PERFORMANCE ANALYSIS OF AMPLIFY-AND-FORWARD RELAY BASED COOPERATIVE SPECTRUM SENSING IN FADING CHANNELS

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ABSTRACT

Relay based cooperative spectrum sensing elevates the spectrum sensing problems in harsh propagation conditions if the relay nodes are positioned suitably for the dynamic spectrum access systems. This article investigates the performance of relay based spectrum sensing (energy detection based) in myriad fading environments using the area under the receiver operating characteristic curve (AUC) performance metric. AUC is a single statistical parameter, which provides better insight of detection probability compared to the receiver operating characteristic curve (ROC). The article includes the performance analysis of two-hop multi-relay amplify-and-forward (AF) relay network with channel state information (CSI) assisted relaying (variable relay gain) and blind (fixed) relay gain. A unified model is also investigated to describe the relay network for variable and fixed relay gain. Various fading and relay configurations are illustrated to show the efficacy of the energy detection based collaborative spectrum sensing in AF relay configuration using the AUC performance metric. Performance of the variable and fixed relay gain based AF relay network are analysed and compared considering different factors affecting energy sensing such as the per-hop signal-to-noise ratio (SNR), fading index, and the time-bandwidth product. The results can be readily used for the design of the cognitive relay network.

KEYWORDS

Area under Curve, Amplify and Forward, Cooperative Relay Network, Diversity Combining, Energy Detector, Spectrum Sensing

1. INTRODUCTION

Key to the emerging paradigm of opportunistic dynamic spectrum access (DSA) is the cognitive radios which can dynamically sense the environment and rapidly tune their transmission parameters to best utilize the vacant/underutilized spectrum bandwidth. The first key requirement for DSA system requires robust spectrum sensing capability in a manner that the DSA systems do not create interference for the incumbent users. Among the various known spectrum sensing techniques, blind sensing based on energy detection is perhaps the simplest and most versatile. However, the detection performance of energy detectors is severely limited by harsh propagation environments and becomes unreliable at low signal-to-noise ratio (SNR). This issue is further complicated by the noise uncertainty and shadowing problem.

A relay node between a primary signal emitter and a detector can be placed to increase the range of detection or to tackle the shadowing scenarios. Among all relaying protocol, amplify and forward (AF) or non-regenerative protocol is the simplest in which relay-nodes just amplify and forward received signal [1]. AF can be further divided into two groups, namely, i) CSI DOI: 10.5121/ijwmm.2012.4508
assisted/variable relay gain [2], in which relay node adapts the relay gain based on the CSI to cancel out the fading effect, and ii) blind/fixed gain relaying, in which relay gain is chosen arbitrarily or sometimes semi-blindly based on the CSI statistics [3]. Blind/fixed relaying does not require monitoring the CSI of the 1st hop at the relay and hence with the fixed gain, relay-node outputs signal with variable power. Systems with blind relaying mechanism in AF model are more attractive from the practical implementation perspective for its lower complexity and ease of deployment. In relay based cooperative spectrum sensing, the fusion center (FC) can make the decision of the presence of primary signal either based on the data fusion (soft) or decision (hard) fusion. The FC can take the advantages of receiver combining techniques such as the maximum ratio combining (MRC), square law combining (SLC) techniques to improve the performance of the spectrum sensing for data fusion. In decision fusion, local relay nodes take initial decisions based on the local observations and forward "1" or "0" to the FC for fusion over the narrow band control channel. "1" indicates the presence of the primary signal whereas "0" indicates the absence of the primary signal. The fusion can be done using the $k$-out-of-$N$ system. This structure is a very popular type of redundancy in fault tolerant systems. The FC declares the presence of the active signal if at least $k$-of-the-$N$ CRs detect the presence of the signal. Following the $k$-out-of-$N$ rule, “OR”, “AND”, and “Majority” fusion rule can be found in literature (e.g., [1], [4] - [10]). Data fusion shows better spectrum sensing compared to decision fusion but decision fusion/hard fusion requires less communication overhead over the reporting channels (between CR and FC), which is attractive for practical implementation.

Recent works [1] and [5] have shown the performance of signal detection improves many times with the collaborative cooperation even at the low SNRs. But the existing relay based analytical frameworks have limitations for performance measurement in fading channels as well as in shadowing effects. Thus, there is a need to obtain a better analytical framework for analysing the energy-detection-performance using the relay concept. In [2] and [11], CSI assisted relaying protocol is utilized to determine the energy detector’s performance but the development is not in generalized form. Atapattu [12] analysed energy detection performance for a fixed gain relay network over only Rayleigh channels using complementary AUC analysis, but the derivation is not in generalized form and the solution is intricate. In [13] and [14], detail performance studies of dual-hop end-to-end transmission system with fixed relay gain are found but not the energy detector’s performance. The existing solutions mainly deal with the perfect $i.i.d$. Rayleigh channels, which is clearly not the case in practical scenarios. Also most studies are found to deal with network performance characterization and not in terms of the energy detector’s performance.

Thus, motivated by the current limitations, we seek simple framework for analysing the CSS relay network in $i.i.d./i.n.d.$ channel conditions subject to multipath fading and shadowing effects. We analyse energy detector’s performance for the simple two-hop relay network using our prior developed unified analytical framework of energy detection using the AUC performance metric [15]. Since AUC is the portion of the area of a unit square, the value ranges from 0 to 1 but in practice, signal detection can be considered as the flipping of a coin. The random guessing produces the diagonal ROC line between (0, 0) and (1, 1) which has an area of 0.5. Thus, for practical detectors, the AUC value will not be less than 0.5 [16]. Thus, AUC is a good measure of detection probability; it represents the probability that choosing the correct decision at the detector is more likely than choosing the incorrect decision. We utilize the AUC metric to characterize the detection performance for both the variable and blind (fixed) AF relaying protocols. We develop generalized frameworks to characterize the AF relay based energy detection performance in myriad of fading channels. In particular, we illustrate examples with Nakagami-m and Rayleigh channels for variable and fixed relay gain network respectively. We also develop a unified solution using the AUC metric both for the variable and fixed relay gain system and include results over Nakagami-m fading statistics. We use the proposed frameworks subsequently to study the efficacies of diversity combining schemes such as the maximum ratio combining (MRC), square law combining (SLC) in multi-relay network.
To our best knowledge, energy detector’s performance study for the AF relay network with the variable relay gain and unified model with the AUC measurement parameter is new while for fixed relay gain our framework with AUC metric is in generalized and simple form. The analysis could be readily used to choose right parameters for energy detectors in cognitive radio system. Our framework circumvents the need of sophisticated computer software to compute the numerical results. Further, the results of this article are more complete analyses of our initial findings that were published in [17] [18]. The major contributions are summarized as follows.

(i) A simple generalized framework is developed using AUC metric to analyse the performance of the energy detector in CSI assisted dual-hop multi-relay network, which can be used in myriad of fading environments.

(ii) A simple generalized framework over myriad of fading channels is developed to analyse the detector’s performance in blind AF two-hop multi-relay network.

(iii) A unified model is developed to handle the variable/blind AF protocol for the spectrum sensing analysis.

The rest of the paper is organized as follows. Section 2 presents the system model for the AF relay based cooperative spectrum sensing using the energy detection scheme. Section 3 includes proposed framework of performance analysis with the AUC metric for the CSI assisted relay network. In Section 4, blind AF relay network is investigated. Section 5 includes the analysis of the unified performance model. Section 6 includes the numerical results while Section 7 includes the concluding remarks.

2. SYSTEM MODEL

Let us consider a two-hop \( N \) relay network (\( i = 1: N \) relay nodes) with variable/fixed relay gain as shown in Fig.1. \( S \) is the primary signal source and \( D \) is the relay coordinator. We also consider that the relay nodes amplify and forward the received noisy signal in orthogonal channels to the central relay coordinator for the soft / data fusion. Also a direct link is possible between \( S \) and \( D \). \( h_{S,D} \), \( h_{S,i} \) and \( h_{i,D} \) are the channel coefficients between source (\( S \)) & relay coordinator (\( D \)); \( S \) & \( i \)-th relays node; and \( i \)-th relay node & \( D \) respectively. Let us now consider a single relay system. The received primary signal at the \( i \)-th relay node and at the relay coordinator (\( D \)) can be given as

\[
y_{S,i} = \sqrt{P_{S,i}}h_{S,i}x + n_{S,i}
\]
where, \( P_{S,j} = P_{S,D} \) is the power of the transmitted source signal \( x \) while \( n_{S,j} \) and \( n_{S,D} \) are the additive white Gaussian noise (AWGN) introduced between the \( S \) and \( i \)-th relay node, and \( S \) and \( D \). The relayed signal from the \( i \)-th relay to \( D \) is given by

\[
y_{i,D} = G_i h_{i,D} y_{S,j} + n_{i,D}
\]

where, \( G_i \) is the amplifier gain of the \( i \)-th relay node and \( n_{i,D} \) is the additive noise introduced between the \( i \)-th relay node and the relay coordinator (\( D \)). The gain can be variable or fixed gain.

We assume the detection at the relay coordinator can be described by the binary hypothesis \((H_0,\ H_1)\) where \( H_0 \) means that the signal is absent whereas \( H_1 \) implies that the signal is present. The energy detection is performed by comparing the measured energy of the observed signal in the observation time interval \( T \) with the energy threshold \( \lambda \) [19]. The decision statistic \((Y)\) is represented by the Chi-square distribution \( \chi^2_{2u} \) under \( H_0 \) and a noncentral Chi-square distribution \( \chi^2_{2u}(2\gamma) \) with \( 2u \) degree of freedom and noncentrality parameter of \( 2\gamma \) under hypothesis \( H_1 \) [19]. \( \gamma \) is the end-to-end instantaneous SNR and \( u = TW \) is the time-bandwidth product \((W \) is the filter bandwidth\). Thus, the probability of the false alarm \( P_f = Pr \{ Y > \lambda \mid H_0 \} \) and the probability of the detection \( P_d = Pr \{ Y > \lambda \mid H_1 \} \) can be given as [20]

\[
P_f(\lambda) = \frac{\Gamma(u, \frac{\lambda}{2})}{\Gamma(u)}
\]

\[
P_d(\gamma, \lambda) = Q_u(\sqrt{2\gamma}, \sqrt{\lambda})
\]

where, \( Q_u(.,.) \) is the generalized \((u\)-th order\) Marcum Q-function and \( \Gamma(.,.) \) is the upper incomplete Gamma function. In terms of AUC metric, the average probability of false alarm and the detection probability can be given as [15] (see Appendix A for derivation)

\[
\overline{A}_{\text{gen}}(\gamma) = \frac{1}{u^{2u} \Gamma(u)} \sum_{k=0}^{\infty} \frac{\Gamma(k+2u) \cdot F_1(1, k+2u; 1+u; 0.5)}{\Gamma(k+u) k! 2^{(u+k)}} (-1)^k \phi^k_\gamma(s) \big|_{s=1}
\]

\[
\overline{A}_{\text{gen}}(\gamma) = 1 - \frac{1}{\Gamma(u)} \sum_{k=0}^{\infty} \frac{\Gamma(k+2u) \cdot F_1(1, k+2u; 1+u+k; 0.5)}{(u+k) \Gamma(k+u) k! 2^{(u+k)}} (-1)^k \phi^k_\gamma(s) \big|_{s=1}
\]

where, \( \phi^k_\gamma(s) \) is the \( k \)-th order derivative of the moment generating function (MGF) of the effective end-to-end signal-to-noise ratio (SNR). Equation (7) is an alternate solution to (6). These frameworks have advantage over any existing energy detection performance measuring framework since most common MGFs are available in literature [see 15, Table 1]. Also these frameworks can handle half odd integer value of \( u \) (bandwidth time product) and fading index \( m \) of Nakagami-m distribution as well as converge fast with nominal number of terms [15].
3. RELAY SCHEME WITH VARIABLE RELAY GAIN

3.1. Single Relay with Two-hop network

Let us consider \( i \)-th relay network. Assuming flat fading and the variable relay gain \( G_r^i = 1 / h_{r,i}^2 \), the overall “exact” SNR of the harmonic means at the relay coordinator (D) can be given as [2]

\[
\gamma_i = \frac{\gamma_{s,i} \gamma_{i,d}}{\gamma_{s,i} + \gamma_{i,d}} \tag{8}
\]

where, \( \gamma_{s,i} = (h_{s,i})^2 / N_0 \) and \( \gamma_{i,d} = (h_{i,d})^2 / N_0 \) are the SNR between the S and the \( i \)-th relay node and \( i \)-th relay to D respectively and \( N_0 \) is the one sided power spectral density (PSD) of the AWGN. If \( \gamma_{s,d} = (h_{s,d})^2 / N_0 \) is the SNR of this direct-path, between S and D, then the equivalent effective SNR at the relay coordinator with the direct-path can be given as

\[
\gamma = \gamma_{s,d} + \gamma_i \tag{9}
\]

Thus, the performance of the energy detection of the radio coordinator in the AF relay network with variable relay gain can be easily given by using the \( k \)-th order derivative of the MGF of the effective SNR (\( \gamma \)) in (6) or (7), which are in generalized forms.

The MGF of the two-hop \( i \)-th relay SNR at the receiver is available in literature. For instance, the MGF of tight bounded \( \gamma_i \) over Rayleigh i.n.d. fading statistics is well-known, and is given as [2]

\[
\phi_{\gamma} (s) = \frac{16}{3 \Omega_{s,i} \Omega_{i,d} (A_i + s)^2} \left[ \frac{4 \left( \frac{1}{\Omega_{s,i}} + \frac{1}{\Omega_{i,d}} \right)}{\left( A_i + s \right)^2} \right] F_2 \left( \begin{array}{c} 3, 3; \frac{3}{2}, \frac{5}{2}; A_s + s \\ 2, 2; A_i + s \end{array} \right) + F_2 \left( \begin{array}{c} 2, 1; \frac{3}{2}, \frac{5}{2}; A_s + s \\ 2, 2; A_i + s \end{array} \right) \tag{10}
\]

where, \( A_1 = \frac{1}{\Omega_{s,i}} + \frac{1}{\Omega_{i,d}} + \frac{2}{\sqrt{\Omega_{s,i} \Omega_{i,d}}} \), \( A_2 = \frac{1}{\Omega_{s,i}} + \frac{1}{\Omega_{i,d}} - \frac{2}{\sqrt{\Omega_{s,i} \Omega_{i,d}}} \). \( \Omega_{s,i} , \Omega_{i,d} \) are the mean SNR between S & \( i \)-th relay-node and the \( i \)-th relay-node & D respectively and \( F_2 (...) \) is the Gaussian hypergeometric series. The simplified compact form of (10) for \( i.i.d. \) case can be given as (11).

\[
\phi_{\gamma} (s) = F_2 \left( 1, 2; \frac{3}{2}; -s \Omega_s \right) \tag{11}
\]

where, \( \Omega_s \) is the per hop \( i \)-th relay mean SNR. Also the MGF of \( \gamma_i \) of Nakagami-m channel with \( i.i.d. \) fading statistics can be is given as [21] in (12).

\[
\phi_{\gamma} (s) = F_1 \left( m, 2m; m + \frac{1}{2}; -s \Omega_s \right) \tag{12}
\]
where, \( m \) is the Nakagami-\( m \) fading index. An alternative simple closed form of MGF of Rayleigh i.n.d. cases can also be found in [22, (55)] but (10) is more tractable for deriving the \( k \)-th derivative. The \( k \)-th derivative of (10) and (12) can be obtained easily and are given as (13) and (14) respectively by using the Leibnitz differential rule, identity [23, (0.430-1)] and
\[
\frac{\partial^k}{\partial s^k}(A + s)^{-\alpha} = -(1)^n(a)_n(A + s)^{(-\alpha - n)}, \text{where, } (a)_n = a(a+1)...(a+n-1) \text{denotes the Pochhammer symbol.}
\]

\[
\phi_{\gamma_i}^{(k)}(s) = \frac{64(\Omega_{S,j} + \Omega_{i,D})}{3(\Omega_{S,j} + \Omega_{i,D})^2} \sum_{n=0}^{k} \binom{k}{n} (3)^{n} (A_{i} + 1)^{-(3k-n)} \times \sum_{r=1}^{n} U_{r} \frac{(3/2)_r}{(5/2)_r} \cdot \frac{16}{3(\Omega_{S,j} + \Omega_{i,D}} \sum_{n=0}^{k} \binom{k}{n} (2k-n) (A_{i} + 1)^{-(2k-n)}
\]

\[
\sum_{r=1}^{n} U_{r} \frac{(2/2)_r}{(5/2)_r} \cdot \frac{16}{3(\Omega_{S,j} + \Omega_{i,D}} \sum_{n=0}^{k} \binom{k}{n} (2k-n) (A_{i} + 1)^{-(2k-n)} \times (A_{i} + 1)^{(r-p-n)}(-1)^{n-m}(r-p)^{-n} (A_{i} + 1)^{-(r-p+n-m)}
\]

and
\[
\phi_{\gamma_i}^{(k)}(s) = \frac{-\Omega_{i,j}}{4m} \binom{m}{k} \frac{(2m)_k}{(m+1/2)_k} \times \sum_{n=0}^{k} \binom{n}{m} \frac{(r-p)!}{(r-p-m)!} \times (A_{i} + 1)^{(r-p-n)}(-1)^{n-m}(r-p)^{-n} (A_{i} + 1)^{-(r-p+n-m)}
\]

Therefore, by using (13), and (14) in (6) or (7), AUC based summary detection performance of the coordinator for a single 2-hop relay system can be easily obtained for i.n.d. Rayleigh and i.i.d. Nakagami-m fading channels respectively. In case of i.n.d. Nakagami-m channels, the MGF of upper bounded \( \gamma_i \) (\( \gamma_i = \min[\gamma_{P,j}, \gamma_{i,D}] \)) can be given as [24, (11)] in (15)

\[
\phi_{\gamma_i}^{(k)}(s) = \sum_{k \in \{P,j\},(i,D)} \frac{(m + m_j)}{(m + m_k)} \frac{\Omega_{m_k}}{\Omega_{m_j} + \Omega_{m_k} + \Omega_{m_j}} \cdot \frac{\Omega_{m_k}}{\Omega_{m_j} + \Omega_{m_k} + \Omega_{m_j}}
\]

\[
\times \sum_{k \in \{P,j\},(i,D)} \frac{\Omega_{m_k}}{\Omega_{m_j} + \Omega_{m_k} + \Omega_{m_j}} \cdot \frac{\Omega_{m_k}}{\Omega_{m_j} + \Omega_{m_k} + \Omega_{m_j}}
\]

where, \( k \) is selected between primary source (\( P \)) and \( i \)-th relay node and \( j \) is selected between \( i \)-th relay and relay coordinator (\( D \)). We can easily get the \( k \)-th derivative of (15) by few algebraic steps or by using the Mathematica.
3.2. Multi-Relay with Two-hop network

Relay based cognitive/DSA radio network may have N relay systems and the radio coordinator can use various combining techniques such as maximum ratio combining (MRC) and square law combining (SLC) to coordinate the primary user’s presence. It is assumed that, all the relay nodes forward and amplified primary user’s signal through orthogonal channels such as the TDMA or FDMA techniques. Combining can be done either before (MRC method) or after detection (SLC method). Practical implementation of MRC is difficult to achieve as the coordinator requires the CSI information of each diversity branch in order to achieve coherent detection. This is not required in the case of SLC as the signals are non-coherently combined after detection; leading to twice the degree of freedom of the MRC; ultimately reduces the detection probability compared to MRC. For illustration, MRC and SLC are discussed below.

3.2.1 Maximal Ratio Combining (MRC)

MRC is a pre-combiner that linearly combines the signals from all diversity branches by first co-phased and weighted in proportional to their SNR. The approximated output SNR, $\gamma_{MRC}$ of the MRC combiner for 2-hop N relay with the direct path can be given as

$$\gamma_{MRC} = \gamma_{S,D} + \sum_{i=1}^{N} \frac{\gamma_{S,i} \gamma_{i,D}}{\gamma_{S,i} + \gamma_{i,D}}$$  (16)

The MRC decision statistics follows $i.i.d. \chi^2_{2n}$ distribution under $H_0$ and $\chi^2_{2n}(\gamma_{MRC})$ under $H_1$ Hypothesis. Therefore, the $P_f$ and the $P_d$ at the MRC output for AWGN channels can be evaluated by (4) and (5) respectively. Thus, (9) is directly applicable to MRC technique for deriving the average AUC or the probability of detection by using the $k^{th}$ order derivative of the combined MGF of $\gamma_{MRC}$. The combined MGF is given as

$$\phi_{MRC}(s) = \phi_{S,D}(s) \prod_{i=1}^{N} \phi_i(s)$$  (17)

where, $\phi_{S,D}(s)$ and $\phi_i(s)$ are the MGFs due to the SNR of direct path and the $i$-th relay part respectively. Using the Leibnitz’s differential product rule, the $k$-th derivative of (17) can be given as

$$\phi_{MRC(N+1)}^{(k)}(s) = \sum_{n_0=0}^{k} \sum_{n_1=0}^{n_0} \sum_{n_2=0}^{n_1} \cdots \sum_{n_{N-1}=0}^{n_{N-2}} \binom{k}{n_0} \binom{n_0}{n_1} \binom{n_1}{n_2} \cdots \binom{n_{N-2}}{n_{N-1}} \phi_{S,D}^{(k-n_0)}(s) \phi_1^{(n_0-n_1)}(s) \phi_2^{(n_1-n_2)}(s) \cdots \phi_{N-1}^{(n_{N-2}-n_{N-1})}(s) \phi_N^{(n_{N-1})}(s)$$  (18)

where, $\phi_i(s) = \phi_{S,D}(s)$ and its $k$-th derivative over myriad fading statistics can be found in literature [see 15, TABLE-3-1]. $\phi_i^{(k)}(s)$ is the $k$-th derivative of the MGF $\phi_i(s)$ is due to the $i$-th relay parts and can be given either by (13) or (14). $(N+1)$ in (18) indicates the $N$ relays with the direct path between $S$ and $D$. $k$-th derivative of (15) can also be used for representing $i.n.d.$ Nakagami-m statistics. For illustration, considering the practical cases of $N = 2$ and 3 relays, (18) can be written as (19) and (20) respectively.
\[
\phi_{MRC(2+1)}^{(k)}(s) = \left( \sum_{n_0=0}^{n_0} \sum_{n_1=0}^{n_1} \left( \sum_{n_2=0}^{n_2} \sum_{n_3=0}^{n_3} \cdots \sum_{n_k=0}^{n_k} \frac{k!}{n_0! n_1! n_2! \cdots n_k!} s^{k-n_0-n_1-n_2-\cdots-n_k} \right) \phi_{H_0}^{(k-n_0)}(s) \Big|_{s=1} \phi_{H_1}^{(n_0-n_1)}(s) \Big|_{s=1} \phi_{H_2}^{(n_1-n_2)}(s) \Big|_{s=1} \right) .
\]
\[\tag{19}\]
\[
\phi_{MRC(3+1)}^{(k)}(s) = \left( \sum_{n_0=0}^{n_0} \sum_{n_1=0}^{n_1} \sum_{n_2=0}^{n_2} \sum_{n_3=0}^{n_3} \cdots \sum_{n_k=0}^{n_k} \frac{k!}{n_0! n_1! n_2! \cdots n_k!} s^{k-n_0-n_1-n_2-\cdots-n_k} \right) \phi_{H_0}^{(k-n_0)}(s) \Big|_{s=1} \phi_{H_1}^{(n_0-n_1)}(s) \Big|_{s=1} \phi_{H_2}^{(n_1-n_2)}(s) \Big|_{s=1} \times \phi_{H_3}^{(n_2-n_3)}(s) \Big|_{s=1} \phi_{H_4}^{(n_3-n_4)}(s) \Big|_{s=1} \phi_{H_5}^{(n_4-n_5)}(s) \Big|_{s=1} .
\]
\[\tag{20}\]

Thus, using (19) and (20) in (6) or (7), average AUC measurement can be obtained for dual hop, 2 and 3 relays with direct path respectively over the Rayleigh or Nakagami-m statistics.

### 3.2.2 Square-Law Combining (SLC)

SLC is a post combiner technique that performs square-and-integrate operation over per branch-output and added to yield a new decision static 

\[ Y_{SLC} \]

For \( N \) relays with one direct path, \( Y_{SLC} \) is the sum of \( (N+1) \) i.i.d. \( \chi^2 \) for \( H_0 \) and \( \chi^2(\epsilon_j) \) for \( H_1 \), where the noncentrality \( \epsilon_j = 2 \gamma_{SLC} \), hence the \( P_f \) and the \( P_d \) can be expressed by (4) and (5) respectively by replacing \( u \) with \( u(N+1) \). Therefore, for SLC scheme, (6) or (7) can be re-written by replacing \( u \) with \( u(N+1) \). Then (19) and (20) can be used with the updated expressions of (6) or (7) to obtain the AUC analysis for SLC scheme for 2 and 3 relay networks.

### 4. Relay Scheme with Blind (fixed) Relay Gain

We consider blind/fixed relay gain in this section and follow the model description of Section 2. Let us consider for simplicity \( \gamma_i \) (\( i = 1,2 \)) is the per-hop SNR for a dual hop relay network, where 1 indicates the path from \( S \) to \( i \)-th relay node and 2 indicates the path between \( i \)-th relay node and \( D \). In [3], detail study of the dual-hop transmission with fixed relay gain can be found and for a single relay with two-hop network, the equivalent end-to-end SNR at the coordinator can be written as

\[ \gamma_{eq} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + C} \]
\[\tag{21}\]

where, \( C \) is a constant with fixed gain \( G \) such that \( C = (\epsilon_2 / G^2 N_{0i}) \) and \( \epsilon_i \) is the source (\( i = 1 \)) and relay (\( i = 2 \)) transmitted signal power, \( N_{0i} \) is the AWGN signal power per hop. Thus, if we know the MGF of the equivalent SNR (21), we can find the \( k \)-th derivative for it. We can use this \( k \)-th derivative directly in (6) or (7) to obtain the average AUC.

For instance, if the two hops are assumed to suffer independent and not necessarily identically distributed Rayleigh fading, then \( \gamma_i \) is exponentially distributed with parameter \( \bar{\gamma}_i = \epsilon_i \Omega_i / N_{0i} \) (\( i = 1,2 \)), where \( \Omega_i = h_i^2 \) is the average fading power on the \( i \)-th hop. Then, \( C \) and the MGF of the equivalent SNR over the Rayleigh fading channels (i. n. d.) with fixed relay gain is given in compact closed-form as [3]
\[ C = \frac{1}{\bar{F}_1} e^{\frac{1}{\bar{F}_1}} E_1 \left( \frac{1}{\bar{F}_1} \right) \]  

(22)

\[ \phi_{I_{n}}(s) = \frac{1}{\bar{F}_1 s + 1} + \frac{C \bar{F}_1 s e^{\frac{1}{\bar{F}_1} (\bar{F}_1 + 1)}}{\bar{F}_2 (\bar{F}_1 s + 1)^2} E_1 \left( \frac{C}{\bar{F}_2 (\bar{F}_1 s + 1)} \right) \]  

(23)

where, \( E_1(.) \) in (22) is the exponential integral function defined as [25, (5.1.1)]. We use \( \frac{\partial^n}{\partial s^n} (A + s)^{-n} = (-1)^n (a)_n (A + s)^{-(a+n)} \) (where \( (a)_n \) denotes Pochhammer symbol), identity [26, (0.430-1)] and the Leibniz’s differential rule, to obtain the \( k^{th} \) order derivative of (23) as

\[ \phi_{I_{n}}^{(k)}(s) = A + \sum_{n=1}^{k} \frac{U_n}{n!} F^{(n)}(y) \]  

(24)

where, \( A = \frac{1}{\bar{F}_1} (-1)^k k! (s + 1)^{-(k+1)} \), \( y = \frac{C}{\bar{F}_2 (\bar{F}_1 s + 1)} \)

\[ F^{(n)} = \sum_{p=0}^{n} \binom{n}{p} y^p (1 - y - 2n + 2p) e^y (n - p)(2n + p) \]

\[ B = (-1)^k E_{1-k}(y) \]

\[ U_n = \sum_{r=0}^{n} (-1)^r \binom{n}{r} y^r \frac{d^k}{ds^k} (y^{n-r}) \]

\[ \frac{d^k}{ds^k} (y^{n-r}) = (-1)^k \binom{n-r}{k} (n-r)_k (s + 1)^{-(n-r-k)} \]

Now substituting (24) into (6) or (7) and evaluating at \( s = 1 \), we obtain the average AUC for the two-hop relay network with fixed relay gain over the independent and not necessarily identically distributed Rayleigh channels.

Our two-hop single relay model can be easily extended to two-hop multi-relay system to utilize the diversity reception concept. Using the \( k^{th} \)-derivatives of the MGF of the received SNR for fixed relay gain with (18), we can simply derive coordinator’s detection performance with fixed relay gain in i.n.d. fading environments for MRC or SLC scheme. Considering the relay part, for MRC and SLC, we can write following generalized schemes

\[ \bar{A}_{\text{gen}}^{\text{MRC}} (\gamma) = 1 - \frac{1}{\Gamma(u)} \sum_{k=0}^{n} \sum_{n_{k-1}=0}^{n} \sum_{n_{k-2}=0}^{n_{k-1}} \cdots \sum_{n_{2}=0}^{n_{1}} \sum_{n_{1}=0}^{n} \frac{\Gamma(k + 2u) \Gamma_1(1, k + 2u; 1 + u + k; 0.5)}{(u + k) \Gamma(k + u) k! 2^{2u+k}} \]  

(25)

\[ \times \phi_{I_{n}}^{(k-n_{1})}(s) \bigg|_{r=1} \phi_{I_{n_{2}}-n_{2}}^{(k-n_{1})}(s) \bigg|_{r=2} \cdots \phi_{I_{n_{N-1}}-n_{N-2}}^{(k-n_{1})}(s) \bigg|_{r=N-1} \]


\[ A_{\text{Avg}}(\gamma) = 1 - \frac{1}{\Gamma(uN)} \sum_{k=0}^{\infty} \frac{\Gamma(k + 2uN) \cdot F_1(1, k + 2uN; 1 + uN + k; 0.5)}{(uN + k) \Gamma(k + uN) k! 2^{2uN+k}} (-1)^k \times (-1)^k \sum_{n=0}^{k} \sum_{n_1=0}^{n} \ldots \sum_{n_{N-1}=0}^{n_{N-2}} \left( \begin{array}{c} k \\ n_1 \\
_2 \\
_{N-1} \end{array} \right) \left( \begin{array}{c} n_1 \\ n_2 \\
_3 \\
_{N-2} \end{array} \right) \ldots \left( \begin{array}{c} n_{N-2} \\ n_{N-1} \end{array} \right) \phi_{1}^{(k-n_0)}(s) \phi_2^{(n-n_2)}(s) \ldots \phi_{N}^{(n_{N-1})}(s) \right|_{u=1} \] \hfill \quad (26) \hfill 

where \( \phi_{1...N}(s) \) are the MGFs of the 1...N relay parts and \( \phi^{(k)}(s) \) is the \( k \)-th order derivative of \( \phi(s) \).

5. **Unified Performance Analysis for the AF Relaying Network**

In this section we include the unified model to analyze energy detector’s performance for the simple dual hop AF relay network. CSI assisted/variable relay gain and blind/fixed relay gain based system analysis are found in the literature and we also developed simple frameworks for variable and fixed relay gain using AUC metric in prior sections. Only limited studies are found regarding the unified model. In recent time, dual hop performance analysis based on the unified model can be found only in [13] and [14] but these did not analyze the energy detector’s performance. Their frameworks are rather complicated and cannot be easily generalized. In this section we develop a simple unified model to analyze the energy detector’s performance using our prior developed AUC approach. Our proposed detection performance analysis framework is capable of handling both the variable and fixed gain AF relay network. To our best knowledge, energy detector’s performance study for the AF dual-hop relay network with the unified SNR model based on the AUC metric is new.

5.1 Unified Model Scheme

We consider the same relay network given by Fig. 1 but relay gain can be fixed or variable with per-hop SNR, \( \gamma_i \). For dual-hop simple relay network, a general/unified model of the received SNR at the destination can be defined as [13]

\[ \gamma_{eq} = \frac{\gamma_1 \gamma_2}{a \gamma_1 + b} \] \hfill \quad (27) \hfill 

where, \( a, b \) can take either 0 or 1. With the pair of \( a \& b \), special cases of AF relaying protocol can be defined. Variable gain and fixed gain relay configuration can be defined by \( (a, b) \) with (1, 0) and (0, 1) respectively. The relay gain \( (G_n) \) of \( n^{th} \) relay can be determined as [13]

\[ G_n^2 = \frac{1}{ah_{n}^2} + bN_0 \] \hfill \quad (28) \hfill 

where, \( h_{n}^2 \) is for the \( n^{th} \) hop relay. Thus, in general, deriving the MGF of (27) and taking its the higher order derivative and using it with (6) or (7), performance analysis can be obtained for the two-hop relay network either for CSI assisted or blind relay configuration where \( G \) can be set using (28).
In particular, we consider the two hops are assumed to suffer independent and not necessarily identically distributed Nakagami-\(m\) fading and thus \(\gamma_i\) is gamma distributed with parameter \(\alpha_i\) and \(\beta_i\) \((i = 1, 2)\). MGF of the unified SNR over the Nakagami-\(m\) channels \((i. n. d.)\) can be found in [13]. As the special case when \(a = 1\) and \(b = 0\), the MGF \((\phi^v_{\text{var}}(s))\) based on the variable relay gain and when \(a = 0\), \(b = 1\), the MGF \((\phi^v_{\text{fixed}}(s))\), based on the fixed relay gain, can be given as (29) and (30) respectively.

\[
\phi^v_{\text{var}}(s) = 1 - 2s \sum_{n=0}^{\alpha-1} \sum_{k=0}^{\alpha-1} \sum_{m=0}^{k} C_1(n,k,m)J_1(n,k,m) \frac{n+m+1}{2} \tag{29}
\]

where, \(C_1(n,k,m) = \frac{a^{n-m-2\alpha_i}}{m!(k-m)!n!(\alpha_i - n - 1)!} \beta_i^{\frac{m-n-2k}{2}} \beta_2^{\frac{n-m-1}{2}}\); and

\[
J_1(n,k,m) = \frac{\sqrt{\pi} \Gamma(\alpha_1 + k + n - m + 2) \Gamma(\alpha_1 + k - n + m)}{\Gamma(\alpha_1 + k + 3 / 2)} \times \left( \frac{16a}{\beta_1 \beta_2} \right)^{n-m+1 / 2} \\
\times 2 F_{1}(\alpha_1 + k + n - m + 2, n - m + 3 / 2; \alpha_1 + k + 3 / 2; \bar{s}) \\
\times \left[ s + \left( \frac{1}{\beta_1} + \frac{1}{\beta_2} \right) \right]^{\alpha_1+k+n-\alpha_1}
\]

\[
\phi^v_{\text{fixed}}(s) = 1 - 2s \sum_{n=0}^{\alpha-1} \sum_{k=0}^{\alpha-1} \sum_{m=0}^{k} C_2(n,k)J_2(n,k) \tag{30}
\]

where, \(C_2(n,k) = \frac{\beta_1^{\frac{n-m-2\alpha_i}{2}} \beta_2^{\frac{n+m+1}{2}}}{k!(\alpha_i - n - 1)!}\), and

\[
J_2(n,k) = \frac{\Gamma(\alpha_1 + 1) \Gamma(\alpha_1 + k - n)}{2\sqrt{b l(\beta_1 \beta_2)}} \times \exp[b l(\beta_1(1 + \beta_1s))] W_{n-2\alpha_i-k-n+1}(b l(\beta_2(1 + \beta_2 s)))
\]

where, \(W_{\mu, \nu}(\cdot)\) denotes the Whittaker \(W\) function [26, (922)].

The \(k\)-th order derivative of (29) and (30) can be easily derived. Taking the \(k\)-th order derivative of (29) and evaluating at \(s = 1\) and inserting in (6) or (7) gives the average AUC for the variable relay gain configuration and similarly with the \(k\)-th order derivative of (30) and evaluating at \(s = 1\) and after insertion in (6) or (7) gives the average AUC, the detection performance measure, of the energy detector for the fixed relay gain based two-hop AF relay network. Also the \(k\)-th derivative of (29) and (30) can be used with (25) and (26) to obtain the analytical results for MRC and SLC combining schemes.
6. NUMERICAL RESULTS

This section includes the numerical results for various AF relay configurations. The numerical results are given in three sub-sections for CSI assisted, blind relay, and unified performance analysis.

![Graph showing Average AUC vs. mean SNR for N-relay network with source to destination direct path over i.i.d. Nakagami-m Channels (m = 2 and u = 3.5).]

6.1 Numerical Results for CSI Assisted AF Relay Network

We have shown performance using the AUC metric using our simple yet useful generalized framework (6) or (7). Our generalized frameworks with appropriate higher order derivative of the equivalent MGF for the two-hop N relay model can be used over myriad of fading statistics. For illustration we have used Nakagami-m fading channels. We investigated relay based performance with or without direct path and also only direct path alone considering various parameters such as fading severity, number of relay paths, and time-bandwidth product. In particular, Fig.2 illustrates the performance of the detector for “only direct path”, “only single relay”, and “relay with direct path (N+1)” (N is the number of relays) cases. It is apparent that the performance of direct path (source to destination) is better than only the single relay configuration. Similar results were also found for the similar relay model for the symbol error rate in [27]. But the motivation of using relay network is to reduce the path loss when common receiver is at a disadvantage position such as the distance between transmitter and the receiver and at the hidden position/obstacle by large objects.
Figure 3. Average AUC vs. mean SNR for a relay network with source to destination direct path over i.i.d. Nakagami-m Channels, with $m = 2$ and $u = 3.5$.

Figure 4. Average AUC vs. mean SNR for two hop fixed gain AF relaying over i.i.d. / i.n.d. Rayleigh fading with MRC illustration ($u = 2$).
Fig. 2 also illustrates the diversity combining results for MRC as well as for the SLC scheme. MRC shows the optimum performance as expected because of the higher end-to-end SNR associated with MRC. It is noticeable that with the diversity technique, detection capability improves greatly where direct-path gives a major impact on the detection performance. The figure also shows that the detection performance for “only-direct-path” outperforms the detection performance for “only single-relay” case even if the direct path’s fading severity is higher. Overall, we see better performance with diversity combining compared to single relay reception at low SNR. The figure indicates that the relay based cooperative spectrum sensing can achieve a target robustness (say $P_d = 0.5$ and $P_f = 0.1$) even at low SNR.

In Fig. 3, the impact of the fading severity on the detection performance over i.i.d. Nakagami-m channels is illustrated. Only single relay is used to obtain the complementary AUC curves on the semi-log scale. From the figure it can be seen that the performance increases with order of $m$ as $m$ increases from 1 to 6. To show the advantages of our approach that it can handle the half odd-integer values of $u$ and $m$, we also plotted AUC curves with half-odd integer $u$ and half-odd integer $m$ values in Fig. 2 & 3 respectively.

Figure 5. Comparison of fixed and variable relay-gain over i.i.d. Rayleigh channels for a dual-hop relay network ($u = 2$).

6.2 Numerical Results for Blind AF Relay Network

This section includes the performance of the relay coordinator based on the blind/fixed relay gain. Our result is much better than that of [12] since we have used the overall “exact” SNR of the harmonic means at the coordinator, whereas Atapattu [12] considered upper bounded SNR.

In Fig. 4, average AUC curves are plotted against the mean SNR for single relay and dual-relay MRC scheme for the blind AF dual hop relay network over the i.n.d. Rayleigh channels. For the second hop of each relay, SNR value is varied. With higher SNR value for the second hop, detector’s performance improves considerably. A variable gain based average AUC for two-hop single relay and SLC ($L = 2$) scheme are derived and compared with the fixed gain based average AUC curve for i.i.d. Rayleigh fading in Fig. 5.

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Figure 6. Complementary AUC. Comparison of MRC with SLC diversity combining schemes for a fixed AF dual-hop relay network over Rayleigh fading ($u = 2, \gamma_2 = 5\gamma_1$).

Figure 7. Fixed and variable relay gain performance over fading channels using unified SNR model with $u = 2$. 
The result shows comparable performance and supports the results stated in [3]. Though, it is not shown here, we obtained that the increase of the time-bandwidth product ($u$) does not increase the AUC value for fixed relay number. Large sample ($u$) values do not necessarily increase the Average AUC; as $u$ increases, the false alarm increases faster with larger samples, which is supported by [16]. Finally, in Fig. 6, average complementary AUC curves are shown on the semi-log plot for the MRC and SLC schemes for the fixed gain AF relay system over i.n.d. Rayleigh channels with $u = 2$. Clearly MRC outperforms SLC combining technique because of the higher end-to-end SNR associated with MRC. The curves confirmed that the higher number of relays ($L$) provide better detection performance.

6.3 Numerical Results for Unified AF Relay Network

This section includes the performance of relay coordinator for primary signal detection using the unified approach of equivalent SNR model. In Fig. 7, average AUC curves are plotted for a single relay system for the variable and fixed relay gain over the Rayleigh and Nakagami-$m$ channels. The results shown are also supported by Fig. 5 and [3]. In Fig. 8, we compared variable and fixed relay gain based spectrum sensing performance for i.n.d. dual-hop single relay network. When first hop of the dual hop relay has significantly higher SNR than that of the second hop, it is observed that the detection probability is much higher for the fixed relay gain compared to the variable relay gain based configuration. Finally, in Fig. 9, using the unified model, average AUC curves of MRC and SLC schemes are compared for the dual-hop, dual-branch with direct path blind AF relay system over Nakagami-$m$ channels with $\alpha_1 = 3$ and $\alpha_2 = 4$. Clearly, the curves follow the results obtained in Section 6.2.

Figure 8. Comparison of variable relay gain and fixed relay gain based spectrum sensing
7. CONCLUSIONS

We have developed analytical frameworks to study the efficacies of cooperative spectrum sensing for the two-hop amplify-and-forward multi-relay network with variable and fixed-gain configurations. Even though we considered this as the relay coordinator’s performance analysis, it is basically the concept of data fusion. Cooperative relaying techniques are of particular interest since such approaches could overcome the practical implementation issue of packing a large number of antenna elements on small-sized wireless devices while improving the detection performance by harnessing the benefits of distributed spatial diversity. We also developed two-hop unified model to analyze the fixed or variable gain based relay network. We have articulately shown the versatility of our frameworks for various scenarios. We considered practical parameters such as the fading severity and relay node number and illustrated that the receiver combining techniques with the relay network can achieve better detection probability and capable of satisfying target robustness.

The significant contribution of this work is the simplicity of the proposed method in analyzing the energy detector’s performance in relay network. Also our method is capable of handling half odd-integer bandwidth-time product \(u\) and the half odd-integer Nakagami-\(m\) fading parameter \(m\). To the best of our knowledge, no prior work of CSI assisted AF relay network dealt with Nakagami-\(m\) channels. Also none of the prior work used AUC metrics except [12] but this cannot treat the half-integer \(u\) or \(m\) values. Also, their framework cannot be generalized for myriad of fading environments as well as their model only considered upper bounded equivalent SNR whereas our approach provides generalized frameworks with overall “exact” received SNR at the relay coordinator. This article has established the fast converging series-solutions of the energy detection performance using the MGF based frameworks for the two-hop AF relay network. The analytical results can be utilized readily for the practical implementation of the relay based cognitive radio network.
APPENDIX-A

With the alternative representation of the generalized Marcum Q-function [28], (5) can be written as

\[ P_q(y, \lambda) = Q_u\left( \sqrt{2\gamma} \sqrt{\lambda} \right) = \sum_{k=0}^{\infty} \frac{\gamma^k e^{-\gamma}}{k!} \frac{\Gamma\left(u+k, \frac{\lambda}{2}\right)}{\Gamma(u+k)} \]  \hspace{1cm} (A-1)

Substituting \( G(a, z) = \Gamma(a) - \Gamma(a, z) \) in (A-1) yields an alternate series as

\[ P_q(y, \lambda) = Q_u\left( \sqrt{2\gamma} \sqrt{\lambda} \right) = 1 - \sum_{k=0}^{\infty} \frac{\gamma^k e^{-\gamma}}{k!} \frac{G\left(u+k, \frac{\lambda}{2}\right)}{\Gamma(u+k)} \]  \hspace{1cm} (A-2)

where, \( G(a, z) \) is the lower incomplete Gamma function. Now \( P_q(y, \lambda) \) can be obtained by averaging (A-1) over the PDF of channel SNR \( f(\gamma) \) as

\[ \overline{P}_{\text{d-Gen}} = \sum_{k=0}^{\infty} \frac{1}{k!} \frac{\Gamma\left(u+k, \frac{\lambda}{2}\right)}{\Gamma(u+k)} \int_{0}^{\infty} e^{-\gamma} \gamma^k f(\gamma) d\gamma \]  \hspace{1cm} (A-3)

Now, using following Laplace-transform identity, we obtain (6)

\[ \int_{0}^{\infty} \gamma^k \exp(-\lambda \gamma) f(\gamma) d\gamma = (-1)^k \phi_{f}^{(k)}(s) \bigg|_{s=\lambda} \]  \hspace{1cm} (A-4)

Similarly, we can obtain (7) by averaging (A-2) and using the Laplace-transform (A-4).

REFERENCES


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