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PAPR REDUCTION OF OFDM SIGNALS USING SELECTIVE MAPPING WITH TURBO CODES

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ABSTRACT

Multiple inputs multiple output orthogonal frequency division multiplexing (MIMO-OFDM) is an attractive transmission technique for high bit-rate communication systems. MIMO-OFDM has become a promising candidate for high performance 4G broadband wireless communications. One main disadvantage of OFDM is the high peak-to-average power ratio (PAPR) of the transmitter’s output signal. Selected-Mapping (SLM) scheme which does not require the transmission of side information and can reduce the peak to average power ratio (PAPR) in turbo coded orthogonal frequency division multiplexing (OFDM) system is proposed. Simulation results show that the system can achieve significant reduction in PAPR and satisfactory bit error rate performance over AWGN channels.

KEYWORDS

Orthogonal Frequency Division Multiplexing (OFDM), peak-to-average-power-ratio (PAPR), Selective Mapping (SLM), Complementary Cumulative Distribution Function (CCDF), Bit Error Rate (BER).

1. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) has drawn significant interests over past decade [1], [2] for its robustness against the multipath fading channels. It is an effective high-speed data transmission scheme without using very expensive equalizers and it has been proposed as the air interface for broadband wireless applications such as wireless local area networks (WLANs). One of the major drawbacks of OFDM systems is that the OFDM signal exhibits a high peak-to-average power ratio (PAPR). Such a high PAPR necessitates the linear amplifier to have large dynamic range which is difficult to accommodate.

On the other hand, an amplifier with nonlinear characteristics will cause undesired distortion of the in-band and out-of-band signals. By now, many techniques have been proposed for relieving the PAPR problem in the OFDM, which can be roughly divided into two classes, the distortion-based techniques and the redundancy-based techniques. The distortion-based techniques reduce the PAPR of the OFDM symbol with the price of adding distortion to the signal points in the subcarriers. Direct clipping [1] simply suppresses the time-domain OFDM signals of which the signal powers exceed a certain threshold. The penalty is the significant increase of out-of-band energy. Peak windowing [2] or filtering after direct clipping [3] can be used to reduce the out-of-band energy. After the filtering operation, the peak of the time-domain signal may regrow. Hence, recursive clipping and filtering (RCF) [4] can be used to suppress both the out-of-band energy and the PAPR. RCF can be modified by restricting the region of distortion [5] to obtain...
improved error performance. On the other hand, estimation of the clipping noise at the receiver [6] can be used to improve the error performance of direct clipping or RCF.

The redundancy-based technique includes coding, selective mapping (SLM), partial transmit sequences (PTS), tone reservation (TR) and tone injection (TI) [7-13], etc. For the redundancy-based technique, the undesired effects occurring to the distortion based techniques can be alleviated while the penalty is the reduced transmission rate or increased average power due to the introduction of redundancy. A block coding technique [14] is to transmit only the code words with low PAPR. Such coding techniques offer good PAPR reduction performance and coding gain. Significant advance of the coding approach for PAPR control using generalized Reed-Muller codes is summarized in [15]. The critical problem for the coding approach is that for the OFDM system with large number of subcarriers, either it encounters design difficulties or the consequent coding rate becomes prohibitively low.

The basic idea of SLM technique is to generate several OFDM symbols as candidates and then select the one with the lowest PAPR for actual transmission. Conventionally, the transmission of side information is needed so that the receiver can use the side information to tell which candidate is selected in the transmission. In [16] and [17], the side information for a channel coded SLM appears explicitly in the data sequence to be encoded so that the side information is protected by the same channel code. The advantage of such an arrangement is that no additional protection is needed for side information and the rate loss due to the side information is small. However, once the side information is incorrectly decoded, the number of error bits in the erroneously decoded codeword can be great. In [12], an SLM technique (for either coded or uncoded cases) which does not need the transmission of side information was proposed, where the discrimination of the desired candidate against the undesired candidates is obtained by specially arranging the constellations for the subcarriers of each candidate so that the modulated signal points for the subcarriers of each pair of candidates are widely different.

In this paper we propose and examine a technique for reducing the probability of a high PAPR, based on part on a method proposed in [18] and [19]. This technique is a variation of selective mapping (SLM) [18], in which a set of independent sequences are generated by some means from the original signal, and then the sequence with the lowest PAPR is transmitted. To generate these sequences we use turbo encoder. Using turbo coding will offer two advantages, significant PAPR reduction and astonishing bit error rate (BER) performance.

2. PAPR PROBLEM AND SLM SCHEME

In the discrete time domain, an OFDM signal $x_n$ of $N$ subcarriers can be expressed as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, \quad 0 \leq n \leq N - 1$$

(1)

Where $X_k$, $k = 0,1,2,3,..., N-1$ are input symbols modulated by BPSK, QPSK or QAM and $n$ is the discrete time index.

The PAPR of an OFDM signal is defined as the ratio of the maximum to the average power of the signal, as follows

$$PAPR(x) = 10 \log_{10} \frac{\max\{|x_n|^2\}}{E[|x|^2]} \quad 0 \leq n \leq N - 1$$

(2)

Where $E[.]$ denotes the expected value operation and $x = [x_1, x_2, x_3, ..., x_{N-1}]^T$. 

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As to the discrete-time signals, since symbol-spaced sampling may sometimes miss some of the signal peaks, signal samples are obtained by oversampling by a factor of \( L \) to better approximate the true PAPR. Oversampled time-domain samples are usually obtained by \( LN \)-point IFFT of the data block with \((L - 1)N\) zero-padding. It is shown in [20] that \( L = 4 \) is sufficient to capture the peaks.

When the OFDM signal with high PAPR passes through a non-linear device, (power amplifier working in the saturation region), the signal will suffer significant non-linear distortion [20]. This non-linear distortion will result in in-band distortion and out-of-band radiation. The in-band distortion causes system performance degradation and the out-of-band radiation causes adjacent channel interference (ACI) that affects systems working in the neighbour bands. To lessen the signal distortion, it requires a linear power amplifier with large dynamic range. However, this linear power amplifier has poor efficiency and is so expensive. Obviously, the distribution of PAPR bears stochastic characteristics in a practical OFDM system. Usually, Complementary Cumulative Distribution Function (CCDF) can be used to evaluate the performance of any PAPR reduction schemes, given by [17].

\[
CCDF(N, \text{PAPR}_0) = 1 - (1 - e^{-\text{PAPR}_0})^N
\]  

(3)

In the SLM approach, \( U \) statistically-independent phase sequences, say, \( p^{(u)} = [p_0^{(u)}, p_1^{(u)}, p_2^{(u)}, \ldots, p_{N-1}^{(u)}]^T \) are generated, where \( p_k^{(u)} = \exp(j\phi_k^{(u)}) \), \( \phi_k^{(u)} \in [0, 2\pi] \), \( k = 0, 1, 2, \ldots, N-1 \), \( u = 1, 2, 3, \ldots, U \). Then the data block \( X = [x_0, x_1, x_2, \ldots, x_{N-1}]^T \) is multiplied component-wise with each one of \( U \) different phase sequence \( p^{(u)} \), resulting in a set of \( U \) different data blocks \( X^u = [x_0p_0^{(u)}, x_1p_1^{(u)}, x_2p_2^{(u)}, x_3p_3^{(u)}, \ldots, x_{N-1}p_{N-1}^{(u)}]^T \), \( u = 1, 2, 3, \ldots, U \). Then, all \( U \) alternative data blocks (one of the alternative subcarrier sequences must be the unchanged original one) are transformed into time domain to get transmitted symbols \( x^u \), \( u = 1, 2, 3, \ldots, U \) by IFFT, where \( x^u \), \( u = 1, 2, 3, \ldots, U \) are defined as the candidate signals. Finally, the one with the minimum PAPR is selected for transmitting, shown in Fig. 1.

At the side of receiver, in order to recover the received signals successfully, the side information is required. This information must be transmitted accompanying with the transmitted signal. When binary symbols are used, \( \lceil \log_2 U \rceil \) bits are required to represent this side information [2, 3], where operation \( \lceil . \rceil \) rounds the elements to the nearest integers toward infinity.

![Figure 1. The block diagram of SLM scheme](image)
2.1. Selective mapping using Turbo Coding

The probability that, the PAPR of the OFDM signal exceeds a certain threshold $Y$ is given by

$$P_r\{PAPR > Y\} = 1 - (1 - e^{-Y})^N$$  \hspace{1cm} (4)$$

In SLM it is assumed that, $U$ statistically independent alternative sequences, which represent the same information, are generated by some suitable means. The sequence with the lowest PAPR is selected for transmission.

The probability that, the lowest PAPR $Y_i$ exceeds a certain threshold $Y$ is given by

$$P_r\{Y_i > Y\} = (P_r\{PAPR > Y\})^U$$  \hspace{1cm} (5)$$

To generate these sequences linear feedback shift register (LFSR) is used [11]. A LFSR is used to transform the data before it is mapped to the orthogonal channels. Different sequences are generated by inserting different bits labels at the beginning of the data. This results in $U = 2^m$ different sequences, where $m$ is the length of the inserted bits.

Turbo codes [21] are parallel concatenated convolutional codes in which the information bits are first encoded by a recursive systematic convolutional (RSC) code and then, after passing the information bits through an interleaver, are encoded by a second RSC code. Turbo decoder is used to recover the transmitted signal at the receiver side. The Turbo decoder consists of two soft input soft output (SISO) modules [22], an interleaver and de-interleaver. Figure 2 shows a turbo system, turbo encoder and decoder.

In this paper, instead of using LFSR, we use turbo encoder to generate different sequences and the sequence with the lowest PAPR is selected for transmission. The different sequences are generated by inserting different bits labels at the beginning of the data. Figure 3 shows the transmitter side of an OFDM system, where the turbo coding and SLM are used for PAPR reduction. For each bits labels $b_i$, $i = 1, 2, 3 \ldots U$ where $b_i$ a sequence of $m$ bits, the turbo encoder will generate a sequence $x_i$, $i = 1, 2, \ldots U$. The sequence that has the lowest PAPR will be selected for transmission. At the receiver side, the receiver does not need any side information, and the bits labels are discarded after decoding.
2.2. Simulation Results

The PAPR reduction and BER performances of the proposed scheme are examined by computer simulation. In the simulation we consider an OFDM signal with $N = 128$ subcarriers, 16-Quadrature Amplitude Modulation (16-QAM) mapping, and turbo code with two RSC encoders each has a constraint length $K = 4$ with generator polynomial $15/17$, where $g_1/g_2$ represents the forward/backward generator polynomials in octal base. Puncturing is used to increase the overall code rate to $R = 1/2$. To obtain accurate PAPR calculations the signal is over sampled, $L = 8$. Additive white Gaussian noise (AWGN) channel is assumed. At the receiver side logarithmic maximum a posterior probability (log-MAP) algorithm is used to implement the SISO modules.

Figure 4. shows the complementary cumulative distribution function (CCDF) of the PAPR of the OFDM signal, where turbo coding and SLM are used for PAPR reduction. The CCDF of the PAPR without SLM is also shown ($U=1$). Clearly, the probability of high PAPR is reduced significantly as the number of sequences increases. When $U = 1$, the PAPR is approximately 9.7 dB and with turbo coding and selective mapping, with $U = 32$, the PAPR of the OFDM signal is nearly 6.8 dB. The simulation results agree with the results obtained by the approximation in equation (4) with small differences. These small differences due to the fact that, equation (4) was derived with the assumption that, the samples are mutually uncorrelated which is not true anymore when over sampling is applied. The BER performance of the
The proposed scheme is also shown in Figure 5. For number of iterations 1, 2 and 5, the probability of bit error is reduced up to $8 \times 10^{-5}$.

**Figure 4. PAPR Performance with 16 QAM**

**Figure 5. Bit Error Rate (BER) Vs. SNR**
2.3. CONCLUSIONS

We have shown that, Turbo coding and SLM can be combined to reduce the PAPR of OFDM signal with quite moderate additional complexity. The advantage of the proposed scheme is that, the Turbo encoder is used for two purposes, error correction and PAPR reduction. This reduces the hardware complexity of the system.

REFERENCES