=1\) to \(k\)
                    Use \(\mu_{j}\) to probabilistically decide whether to emigrate to \(I S I_{i}\)
                        If \(I S I_{j}\) is selected
                        Randomly select an \(S I V \sigma\) from \(I S I_{j}\)
                                Replace a random \(S I V S\) in \(I S I_{i}\) with \(S I V \sigma\)
                        end if
                end for
            end if
        next SIV
next island
```


### 3.1.2. SMB-BBO Model:

Let $I S I_{i}$ denote the $i$ th population member and contains $n$ features
For each island $I S I_{i}$ (where $i=1,2,3, \ldots, k$ )
Use $\lambda_{i}$ to probabilistically select the immigrating island $I S I_{i}$ If rand $<\lambda_{i}$

Pick a random SIV $s$ (where $s=1,2,3, \ldots, n$ )
For $j=1$ to $k$
Use $\mu_{j}$ to probabilistically decide whether to emigrate to $I S I_{i}$
If $I I_{j}$ is selected
Randomly select an $S I V \sigma$ from $I S I_{j}$
Replace a random $S I V s$ in $I S I_{i}$ with $S I V \sigma$
end if
end for
end if
next island

The simplified models (SPMB and SSMB) are similar to the previous normal models (PMB and SMB), except that the simplified models will always use the best obtained solution as the emigrating island instead of doing an internal loop checking. It is apparent that the simplified models have two conflicting issues. They are faster (less CPU time) because the internal looping is eliminated. However, they could trap in a local minima because they always depend on the best solution, and consequently the probability of finding other better solutions reduces.

### 3.1.3. SPMB-BBO Model:

```
Let }IS\mp@subsup{I}{i}{}\mathrm{ denote the }i\mathrm{ th population member and contains }n\mathrm{ features
For each island ISI (where i=1,2,3,\ldots,k)
    For each SIV s (where s=1,2,3,\ldots,n)
        Use}\mp@subsup{\lambda}{i}{}\mathrm{ to probabilistically select the immigrating island }IS\mp@subsup{I}{i}{
        If rand < 勆
            Select the best obtained solution as the emigrating island ISI best
            If }IS\mp@subsup{I}{\mathrm{ best }}{}\mathrm{ is selected
                Randomly select an SIV \sigma from }IS\mp@subsup{I}{\mathrm{ best }}{
                    Replace a random SIV s in ISI i}\mathrm{ with SIV }
            end if
        end if
    next SIV
next island
```


### 3.1.4. SSMB-BBO Model:

```
Let }IS\mp@subsup{I}{i}{}\mathrm{ denote the }i\mathrm{ th population member and contains }n\mathrm{ features
For each island ISI (where i=1,2,3,\ldots,k)
    Use }\mp@subsup{\lambda}{i}{}\mathrm{ to probabilistically select the immigrating island }IS\mp@subsup{I}{i}{
    If rand < 勆
        Pick a random SIV s (where s=1,2,3,\ldots,n)
        Select the best obtained solution as the emigrating island ISI best
        If }IS\mp@subsup{I}{\mathrm{ best }}{}\mathrm{ is selected
            Randomly select an SIV \sigma from ISI Iest
            Replace a random SIV s in }IS\mp@subsup{I}{i}{}\mathrm{ with SIV }
        end if
    end if
next island
```


### 3.2. Mutation

The features available on an island (i.e., $n$-SIV) can be changed dramatically due to random events called mutations [31], which forces $\hat{S}$ to deviate from its equilibrium value [1].

In nature, the mutation process could happen on one feature or multiple of features. Most observed mutations are harmful, like predators from other islands, tsunamis, volcanos, diseases or earthquakes, which are not directed to be useful [17]. On the other hand, there are some useful events that can enhance those $n$-SIV to give better solutions, such as wind-carrying seeds (wind pollination) or flotsams (shipwreck) [18].

In BBO, this mutation process is modeled as $S I V$ mutation, where the mutation rate $m$ can be determined by involving species count probabilities $P_{s}$ into the following equation:

$$
\begin{equation*}
m=m_{\max }\left(1-\frac{P_{s}}{P_{\max }}\right) \tag{15}
\end{equation*}
$$

where $P_{\max }=\max \left(P_{s}\right)$ and $m_{\max }$ is a user-defined maximum mutation rate that $m$ can reach. From Eq. 15, $m$ reaches to its minimum "zero" at the maximum value of $P_{s}$, and vice versa. Thus, $m$ is inversely proportional to $P_{s}$.

The objective of using mutation rate is to set the low and high $I S I$ solutions likely to mutate, which gives them an ability to enhance their results more than what they already have, where the solutions at the equilibrium point are not mutated [1].

The mutation process can be described as:

For $i=1$ to $k$ (where k is the number of islands, see Eq. 14)
Calculate probability $P_{s}$ based on $\lambda_{s}$ and $\mu_{s}$ (by numerical or direct method)
Calculate mutation rate $m$ (using Eq. 15)
Select $I S I_{i}$ with probability proportional to $P_{s}$
If $I S I_{i}$ is selected
Replace $S I V$ of $I S I_{i}$ with a randomly generated $S I V$
end if
end for

### 3.3. BBO Algorithm

The steps of the general BBO algorithm can be listed as:

1. Initialize the BBO parameters ( $S_{\text {max }}, I, E$, $m_{\max }$, etc).
2. Find species count probabilities $P_{s}$ and mutation rate $m$ based on the calculated immigration rate $\lambda_{s}$ and emigration rate $\mu_{s}$ by Eqs. 5 and 6.
3. Generate $k$ random islands, where each island represents one solution to a given problem with $n-S I V$.
4. Sort the solutions $k$-ISI for all islands, so the first best solution should be mapped with the highest number of species and the highest emigration rate $\mu_{s}$ (or the lowest immigration rate $\lambda_{s}$ ), and continue the descending order till reaching to the worst solution.
5. Do elitism process for saving the required best solutions for the next generation; it is an optional step [30].
6. Probabilistically select the source islands based on $\mu_{s}$, and the islands which need to be modified "the recipient islands" based on $\lambda_{s}$, and do the migration process. Then, update all $k-I S I$ before ending this step.
7. Do mutation process for the islands based on their probabilities that are listed in the probability vector after calculated in step (2). Then, update all $k$-ISI once the mutation process is completed.
8. Return to step (4) for the next iteration. This loop can be terminated either if reaching to an acceptable tolerance or after completing the desired number of generations.

## 4. Performance Comparison

The main problem associated with all the modified BBOs is that the modifications were done on an arbitrary selected form of the four original forms. There is no clarification on which form the proposed modification stands on and why.

The four original forms of BBO have been tested through 23 benchmark functions with different dimensions and complexities.

These functions can be classified into three groups: unimodal, multimodal with few local minima and multimodal with many local minima. Functions f01-f13 are high-dimensional problems. Functions f01-05 and f07 are high-dimensional and unimodal, f06 is a high-dimensional step function with one discontinuous minimum. Functions f08-13 are high-dimensional and multimodal with many local minima, and the remaining functions are low-dimensional and multimodal with few local minima [33]. The details of these benchmark functions can be found in the Appendix.

The parameters that have been used here are similar to those used in [25,29]: population size of 50, $I=E=1, m_{\max }=0.01$, generation limit of 20,000 for f01-13 and 1000 for f14-23, elitism parameter of 1 , and Monte-Carlo simulation with 30 trails.

Table 1 summarizes the performance of PMB, SMB, SPMB and SSMB models for 23 benchmark functions. The highlighted cells in the tables represents the best result among the four BBO algorithms. It can be clearly seen that the performance of PMB and SPMB are superior as compared to SMB and SSMB. For high-dimensional problems, PMB wins with 7 best solutions, 10 mean and 8 standard deviation out of 13 ; while SPMB wins with 6 best solutions, 3 mean and 5 standard deviation. On the other hand, for low-dimensional problems, SSMB enters this competition, and gives better Best, Mean and Standard deviation than that of the PMB for the functions $\mathrm{f} 16, \mathrm{f} 17, \mathrm{f} 18$, but it does not win as compared to SPMB. Single and simplified single migration based models are not valid for f21-23, because these problems are 1-dimensional problems, and the migration process is done within only one independent variable.

Table 1. Comparison of the results for 30 trails of the original four BBO models, where Best, Mean, and StdDev stands for the smallest error, the mean of all errors, and the standard deviation, respectively.

|  |  |  |  |  |  |  |  |  |  | ) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| f\# | n | Part | M Migratio | Based | Singl | Migration | Based | Simplified | Partial Mig | ration Based | Simplifi | Single Mig | ion Based |
|  |  | Best | Mean | StdDev | Best | Mean | StdDev | Best | Mean | StdDev | Best | Mean | StdDev |
| f01 | 30 | 1.8518E+00 | 3.8843E+00 | 1.3964E+0 | $1.6999 \mathrm{E}+02$ | $3.1652 \mathrm{E}+02$ | 9.9742E+01 | $2.2588 \mathrm{E}+00$ | 4.3027E+00 | 1.3073E+00 | 8.1588E+01 | 5592E+02 | 1.5209E+02 |
| f02 | 30 | 4.002 | 7.018 | $1.3458 \mathrm{E}-01$ | E+00 | $6.1515 \mathrm{E}+00$ | 1.3 | 5.167 | 7.6268E-01 | 1.0545E-01 | 5.4535E+00 | +00 | 1782 +00 |
| $f 03$ | 30 | E+02 | $1.9527 \mathrm{E}+04$ | 1.1268E+04 | 79E+03 | 3.0638E+04 | 1.3227E+04 | 2048E+03 | 3.2307E+04 | $8.3231 \mathrm{E}+03$ | 4.645E+02 | 7268E+04 | E+04 |
| $f 04$ | 30 | 3.2007E+00 | 6.2387E+00 | $1.1469 \mathrm{E}+00$ | $2.3978 \mathrm{E}+01$ | 3.4745E+01 | 6.0946E+00 | 2.8725E+00 | 5.6856E+00 | $1.2431 \mathrm{E}+00$ | $2.8262 \mathrm{E}+01$ | 3.8791E+01 | $4.1634 \mathrm{E}+00$ |
| f05 | 30 | $1.0517 \mathrm{E}+02$ | $2.6058 \mathrm{E}+02$ | 7.9555E+01 | 5.0807E+03 | $2.3564 \mathrm{E}+04$ | 1.6310E+04 | $1.3597 \mathrm{E}+02$ | $2.7246 \mathrm{E}+02$ | 1.1116E+02 | $7.4418 \mathrm{E}+03$ | 3.9130E+04 | $3.2640 \mathrm{E}+04$ |
| $f 06$ | 30 | 2.0000E+0 | $4.7333 \mathrm{E}+00$ | $1.8245 \mathrm{E}+00$ | 400E+02 | $3.9163 \mathrm{E}+02$ | $2.0465 \mathrm{E}+02$ | 1.0000E+00 | $4.7667 \mathrm{E}+00$ | $2.4315 \mathrm{E}+0$ | 1.3900E+02 | 5263E+02 | $2.8296 \mathrm{E}+02$ |
| f07 | 30 | 2.8562E-07 | 1.2406E-06 | 1.2872E-06 | 7.6867E-04 | 1.2135E-02 | $1.4628 \mathrm{E}-02$ | 1.9837E-07 | 1.3833E-06 | 1.1343E-06 | 1.0547E-03 | 1.3451E-02 | 1.4276E-02 |
| $f 08$ | 30 | 4.4291E-06 | 1.3876E-05 | 5466E-06 | 5004E-04 | 1.6463E-03 | 1.2669E-03 | 5.0828E-06 | 1.3322E-05 | 7.4610E-06 | 5.9656E-04 | 1.8382E-03 | 1.0377E-03 |
| f09 | 30 | 9.4594E-01 | $1.9351 \mathrm{E}+00$ | 6.4580E-01 | 2.3807E+01 | 3.3102E+01 | 5.6152E+00 | 1.0236E+00 | $1.9211 \mathrm{E}+00$ | 5.7860E-01 | 2.6082E+01 | 3.4796E+01 | 6.4496E+00 |
| $f 10$ | 30 | 6.3026E-01 | 9.9236E-01 | 2.3065E-01 | $4.3719 \mathrm{E}+00$ | 5.7651E+00 | 6.6010E-01 | 4.9817E-01 | $1.0345 \mathrm{E}+00$ | $2.5990 \mathrm{E}-01$ | 4.6872E+00 | 5.9257E+00 | 6.9267E-01 |
| $f 11$ | 30 | 08E-01 | $1.0263 \mathrm{E}+00$ | 402 | 095E+00 | $4.4318 \mathrm{E}+00$ | $1.3255 \mathrm{E}+00$ | 9.4245E-01 | 1.0357E+00 | 2.7730E-02 | $2.1708 \mathrm{E}+00$ | . $0205 \mathrm{E}+00$ | 1.9755E+00 |
| $f 12$ | 30 | 5.0934E-03 | 2.9591E-02 | 3.2559E-02 | 493E+00 | $3.5616 \mathrm{E}+00$ | 3.8957E+00 | 3.8794E-03 | 3.1228E-02 | 3.5340E-02 | $1.4325 \mathrm{E}+00$ | $6.3746 \mathrm{E}+01$ | $3.2648 \mathrm{E}+02$ |
| $f 13$ | 30 | 001E-02 | 1.6875E-01 | 6.0757E-02 | 79E+00 | 3.1883E+03 | 1.3051E+0. | 8.1548E-02 | 2.0015E-01 | 8.0590E-02 | 6.9544E+00 | $2.3081 \mathrm{E}+03$ | 8.1029E+03 |
| $f 14$ | 2 | 2.1720E-11 | 8.0558E-08 | 8665-07 | 942E-10 | 595E-04 | 5.4076E-04 | 6.1392E-12 | 3.8090E-10 | 3.2748E-1 | 8.4939E-11 | 1.0718E-07 | OE-07 |
| $f 15$ | 4 | 3.9927E-04 | 8.0573E-04 | 3.2529E-04 | 5.6999E-04 | 1.3729E-03 | 4.5011E-04 | 1.8923E-04 | 7.0160E-04 | 3.1929E-04 | 4.5865E-04 | 1.3658E-03 | 5.8181E-04 |
| f16 | 2 | 2.3455E-07 | 9.1217E-05 | 1.2481E-04 | 1.6640E-05 | 4.1018E-04 | 4.3907E-04 | 3.5869E-10 | 4.8485E-06 | 4.7243E-06 | 5.7770E-08 | 2.7089E-05 | 3.9141E-05 |
| $f 17$ | 2 | 80E-0 | 9.6466E-05 | 1.6743E-0 | E-06 | 3.4432E-04 | 3.9477E-04 | 2486E-07 | 1.9438E-05 | 2.708 | 4.129E-7 | 7.3364E-05 | 9.1485E-05 |
| f18 | 2 | E-05 | E-03 | 1.6278E-03 | E-05 | 5.8628E-03 | 6.6169E-03 | 1.2784E-07 | 8.1736E-05 | 8.7004E-0 | 9.2851E-06 | 2.9315E-04 | 3.7882E-04 |
| $f 19$ | 3 | 7177E-06 | 5.0374E-04 | 11E- | 07E-04 | 1.6343E-0 | 888E-03 | 5.9005E-07 | 5.0688E-05 | 5.9545E-05 | 327E | 2029E-0 | 4.9816E-04 |
| f20 | 6 | 1.8781E-03 | 6.9158E-02 | 6.0706E-02 | $3.6659 \mathrm{E}-02$ $1.6534 \mathrm{E}-01$ $7.1746 \mathrm{E}-02$ |  |  | 3.3727E-03 | 7.9192E-02 | 5.6884E-02 | 2.1696 | 1.5974E-01 | 6.5199E-02 |
| f21 | 1 | 9.8030E-08 | 2.8723E-05 | 4.2939E-05 | For 1-dimensional problems, SMB-BBO is not applicable; (SMB=PMB) |  |  | 3.8113E-09 | 3.3121E-05 | 6.0656E-05 | For 1-dimensional problems, SSMB-BBO is not applicable; (SSMB=SPMB) |  |  |
| f22 | 1 | 8.1958E-10 | 2.1546E-05 | 5.2185-05 |  |  |  | 2.2393E-08 | 3.4060E-05 | 4.6055E-05 |  |  |  |
| $f 23$ | 1 | 1.7454E-08 | 7.4624E-05 | 1.8726E-04 |  |  |  | 1.0113E-07 | 3.7714E-05 | 5.3672E-05 |  |  |  |

Although, in overall, SSMB has respectively the first and second worst performance for high and low-dimensional problems, it achieved the fastest algorithm as shown in Table 2. This is logical, because of two reasons. First, it does a migration on one randomly selected SIV for each island rather than all $n-S I V$ as in PMB and SPMB. Second, it will always select the best found solution as a source island for migration instead of doing a loop checking as in SMB. This is why the simplified versions of PMB and SMB trip in local minima particularly as the complexity, side constraints and/or dimensions increases and as the number of islands or population size decreases. In this situation, PMB has the best exploration and exploitation, especially when the mutation stage that compensate the weakness of the migration algorithm is absent, in which the performance of SPMB is significantly decreased, as shown in Table 3.

Table 2. Normalized CPU time for the high-dimensional problems f01-f13

| f \# | BBO Models |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | PMB-BBO | SMB-BBO | SPMB-BBO | SSMB-BBO |
| f01 | $1.4804 \mathrm{E}+00$ | $1.0084 \mathrm{E}+00$ | $1.3181 \mathrm{E}+00$ | $1.0000 \mathrm{E}+00$ |
| f02 | $1.4687 \mathrm{E}+00$ | $1.0086 \mathrm{E}+00$ | $1.3103 \mathrm{E}+00$ | $1.0000 \mathrm{E}+00$ |
| f03 | $1.1672 \mathrm{E}+00$ | $1.0082 \mathrm{E}+00$ | $1.1104 \mathrm{E}+00$ | $1.0000 \mathrm{E}+00$ |
| f04 | $1.5018 \mathrm{E}+00$ | $1.0086 \mathrm{E}+00$ | $1.3355 \mathrm{E}+00$ | $1.0000 \mathrm{E}+00$ |
| f05 | $1.4717 \mathrm{E}+00$ | $1.0074 \mathrm{E}+00$ | $1.3133 \mathrm{E}+00$ | $1.0000 \mathrm{E}+00$ |
| f06 | $1.4721 \mathrm{E}+00$ | $1.0079 \mathrm{E}+00$ | $1.3113 \mathrm{E}+00$ | $1.0000 \mathrm{E}+00$ |
| f07 | $1.4089 \mathrm{E}+00$ | $1.0070 \mathrm{E}+00$ | $1.2769 \mathrm{E}+00$ | $1.0000 \mathrm{E}+00$ |
| f08 | $1.4521 \mathrm{E}+00$ | $1.0074 \mathrm{E}+00$ | $1.2969 \mathrm{E}+00$ | $1.0000 \mathrm{E}+00$ |
| f09 | $1.4621 \mathrm{E}+00$ | $1.0068 \mathrm{E}+00$ | $1.3041 \mathrm{E}+00$ | $1.0000 \mathrm{E}+00$ |
| f10 | $1.4299 \mathrm{E}+00$ | $1.0052 \mathrm{E}+00$ | $1.2815 \mathrm{E}+00$ | $1.0000 \mathrm{E}+00$ |
| f11 | $1.4256 \mathrm{E}+00$ | $1.0070 \mathrm{E}+00$ | $1.2876 \mathrm{E}+00$ | $1.0000 \mathrm{E}+00$ |
| f12 | $1.3703 \mathrm{E}+00$ | $1.0056 \mathrm{E}+00$ | $1.2437 \mathrm{E}+00$ | $1.0000 \mathrm{E}+00$ |
| f13 | $1.3878 \mathrm{E}+00$ | $1.0021 \mathrm{E}+00$ | $1.2505 \mathrm{E}+00$ | $1.0000 \mathrm{E}+00$ |
| Avg CPU Time | $1.4230 \mathrm{E}+00$ | $1.0070 \mathrm{E}+00$ | $1.2800 \mathrm{E}+00$ | $1.0000 \mathrm{E}+00$ |

Table 3. The overall performance of PMB and SPMB with/without mutation stage

| f \# | Function Name | n | Mutation Stage | Biogeography Based Optimization (BBO) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Partial Migration Based |  |  | Simplified Partial Migration Based |  |  |
|  |  |  |  | Best | Mean | StdDev | Best | Mean | StdDev |
| f05 | Generalized Rosenbrock | 30 | Activated | 1.0517E+02 | $2.6058 \mathrm{E}+02$ | 7.9555E+01 | 1.3597E+02 | $2.7246 \mathrm{E}+02$ | 1.1116E+02 |
|  |  |  | Not-Activated | 1.7036E+02 | 6.0492E+02 | 6.4746E+02 | $2.2803 \mathrm{E}+02$ | $7.5845 \mathrm{E}+02$ | 8.8977E+02 |
| f08 | Generalized <br> Schwefel F2.26 | 30 | Activated | 4.4291E-06 | $1.3876 \mathrm{E}-05$ | 5.7466E-06 | 5.0828E-06 | 1.3322E-05 | 7.4610E-06 |
|  |  |  | Not-Activated | 8.1593E-06 | 2.9257E-05 | 1.1255E-05 | 1.2890E-05 | 3.7150E-05 | 1.3349E-05 |
| f14 | Shekel's Foxholes | 2 | Activated | $2.1720 \mathrm{E}-11$ | 8.0558E-08 | $2.1865 \mathrm{E}-07$ | 6.1392E-12 | 3.8090E-10 | 3.2748E-10 |
|  |  |  | Not-Activated | 5.4152E-08 | 2.2467E-03 | 8.6667E-03 | 1.6396E-07 | 7.6697E-03 | $1.3175 \mathrm{E}-02$ |
| f18 | Goldstein-Price | 2 | Activated | $2.7778 \mathrm{E}-05$ | 1.5749E-03 | 1.6278E-03 | 1.2784E-07 | 8.1736E-05 | 8.7004E-05 |
|  |  |  | Not-Activated | 1.4627E-04 | 2.5692E-02 | 2.2876E-02 | 2.0330E-03 | 4.0344E-02 | 5.0549E-02 |

To verify this conclusion, three performance tests have been conducted as shown in Tables 4,5 and 6. Each one of these three tests is focused on one criteria.

Test I is shown in Table 4, and it is used to study the performance of PMB and SPMB algorithms as the problem's dimension decreases. The parameters used for this test are similar to that used in Table 1, except that the generation limit are set as: 1000 for $n=2,4,6 ; 5000$ for $n=10 ; 10,000$ for $n=20$ and 20,000 for $n=30$.

Whereas, Test II shown in Table 5 is used to study the performance of PMB and SPMB algorithms as the number of islands or population size increases for two of low-dimensional problems.

Finally, Test III shown in Table 6 is used to study the performance of PMB and SPMB algorithms under different upper and lower limits of the variable bounds (also known as domain, search space, side constraints, etc).
As can be seen from Table 4, the SPMB performs better as the problem's dimension decreases. But when the population size is small, the PMB will perform better even for the low-dimensional problems, as shown in Table 5. From Table 6, if the search space is large, the PMB wins too.

Fig. 3 shows the curves of fitness functions of PMB, SMB, SPMB, and SSMB for the Schwefel's problem 1.2 "f03", Generalized Rosenbrock's function "f05", Generalized Rastrigin's function "f09" and Generalized Griewank's function " f 11 ". For functions $\mathrm{f} 05, \mathrm{f} 09$ and f 11 , it can be clearly seen that the PMB and SPMB algorithms outperform the SMB and SSMB algorithms, and the gap between the convergences is large. For the function f03, SSMB enters this competition and shows better fitness as compared to SPMB. However, the PMB algorithm is again has the best convergence.

Table 4. Performance Test I-f05 with different dimensions

| f \# | Name | Dimensions | Biogeography Based Optimization (BBO) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Partial Migration Based |  |  | Simplified Partial Migration Based |  |  |
|  |  |  | Best | Mean | StdDev | Best | Mean | StdDev |
| f05a | Generalized <br> Rosenbrock's Function | 2 | 5.0569E-06 | 3.0404E-04 | 2.9421E-04 | 1.1454E-06 | 4.3368E-04 | 7.0482E-04 |
| f05b |  | 4 | $2.6016 \mathrm{E}+00$ | $1.3546 \mathrm{E}+02$ | $1.0058 \mathrm{E}+02$ | 3.8176E-01 | 3.0585E+01 | 3.2854E+01 |
| f05c |  | 6 | $4.5589 \mathrm{E}+01$ | 8.1933E+02 | $7.3780 \mathrm{E}+02$ | $4.0158 \mathrm{E}+01$ | $8.3004 \mathrm{E}+02$ | $7.5869 \mathrm{E}+02$ |
| f05d |  | 10 | 2.8495E+01 | $2.1690 \mathrm{E}+02$ | $1.8675 \mathrm{E}+02$ | $7.1550 \mathrm{E}+01$ | $2.4654 \mathrm{E}+02$ | 1.7535E+02 |
| f05e |  | 20 | 8.7400E+01 | 3.3766E+02 | 2.4278E+02 | $1.3913 \mathrm{E}+02$ | $3.5894 \mathrm{E}+02$ | $4.0126 \mathrm{E}+02$ |
| f05f |  | 30 | $1.0517 \mathrm{E}+02$ | $2.6058 \mathrm{E}+02$ | 7.9555E+01 | $1.3597 \mathrm{E}+02$ | $2.7246 \mathrm{E}+02$ | $1.1116 \mathrm{E}+02$ |

Table 5. Performance Test II - f15 and f16 with different populations

| f | Name | Population Size | Biogeography Based Optimization (BBO) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Partial Migration Based |  |  | Simplified Partial Migration Based |  |  |
|  |  |  | Best | Mean | StdDev | Best | Mean | StdDev |
| f15a | Kowalik's <br> Function | 5 | 3.0566E-04 | 1.1017E-03 | 5.6436E-04 | 4.2186E-04 | $1.4561 \mathrm{E}-03$ | 7.5693E-04 |
| f15b |  | 10 | 5.1379E-04 | 1.2970E-03 | $7.7744 \mathrm{E}-04$ | 2.1192E-04 | 1.0605E-03 | 7.2553E-04 |
| f15c |  | 50 | 3.9927E-04 | 8.0573E-04 | 3.2529E-04 | 1.8923E-04 | $7.0160 \mathrm{E}-04$ | 3.1929E-04 |
| f15d |  | 200 | $1.9935 \mathrm{E}-04$ | 6.2320E-04 | $2.2726 \mathrm{E}-04$ | 1.4958E-04 | $4.4200 \mathrm{E}-04$ | 1.7749E-04 |
| f16a | Six-Hump CamelBack Function | 5 | 1.0557E-05 | 6.5798E-04 | 7.0515E-04 | 1.1389E-05 | $8.7138 \mathrm{E}-04$ | 8.0479E-04 |
| f16b |  | 10 | 1.1365E-07 | $2.9963 \mathrm{E}-04$ | 4.1067E-04 | 2.4790E-06 | $1.8534 \mathrm{E}-04$ | 2.8880E-04 |
| f16c |  | 50 | 2.3455E-07 | 9.1217E-05 | $1.2481 \mathrm{E}-04$ | 5.7770E-08 | 2.7089E-05 | 3.9141E-05 |
| f16d |  | 200 | $1.7238 \mathrm{E}-07$ | $2.2757 \mathrm{E}-05$ | $2.7229 \mathrm{E}-05$ | 5.7634E-09 | 5.9924E-07 | 7.5759E-07 |

Table 6. Performance Test III - f11 with different side constraints

| f\# | Name | Side <br> Constraints | Biogeography Based Optimization (BBO) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Partial Migration Based |  |  | Simplified Partial Migration Based |  |  |
|  |  |  | Best | Mean | StdDev | Best | Mean | StdDev |
| f11a | Generalized Griewank's Function | Xi $\in[-60,60]$ | 8.6106E-02 | 2.3591E-01 | 8.7067E-02 | 5.3899E-02 | 2.1482E-01 | 7.9380E-02 |
| f11b |  | xi $\in[-600,600]$ | 8.7314E-01 | $1.2508 \mathrm{E}+00$ | 2.3871E-01 | 9.6562E-01 | $1.3044 \mathrm{E}+00$ | 2.2903E-01 |
| $f 11 \mathrm{c}$ |  | Xi $\in[-6000,6000]$ | $4.9293 \mathrm{E}+00$ | 3.2079E+01 | $2.4023 \mathrm{E}+01$ | 6.0255E+00 | 3.7802E+01 | $2.7235 \mathrm{E}+01$ |



Figure 3. Curves of fitness functions of $\mathrm{PMB}, \mathrm{SMB}, \mathrm{SPMB}$, and SSMB for some selected functions. (a) f03, (b) f05, (c) f09, (d) f11

## 5. Conclusions and Suggestions

The performance of the four original forms of BBO algorithms (PMB, SMB, SPMB and SSMB) have been extensively tested and compared using 23 benchmark functions of different dimensions and complexities, as well as different scenarios have been done for some selected functions. The results show that the PMB, which is the first presented model in 2008, gives the best performance as the complexity, side constraints and/or dimensions of a given problem increases and as the number of islands or population size decreases. However, the PMB is found to be the slowest algorithm which requires around $42.30 \%$ more CPU time than that of the SSMB as shown in Table 2. To compromise between the cost and convergence speed under the above scenarios, SPMB is found to be the best choice as it requires around $28.00 \%$ more CPU time, but its exploration and exploitation will decrease significantly. The performance of the SMB algorithm found to be the worst as compared to the others.

The results obtained in this paper can be used as a foundation and a first step for enhancing any prospective modification on the BBO algorithm including the existing modifications that are described in literature.

## APPENDIX

This appendix includes a complete list that have been used in this paper. The details of these benchmark functions can be found in [32]. The global minimums of the functions (f08, f14, f15, $\mathrm{f} 16, \mathrm{f} 17, \mathrm{f} 19$, f 20 , and $\mathrm{f} 21-\mathrm{f} 23$ ) are rounded in [32]. In this paper, the correct values are taken instead from [34,35,36,37,38], respectively.

## f01: Sphere Model

$$
f(x)=\sum_{i=1}^{n} x_{i}^{2}
$$

- $-100 \leq x_{i} \leq 100, \quad i=1,2, \ldots, n$
- $f_{\min }\left(X^{*}\right)=0, \quad x_{i}^{*}=0$


## f02: Schwefel's Problem 2.22

$$
f(x)=\sum_{i=1}^{n}\left|x_{i}\right|+\prod_{i=1}^{n}\left|x_{i}\right|
$$

- $-10 \leq x_{i} \leq 10, \quad i=1,2, \ldots, n$
- $f_{\text {min }}\left(X^{*}\right)=0, x_{i}^{*}=0$
f03: Schwefel's Problem 1.2

$$
f(x)=\sum_{i=1}^{n}\left(\sum_{j=1}^{i} x_{j}\right)^{2}
$$

- $-100 \leq x_{i} \leq 100, \quad i=1,2, \ldots, n$
- $f_{\text {min }}\left(X^{*}\right)=0, \quad x_{i}^{*}=0$


## f04: Schwefel's Problem 2.21

$$
f(x)=\max _{i}\left\{\left|x_{i}\right|, 1 \leq i \leq n\right\}
$$

- $-100 \leq x_{i} \leq 100, \quad i=1,2, \ldots, n$
- $f_{\text {min }}\left(X^{*}\right)=0, x_{i}^{*}=0$


## f05: Generalized Rosenbrock's Function

$$
f(x)=\sum_{i=1}^{n-1}\left[100\left(x_{i+1}-x_{i}^{2}\right)^{2}+\left(x_{i}-1\right)^{2}\right]
$$

- $-30 \leq x_{i} \leq 30, \quad i=1,2, \ldots, n$
- $f_{\min }\left(X^{*}\right)=0, x_{i}^{*}=1$


## f06: Step Function

$$
f(x)=\sum_{i=1}^{n}\left(\left\lfloor x_{i}+0.5\right\rfloor\right)^{2}
$$

- $-100 \leq x_{i} \leq 100, \quad i=1,2, \ldots, n$
- $f_{\text {min }}\left(X^{*}\right)=0, x_{i}^{*}=0$
f07: Quartic Function

$$
f(x)=\sum_{i=1}^{n} i x_{i}^{4}
$$

- $-1.28 \leq x_{i} \leq 1.28, \quad i=1,2, \ldots, n$
- $f_{\text {min }}\left(X^{*}\right)=0, x_{i}^{*}=0$


## f08: Generalized Schwefel's Problem 2.26

$$
f(x)=-\sum_{i=1}^{n}\left[x_{i} \sin \left(\sqrt{\left|x_{i}\right|}\right)\right]
$$

- $-500 \leq x_{i} \leq 500, \quad i=1,2, \ldots, n$,
- $f_{\text {min }}\left(X^{*}\right)=-418.982887272433799807913601398 n, x_{i}^{*}=420.968748$


## f09: Generalized Rastrigin's Function

$$
f(x)=\sum_{i=1}^{n}\left[x_{i}^{2}-10 \cos \left(2 \pi x_{i}\right)+10\right]
$$

- $-5.12 \leq x_{i} \leq 5.12, \quad i=1,2, \ldots, n$
- $f_{\text {min }}\left(X^{*}\right)=0, x_{i}^{*}=0$


## f10: Ackley's Function

$$
f(x)=-20 e^{\left(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^{n} x_{i}^{2}}\right)}-e^{\left(\frac{1}{n} \sum_{i=1}^{n} \cos \left(2 \pi x_{i}\right)\right)}+20+e^{(1)}
$$

- $-32 \leq x_{i} \leq 32, \quad i=1,2, \ldots, n$
- $f_{\text {min }}\left(X^{*}\right)=0, x_{i}^{*}=0$


## f11: Generalized Griewank's Function

$$
f(x)=\frac{1}{4000} \sum_{i=1}^{n} x_{i}^{2}-\prod_{i=1}^{n} \cos \left(\frac{x_{i}}{\sqrt{\hat{i}}}\right)+1
$$

- $-600 \leq x_{i} \leq 600, \quad i=1,2, \ldots, n$
- $f_{\text {min }}\left(X^{*}\right)=0, \quad x_{i}^{*}=0$


## f12: Generalized Penalized No. 1 Function

$$
f(x)=\frac{\pi}{n}\left\{10 \sin ^{2}\left(\pi y_{1}\right)+\sum_{i=1}^{n-1}\left(y_{i}-1\right)^{2}\left[1+10 \sin ^{2}\left(\pi y_{i+1}\right)\right]+\left(y_{n}-1\right)^{2}\right\}+\sum_{i=1}^{n} u\left(x_{i}, a, k, m\right)
$$

where

$$
y_{i}=1+\frac{1}{4}\left(x_{i}+1\right), \quad u\left(x_{i}, a, k, m\right)=\left\{\begin{array}{lr}
k\left(x_{i}-a\right)^{m} & x_{i}>a \\
0 & -a \leq x_{i} \leq a \\
k\left(-x_{i}-a\right)^{m} & x_{i}<-a
\end{array}\right.
$$

- $\quad a=10, \quad k=100 \quad \& \quad m=4$
- $-50 \leq x_{i} \leq 50, \quad i=1,2, \ldots, n$
- $f_{\text {min }}\left(X^{*}\right)=0, x_{i}^{*}=1$


## f13: Generalized Penalized No. 2 Function

$$
\begin{aligned}
& f(x)=0.1\left\{\sin ^{2}\left(3 \pi x_{1}\right)+\sum_{i=1}^{n-1}\left(x_{i}-1\right)^{2}\left[1+\sin ^{2}\left(3 \pi x_{i+1}\right)\right]+\left(x_{n}-1\right)^{2}\left[1+\sin ^{2}\left(2 \pi x_{n}\right)\right]\right\} \\
& \quad+\sum_{i=1}^{n} u\left(x_{i}, a, k, m\right)
\end{aligned}
$$

where

$$
u\left(x_{i}, a, k, m\right)=\left\{\begin{array}{lr}
k\left(x_{i}-a\right)^{m} & x_{i}>a \\
0 & -a \leq x_{i} \leq a \\
k\left(-x_{i}-a\right)^{m} & x_{i}<-a
\end{array}\right.
$$

- $a=5, \quad k=100 \quad \& \quad m=4$
- $-50 \leq x_{i} \leq 50, \quad i=1,2, \ldots, n$
- $f_{\text {min }}\left(X^{*}\right)=0, x_{i}^{*}=1$


## f14: Shekel's Foxholes Function

$$
f(x)=\left[\frac{1}{500}+\sum_{j=1}^{25} \frac{1}{j+\sum_{i=1}^{2}\left(x_{i}-a_{i, j}\right)^{6}}\right]^{-1}
$$

where

$$
a_{i, j}=\left[\begin{array}{ccccrrrrrr}
-32 & -16 & 0 & 16 & 32 & -32 & \ldots & 0 & 16 & 32 \\
-32 & -32 & -32 & -32 & -32 & -16 & \ldots & 32 & 32 & 32
\end{array}\right]
$$

- $-65.536 \leq x_{i} \leq 65.536, \quad i=1,2$
- $f_{\text {min }}\left(X^{*}\right) \approx 0.998003837794449325873406851315, x_{i}^{*} \approx-31.97833$


## f15: Kowalik's Function

$$
f(x)=\sum_{j=1}^{11}\left[a_{j}-\frac{x_{1}\left(b_{j}^{2}+b_{j} x_{2}\right)}{b_{j}^{2}+b_{j} x_{3}+x_{4}}\right]^{2}
$$

- $-5 \leq x_{i} \leq 5, \quad i=1,2,3,4$
- $f_{\text {min }}\left(X^{*}\right) \approx 0.0003074859878056042168404344971009$,
- $x_{i}^{*} \approx\left\{\begin{array}{l}0.192833452744335301314942585123 \\ 0.190836242203235800915303666443 \\ 0.123117296029247410738689372920 \\ 0.135765991801668826273045769995\end{array}\right\}$

Table 6. Data for Kowalik's Function

| $j$ | $a_{j}$ | $b_{j}^{-1}$ |
| :--- | :---: | :--- |
| 1 | 0.1957 | 0.25 |
| 2 | 0.1947 | 0.5 |
| 3 | 0.1735 | 1 |
| 4 | 0.1600 | 2 |
| 5 | 0.0844 | 4 |
| 6 | 0.0627 | 6 |
| 7 | 0.0456 | 8 |
| 8 | 0.0342 | 10 |
| 9 | 0.0323 | 12 |
| 10 | 0.0235 | 14 |
| 11 | 0.0246 | 16 |

## f16: Six-Hump Camel-Back Function

$$
f(x)=4 x_{1}^{2}-2.1 x_{1}^{4}+\frac{1}{3} x_{1}^{6}+x_{1} x_{2}-4 x_{2}^{2}+4 x_{2}^{4}
$$

- $-5 \leq x_{i} \leq 5, \quad i=1,2$
- $f_{\text {min }}\left(X^{*}\right)=-1.031628453489877$ "It has four global minimum",
- $x_{i}^{*}=( \pm 0.08984201368301331, \pm 0.7126564032704135)$


## f17: Branin RCOS Function

$$
f(x)=\left(x_{2}-\frac{5.1}{4 \pi^{2}}\right) x_{1}^{2}+\left(\frac{5}{\pi} x_{1}-6\right)^{2}+10\left(1-\frac{1}{8 \pi}\right) \cos \left(x_{1}\right)+10
$$

- $-5 \leq x_{1} \leq 10, \quad 0 \leq x_{2} \leq 15$
- $f_{\min }\left(X^{*}\right)=0.39788735772973816$ "It has three global minimum",
- $x_{i}^{*}=(-\pi, 12.275),(\pi, 2.275),(9.42478,2.475)$


## f18: Goldstein-Price Function

$$
f(x)=f_{1}(x) f_{2}(x)
$$

where

$$
\begin{aligned}
& f_{1}(x)=1+\left(x_{1}+x_{2}+1\right)^{2}\left(19-14 x_{1}+3 x_{1}^{2}-14 x_{2}+6 x_{1} x_{2}+3 x_{2}^{2}\right) \\
& f_{2}(x)=30+\left(2 x_{1}-3 x_{2}\right)^{2}\left(18-32 x_{1}+12 x_{1}^{2}+48 x_{2}-36 x_{1} x_{2}+27 x_{2}^{2}\right)
\end{aligned}
$$

- $-2 \leq x_{i} \leq 2, \quad i=1,2$
- $f_{\text {min }}\left(X^{*}\right)=3, x_{i}^{*}=(0,-1)$


## f19,20: Hartman's Family

$$
f(x)=-\sum_{i=1}^{m} c_{i} \exp \left[-\sum_{j=1}^{n} a_{i, j}\left(x_{j}-p_{i, j}\right)^{2}\right]
$$

- $m=4, n=3,6$ for $f 19$ and f20, respectively
- $0 \leq x_{j} \leq 1, \quad j=1,2, \ldots, n$
- $f 19: f_{\min }\left(X^{*}\right)=-3.86278214782076, \quad x_{i}^{*}=\left\{\begin{array}{c}0.1, \\ 0.55592003, \\ 0.85218259\end{array}\right\}$
- $f 20: f_{\min }\left(X^{*}\right)=-3.32236801141551, \quad x_{i}^{*}=\left\{\begin{array}{l}0.20168952, \\ 0.15001069, \\ 0.47687398, \\ 0.27533243, \\ 0.31165162, \\ 0.65730054\end{array}\right\}$

Table 7. Data for Hartman's Function 1

| $i$ | $a_{i, j}, j$ <br> $=1,2,3$ |  |  | $c_{i}$ | $p_{i, j}, j=1,2,3$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 3 | 10 | 30 | 1 | 0.3689 | 0.1170 | 0.2673 |
| 2 | 0.1 | 10 | 35 | 1.2 | 0.4699 | 0.4387 | 0.7470 |
| 3 | 3 | 10 | 30 | 3 | 0.1091 | 0.8732 | 0.5547 |
| 4 | 0.1 | 10 | 35 | 3.2 | 0.038150 | 0.5743 | 0.8828 |

Table 8. Data for Hartman's Function 2

| $i$ | $c_{i}$ | $a_{i, j}, j=1,2, \ldots, 6$ |  |  |  |  |  | $p_{i, j}, j=1,2, \ldots, 6$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 10 | 3 | 17 | 3.5 | 1.7 | 8 | 0.1312 | 0.1696 | 0.5569 | 0.0124 | 0.8283 | 0.5886 |
| 2 | 1.2 | 0.05 | 10 | 17 | 0.1 | 8 | 14 | 0.2329 | 0.4135 | 0.8307 | 0.3736 | 0.1004 | 0.9991 |
| 3 | 3 | 3 | 3.5 | 1.7 | 10 | 17 | 8 | 0.2348 | 0.1415 | 0.3522 | 0.2883 | 0.3047 | 0.6650 |
| 4 | 3.2 | 17 | 8 | 0.05 | 10 | 0.1 | 14 | 0.4047 | 0.8828 | 0.8732 | 0.5743 | 0.1091 | 0.0381 |

## f21,22,23: Shekel's Family

$$
f(x)=-\sum_{i=1}^{n}\left[\left(x-a_{i}\right)\left(x-a_{i}\right)^{T}+c_{i}\right]
$$

- $n=5,7$ and 10 for $f 21, f 22$ and $f 23$, respectively
- $0 \leq x \leq 10, \quad i=1,2, \ldots, n$
- $f 21: f_{\min }\left(x^{*}\right)=-10.153198755084881776356839400251$,

$$
x^{*}=4.000085212027619539925233402760
$$

- $f 22: f_{\min }\left(x^{*}\right)=-10.402822044707775329070518200751$, $x^{*}=4.000089532152739968028886505640$
- $f 23: f_{\text {min }}\left(x^{*}\right)=-10.536290299294717105427357601002$, $x^{*}=4.0001281610100453290705182007510$

Table 9: Data for Shekel Functions $f 21, f 22, f 23$

| $i$ | $a_{i j}, j=1,2,3,4$ |  |  |  |
| :--- | :--- | :--- | :--- | :---: |
| $c_{i}$ |  |  |  |  |
| 1 | 4 | 4 | 4 | 4 |
| 0.1 |  |  |  |  |
| 2 | 1 | 1 | 1 | 1 |
| 3 | 0.2 |  |  |  |
| 3 | 8 | 8 | 8 | 8 |
| 4 | 6 | 6 | 6 | 6 |
| 0.2 |  |  |  |  |
| 5 | 3 | 7 | 3 | 7 |
| 6 | 2 | 9 | 2 | 9 |
| 7 | 5 | 5 | 3 | 3 |
| 8 | 8 | 1 | 0.6 | 1 |
| 9 | 6 | 2 | 6 | 2 |
| 10 | 7 | 3.6 | 7 | 3.6 |

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#### Abstract

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