ON APPROACH TO OPTIMIZE MANUFACTURING OF A TRANSISTORS WITH TWO SOURCES TO DECREASE THEIR DIMENSIONS

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ABSTRACT

In this paper we introduce an approach to optimize technological process of manufacturing of heterotransistors with two sources to decrease their dimensions. Framework the approach we consider a heterostructure with specific configuration. After manufacturing of the heterostructure we consider doping of several areas of the heterostructure by diffusion or ion implantation. The doping was finished by optimized annealing of dopant and/or radiation defects. We introduce an analytical approach for prognosis of mass transport to obtain the required results.

KEYWORDS

Heterotransistor with two sources; optimization of technological process; analytical approach for prognosis of mass transport.

1. INTRODUCTION

One of intensively solving aims of solid state electronics in the present time is increasing of density of elements of integrated circuits due to decreasing of their dimensions and optimization of technological processes. One can also find increasing of speed of functioning of these elements. Dimensions of these elements could be decreased by laser or microwave types annealing of the considered dopants [1-3]. These types of annealing leads to inhomogeneity of diffusion coefficient and another parameters of processes due to inhomogenous distribution of temperature and Arrhenius law. The inhomogeneity of these parameters gives a possibility to decrease dimensions of elements of integrated circuits. Radiation processing [4,5] and/or inhomogeneity of heterostructures [6-9] could be also used to change properties of doped materials.

In this paper we consider manufacturing a heterotransistor with two sources (see Fig. 1). The transistor has been manufactured framework the heterostructure from Figs. 1. The heterostructure include into itself a substrate and an epitaxial layer. Several sections have been manufactured in the epitaxial layer by using other materials so as it is shown on Figs. 1. We consider doping of these sections by diffusion or ion implantation to manufacture the required types of conductivity. After this doping it is required optimized annealing of dopant and/or radiation defects. Framework the paper we analyzed redistribution of dopant and radiation defects for determination of conditions, which correspond to decreasing of dimension of the considered transistor.

2. METHOD OF SOLUTION

To solve our aim we analyzed distribution of concentration of dopant in space and time. The distribution has been calculated as solution of the following boundary problem

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$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_C\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_C\frac{\partial C(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D_C\frac{\partial C(x,y,z,t)}{\partial z}\right]. \quad (1)$$

Boundary and initial conditions for the equations could be written as

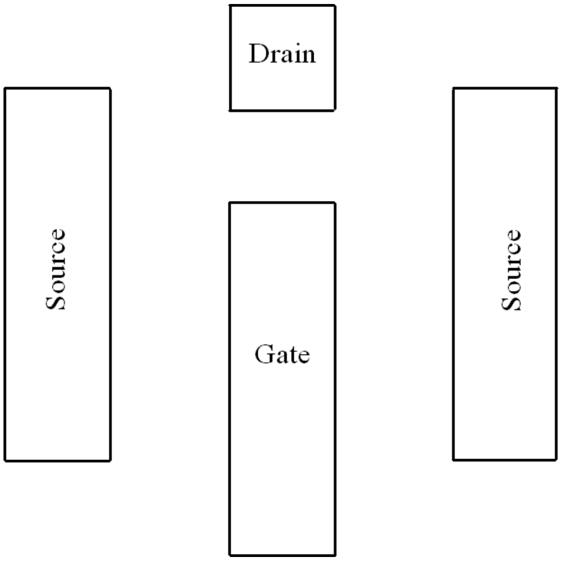

Fig. 1*a*. Structure of transistor with two sources. View from top

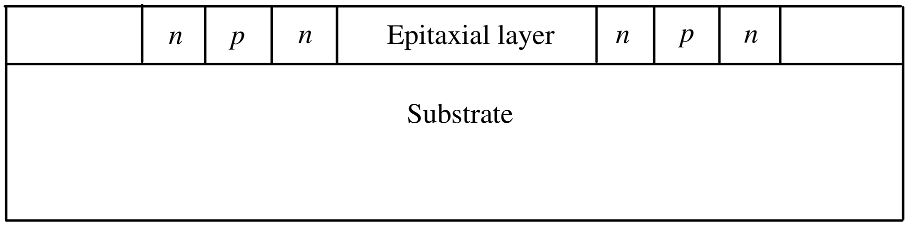

Fig. 1*b*. Heterostructure with two layers and sections in the epitaxial layer

$$\left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{x=L_y}=0,$$

$$\left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{x=L_z}=0,\ C(x,y,z,0)=f(x,y,z). \quad (2)$$

Function $C(x,y,z,t)$ describes the distribution of concentration of dopant in space and time; T is the temperature of annealing; D_C is the dopant diffusion coefficient. Value of dopant diffusion coefficient will be changed with changing temperature of annealing. The value will be also changed with changing of concentrations of dopant and radiation defects. All above dependences could be accounted by the following relation [4,11,12]

$$D_C=D_L(x,y,z,T)\left[1+\xi\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]. \quad (3)$$

The function $D_L(x,y,z,T)$ gives a possibility to take into account dependences of dopant diffusion coefficient on coordinate (due to presents several layers in heterostructure) and temperature (due to Arrhenius law). The function $P(x,y,z,T)$ describes the limit of solubility of dopant. The parameter $1\le\gamma\le3$ describes quantity of charged defects, which were interacted (in average) with each atom of dopant [11]. The function $V(x,y,z,t)$ describes distribution of concentration of radiation vacancies in space and time with equilibrium distribution V^*. It is known that diffusion doping of materials did not leads to generation radiation defects. In this situation $\zeta_1=\zeta_2=0$. We determine spatio-temporal distributions of concentrations of radiation defects by solving the following system of equations [4,12]

$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]-k_{I,I}(x,y,z,T)\times$$
$$\times I^2(x,y,z,t)+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]-k_{I,V}(x,y,z,T)\,I(x,y,z,t)\,V(x,y,z,t) \tag{4}$$
$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]-k_{V,V}(x,y,z,T)\times$$
$$\times V^2(x,y,z,t)+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]-k_{I,V}(x,y,z,T)I(x,y,z,t)\,V(x,y,z,t).$$

Boundary and initial conditions for these equations are

$$\left.\frac{\partial\rho(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial\rho(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial\rho(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial\rho(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,$$
$$\left.\frac{\partial\rho(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial\rho(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,\ \rho(x,y,z,0)=f_\rho(x,y,z). \tag{5}$$

Here $\rho=I,V$. The function $I(x,y,z,t)$ describes variation of distribution of concentration of radiation interstitials in space and time. The function $D_\rho(x,y,z,T)$ describes spatial and temperature dependences of point radiation defects diffusion coefficients. Terms $V^2(x,y,z,t)$ and $I^2(x,y,z,t)$ describes generation simplest complexes of radiation defects (i.e. divacancies and diinterstitials); the function $k_{I,V}(x,y,z,T)$ describes special and temperature dependences of the parameter of recombination of point radiation defects; the function $k_{I,I}(x,y,z,T)$ and $k_{V,V}(x,y,z,T)$ describe special and temperature dependences of parameters of generation of simplest complexes of point radiation defects.

We determine concentrations of divacancies $\Phi_V(x,y,z,t)$ and dinterstitials $\Phi_I(x,y,z,t)$ as functions of space and time by solving the following system of equations [4,12]

$$\frac{\partial\Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi I}(x,y,z,T)\frac{\partial\Phi_I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi I}(x,y,z,T)\frac{\partial\Phi_I(x,y,z,t)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[D_{\Phi I}(x,y,z,T)\frac{\partial\Phi_I(x,y,z,t)}{\partial z}\right]+k_{I,I}(x,y,z,T)I^2(x,y,z,t)-k_I(x,y,z,T)I(x,y,z,t) \tag{6}$$
$$\frac{\partial\Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi V}(x,y,z,T)\frac{\partial\Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi V}(x,y,z,T)\frac{\partial\Phi_V(x,y,z,t)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[D_{\Phi V}(x,y,z,T)\frac{\partial\Phi_V(x,y,z,t)}{\partial z}\right]+k_{V,V}(x,y,z,T)V^2(x,y,z,t)-k_V(x,y,z,T)V(x,y,z,t).$$

Boundary and initial conditions for these equations are

$$\left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,$$

$$\left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,\ \Phi_I(x,y,z,0)=f_{\Phi I}(x,y,z),\ \Phi_V(x,y,z,0)=f_{\Phi V}(x,y,z).\ (7)$$

Here functions $D_{\Phi\rho}(x,y,z,T)$ describe spatial and temperature dependences of the diffusion coefficients of the considered complexes of radiation defects; functions $k_I(x,y,z,T)$ and $k_V(x,y,z,T)$ describe spatial and temperature dependences of the parameters of decay of these complexes. To solve equations for concentrations of dopant and radiation defects we used method of averaging of function corrections [13] with decreased quantity of iteration steps [14]. All calculation procedure is presented in the Appendix.

3. DISCUSSION

Now we analyze distribution of concentration of infused (see Fig. 2*a*) and implanted (see Fig. 2*b*) dopants in space and time in the considered epitaxial layer. Annealing time of dopant for each curve coincides with the same annealing times for another curves framework of each figure. Increasing of number of curve corresponds to increasing of difference between values of dopant diffusion coefficient in layers of heterostructure. Based on these figures one can find that interface between layers of heterostructure gives a possibility to increase absolute value of gradient of concentration of dopant in perpendicular direction to the interface. The variation of the gradient leads to decreasing of dimensions of the considered transistors. At the same time with increasing of absolute value of the above gradient homogeneity of distribution of concentrations of dopants in doped areas increases.

To determine optimal annealing time we take into account variation of the considered gradient of concentration of dopant near interface between epitaxial layer and substrate. With decreasing of value of annealing time one can find increasing of inhomogeneity of distribution of concentration of dopant (see Figs. 3*a* for diffusion type of doping and 3*b* for ion type of doping). As optimal annealing time of dopant and/or radiation defects we determine compromise value of the required time time framework recently introduced criterion [15-20]. Based on the criterion we approximate real distribution of concentration of dopant by idealized step-wise function $\psi(x, y,z)$. After the approximation we determine the optimal value of annealing time by minimization of the mean-squared error

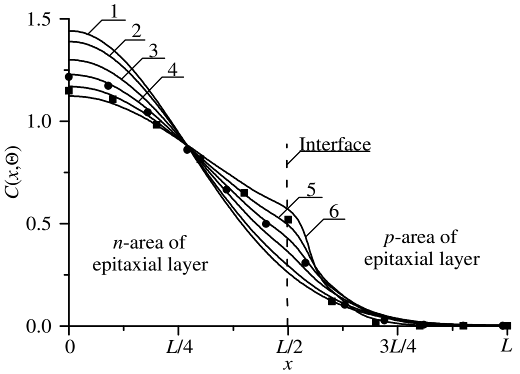

Fig.2*a*. Dependences of concentration of infused dopant on coordinate in the considered heterostructure in direction, which is perpendicular to interface between epitaxial layer substrate. Difference between values

of dopant diffusion coefficient in layers of heterostructure increases with increasing of number of curves. Dopant diffusion coefficient in the substrate is smaller than in epitaxial layer. Squares are the experimental data from [21]. Circles are the experimental data from [22]

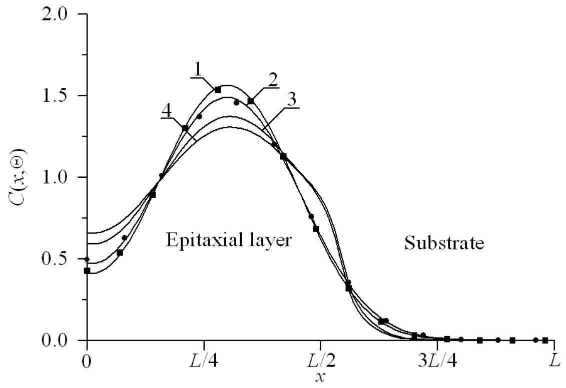

Fig.2*b*. Dependences of concentration of implanted dopant on coordinate in the considered heterostructure in direction, which is perpendicular to interface between epitaxial layer substrate for two annealing times: $\Theta = 0.0048\,(L_x^2 + L_y^2 + L_z^2)/D_0$ (for curves 1 and 3) and $\Theta = 0.0057(L_x^2 + L_y^2 + L_z^2)/D_0$ (for curves 2 and 4). Difference between values of dopant diffusion coefficient in layers of heterostructure increases with increasing of number of curves. Dopant diffusion coefficient in the substrate is smaller than in epitaxial layer. Squares are the experimental data from [23]. Circles are the experimental data from [24]

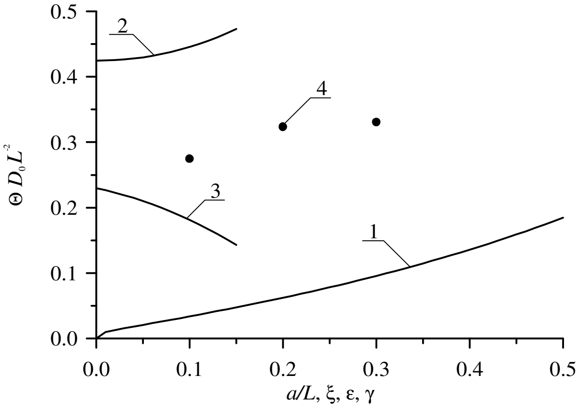

Fig.3*a*. Dependences of dimensionless optimal annealing time of infused dopant. Curve 1 describes dimensionless optimal annealing time as the function of the relation a/L for $\xi = \gamma = 0$ and for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 describes dimensionless optimal annealing time as the function of the parameter ε for a/L=1/2 and $\xi = \gamma = 0$ and for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 3 describes dimensionless optimal annealing time as the function of the parameter ξ for a/L=1/2 and $\varepsilon = \gamma = 0$ and for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 4 describes dimensionless optimal annealing time as the function of the parameter γ for a/L=1/2 and $\varepsilon = \xi = 0$ and for equal to each other values of dopant diffusion coefficient in all parts of heterostructure

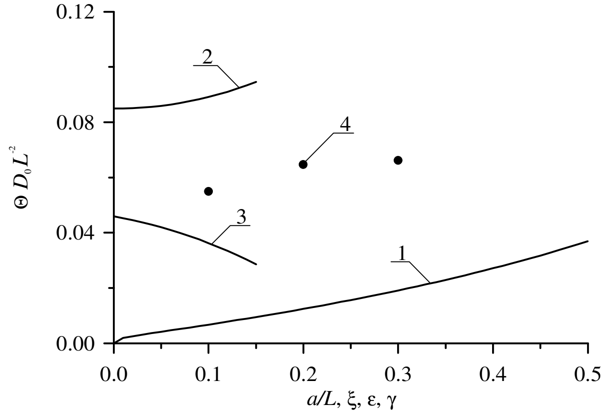

Fig.3*b*. Dependences of dimensionless optimal annealing time of implanted dopant. Curve 1 describes dimensionless optimal annealing time as the function of the relation *a/L* for $\xi = \gamma = 0$ and for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 describes dimensionless optimal annealing time as the function of the parameter ε for *a/L*=1/2 and $\xi = \gamma = 0$ and for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 3 describes dimensionless optimal annealing time as the function of the parameter ξ for *a/L*=1/2 and $\varepsilon = \gamma = 0$ and for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 4 describes dimensionless optimal annealing time as the function of the parameter γ for *a/L*=1/2 and $\varepsilon = \xi = 0$ and for equal to each other values of dopant diffusion coefficient in all parts of heterostructure

$$U = \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \left[C(x,y,z,\Theta) - \psi(x,y,z) \right] d\,z\,d\,y\,d\,x. \tag{8}$$

Dependences of optimal annealing time are presented on Figs. 3 for both types of doping (diffusion and ion types). It is known, that ion doping of materials leads to necessity to anneal of radiation defects. The annealing leads to spreading of concentration of distribution of dopant during this annealing. In the ideal case distribution of dopant achieves appropriate interfaces between materials of heterostructure during annealing of radiation defects. It should be noted, that it is practicably to additionally anneal the dopant in the case when dopant did not achieves any interfaces during annealing of radiation defects. At the same time ion type of doping gives us possibility to decrease mismatch-induced stress in heterostructure [25].

4. CONCLUSIONS

We analyzed redistribution of infused and implanted dopants during manufacturing a heterotransistors with two sources. We formulate several recommendations to optimize manufacturing of the heterotransistors to decrease their dimensions. Analytical approach for prognosis of diffusion and ion types of doping with account simultaneous changing of parameters in space and time has been introduced. At the same time nonlinearity of the doping processes could be taken into account by the approach.

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APPENDIX

We solved the considered boundary problems by method of averaging of function corrections [13] with decreased quantity of iteration steps [14]. Framework the approach we consider the initial iterations of these solutions as solutions of linear Eqs. (1), (4) and (6) with averaged values of diffusion coefficients D_{0L}, D_{0I}, D_{0V}, $D_{0\Phi I}$, $D_{0\Phi V}$:

$$C_1(x,y,z,t)=\frac{F_{0C}}{L_xL_yL_z}+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{nC}c_n(x)c_n(y)c_n(z)e_{nC}(t),$$

$$I_1(x,y,z,t)=\frac{F_{0I}}{L_xL_yL_z}+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{nI}c_n(x)c_n(y)c_n(z)e_{nI}(t),$$

$$V_1(x,y,z,t)=\frac{F_{0C}}{L_xL_yL_z}+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{nC}c_n(x)c_n(y)c_n(z)e_{nV}(t),$$

$$\Phi_{I1}(x,y,z,t)=\frac{F_{0\Phi_I}}{L_xL_yL_z}+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{n\Phi_I}c_n(x)c_n(y)c_n(z)e_{n\Phi_I}(t),$$

$$\Phi_{V1}(x,y,z,t)=\frac{F_{0\Phi_V}}{L_xL_yL_z}+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{n\Phi_V}c_n(x)c_n(y)c_n(z)e_{n\Phi_V}(t),$$

where $e_{n\rho}(t)=\exp\left[-\pi^2n^2D_{0\rho}t\left(\frac{1}{L_x^2}+\frac{1}{L_y^2}+\frac{1}{L_z^2}\right)\right]$, $F_{n\rho}=\int_0^{L_x}c_n(u)\int_0^{L_y}c_n(v)\int_0^{L_z}c_n(v)f_\rho(u,v,w)\,d\,w\,d\,v\,d\,u$, $c_n(\chi)$ = cos (π n χ/L_χ). We consider the above solutions as initial-order approximations of concentrations of dopant and radiation defects.

Approximations of considered concentrations with higher orders (higher than the first-order) could be determine framework standard iterative procedure [13,14]. The procedure based on replacement of the functions $C(x,y,z,t)$, $I(x,y,z,t)$, $V(x,y,z,t)$, $\Phi_I(x,y,z,t)$, $\Phi_V(x,y,z,t)$ in the right sides of the Eqs. (1), (4) and (6) on the following sums $\alpha_{n\rho}+\rho_{n-1}(x,y,z,t)$. The standard iterative procedure leads to equations for the second-order approximations of concentrations of dopant and radiation defects

$$\begin{aligned}\frac{\partial C_2(x,y,z,t)}{\partial t}&=\frac{\partial}{\partial x}\left(\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\times\right.\\&\left.\times D_L(x,y,z,T)\frac{\partial C_1(x,y,z,t)}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_L(x,y,z,T)\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\times\right.\\&\left.\times\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\frac{\partial C_1(x,y,z,t)}{\partial y}\right)+\frac{\partial}{\partial z}\left(D_L(x,y,z,T)\frac{\partial C_1(x,y,z,t)}{\partial z}\times\right.\\&\left.\times\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\right)\end{aligned}\qquad(8)$$

$$\left\{\begin{array}{l}
\frac{\partial I_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial y}\right]+\\
+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial z}\right]-[\alpha_{2I}+I_1(x,y,z,t)][\alpha_{2V}+V_1(x,y,z,t)]\times\\
\times k_{I,V}(x,y,z,T)-k_{I,I}(x,y,z,T)[\alpha_{2I}+I_1(x,y,z,t)]^2\\
\frac{\partial V_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial y}\right]+\\
+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial z}\right]-[\alpha_{2I}+I_1(x,y,z,t)][\alpha_{2V}+V_1(x,y,z,t)]\times\\
\times k_{I,V}(x,y,z,T)-k_{V,V}(x,y,z,T)[\alpha_{2V}+V_1(x,y,z,t)]^2
\end{array}\right. \tag{9}$$

$$\left\{\begin{array}{l}
\frac{\partial \Phi_{I2}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{I1}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\times\right.\\
\left.\times\frac{\partial \Phi_{I1}(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{I1}(x,y,z,t)}{\partial z}\right]+k_{I,I}(x,y,z,T)\times\\
\times I^2(x,y,z,t)-k_I(x,y,z,T)I(x,y,z,t)\\
\frac{\partial \Phi_{V2}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{V1}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\times\right.\\
\left.\times\frac{\partial \Phi_{I1}(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{V1}(x,y,z,t)}{\partial z}\right]+k_{V,V}(x,y,z,T)\times\\
\times V^2(x,y,z,t)-k_V(x,y,z,T)V(x,y,z,t)
\end{array}\right. \tag{10}$$

Integration of the left and right sides of Eqs.(8)-(10) gives us possibility to obtain relations for the second-order approximations of concentrations of dopant and radiation defects in final form

$$\begin{array}{l}
C_2(x,y,z,t)=\frac{\partial}{\partial x}\left(\int\limits_0^t\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\times\right.\\
\left.\times D_L(x,y,z,T)\frac{\partial C_1(x,y,z,\tau)}{\partial x}\right)d\tau+\frac{\partial}{\partial y}\left(\int\limits_0^t\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times\right.\\
\left.\times D_L(x,y,z,T)\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\frac{\partial C_1(x,y,z,\tau)}{\partial y}d\tau\right)+f_C(x,y,z)+\frac{\partial}{\partial z}\left(\int\limits_0^t D_L(x,y,z,T)\times\right.\\
\left.\times\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\frac{\partial C_1(x,y,z,\tau)}{\partial z}d\tau\right)
\end{array} \tag{8a}$$

$$\begin{array}{l}
I_2(x,y,z,t)=\frac{\partial}{\partial x}\left[\int\limits_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial x}d\tau\right]+\frac{\partial}{\partial y}\left[\int\limits_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial y}d\tau\right]+\\
+\frac{\partial}{\partial z}\left[\int\limits_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial z}d\tau\right]-\int\limits_0^t k_{I,I}(x,y,z,T)[\alpha_{2I}+I_1(x,y,z,\tau)]^2d\tau+\\
+f_I(x,y,z)-\int\limits_0^t k_{I,V}(x,y,z,T)[\alpha_{2I}+I_1(x,y,z,\tau)][\alpha_{2V}+V_1(x,y,z,\tau)]d\tau
\end{array} \tag{9a}$$

$$V_2(x,y,z,t)=\frac{\partial}{\partial x}\left[\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial x}d\tau\right]+\frac{\partial}{\partial y}\left[\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial y}d\tau\right]+$$

$$\times\frac{\partial V_1(x,y,z,\tau)}{\partial y}d\tau\Bigg]+\frac{\partial}{\partial z}\left[\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial z}d\tau\right]-\int_0^t k_{V,V}(x,y,z,T)\left[\alpha_{2I}+V_1(x,y,z,\tau)\right]^2 d\tau+$$

$$+f_V(x,y,z)-\int_0^t k_{I,V}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]d\tau$$

$$\Phi_{I2}(x,y,z,t)=\frac{\partial}{\partial x}\left[\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{I1}(x,y,z,\tau)}{\partial x}d\tau\right]+\frac{\partial}{\partial y}\left[\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{I1}(x,y,z,\tau)}{\partial y}d\tau\right]+$$

$$+\frac{\partial}{\partial z}\left[\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{I1}(x,y,z,\tau)}{\partial z}d\tau\right]+\int_0^t k_{I,I}(x,y,z,T)\,I^2(x,y,z,\tau)d\tau+$$

$$+f_{\Phi_I}(x,y,z)-\int_0^t k_I(x,y,z,T)I(x,y,z,\tau)d\tau \qquad (10a)$$

$$\Phi_{V2}(x,y,z,t)=\frac{\partial}{\partial x}\left[\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{V1}(x,y,z,\tau)}{\partial x}d\tau\right]+\frac{\partial}{\partial y}\left[\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{V1}(x,y,z,\tau)}{\partial y}d\tau\right]+$$

$$+\frac{\partial}{\partial z}\left[\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{V1}(x,y,z,\tau)}{\partial z}d\tau\right]+\int_0^t k_{V,V}(x,y,z,T)\,V^2(x,y,z,\tau)d\tau+$$

$$+f_{\Phi_V}(x,y,z)-\int_0^t k_V(x,y,z,T)V(x,y,z,\tau)d\tau$$

Average values of the considered approximations have been determined by the following relations [13,14]

$$\alpha_{2\rho}=\frac{1}{\Theta L_xL_yL_z}\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[\rho_2(x,y,z,t)-\rho_1(x,y,z,t)\right]dz\,dy\,dx\,dt. \qquad (11)$$

Substitution of approximations (8*a*)-(10*a*) into the previous relation gives the possibility to obtain relations for the average values $\alpha_{2\rho}$ in the following final form

$$\alpha_{2C}=\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_C(x,y,z)dz\,dy\,dx, \qquad (12)$$

$$\alpha_{2I}=\frac{1}{2A_{II00}}\Bigg\{\left(1+A_{IV01}+A_{II10}+\alpha_{2V}A_{IV00}\right)^2-4A_{II00}\Bigg[\alpha_{2V}A_{IV10}-A_{II20}+A_{IV11}-$$

$$-\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)dz\,dy\,dx\Bigg]\Bigg\}^{\frac{1}{2}}-\frac{1+A_{IV01}+A_{II10}+\alpha_{2V}A_{IV00}}{2A_{II00}} \qquad (13a)$$

$$\alpha_{2V}=\frac{1}{2B_4}\sqrt{\frac{(B_3+A)^2}{4}-4B_4\left(y+\frac{B_3y-B_1}{A}\right)}-\frac{B_3+A}{4B_4}, \qquad (13b)$$

Here $A_{abij}=\frac{1}{\Theta L_xL_yL_z}\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{a,b}(x,y,z,T)I_1^i(x,y,z,t)V_1^j(x,y,z,t)dz\,dy\,dx\,dt$, $B_4=A_{IV00}^2A_{IV00}^2-$
$-2\left(A_{IV00}^2-A_{II00}A_{VV00}\right)^2$, $B_3=A_{IV00}A_{IV00}^2+A_{IV01}A_{IV00}^3+A_{IV00}A_{II10}A_{IV00}^2-4\left(A_{IV00}^2-A_{II00}A_{VV00}\right)\left[A_{IV01}\times\right.$
$\left.\times 2A_{IV00}+2A_{IV00}\left(1+A_{IV01}+A_{II10}\right)-2A_{II00}\left(A_{IV10}+A_{VV10}+1\right)\right]-4A_{IV10}A_{II00}A_{IV00}^2+2A_{IV00}A_{IV01}A_{IV00}^2$,
$B_2=A_{IV00}^2\left\{\left(1+A_{IV01}+A_{II10}\right)^2+A_{IV00}^2A_{IV01}^2+2A_{IV00}A_{IV00}\left(A_{IV00}+A_{IV00}A_{IV01}+A_{IV00}A_{II10}-4A_{IV10}A_{II00}\right)-\right.$

$$-4A_{II00}\left[A_{IV11}-A_{II20}-\frac{1}{L_xL_yL_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_I(x,y,z)\,d\,z\,d\,y\,d\,x\right]\Bigg\}\Big\{\big[2A_{IV01}A_{IV00}+2A_{IV00}(1+A_{IV01}+A_{II10})-$$

$$-2A_{II00}(A_{IV10}+1+A_{VV10})\big]^2+2\left[A_{IV01}(1+A_{IV01}+A_{II10})+\frac{2}{L_xL_yL_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_V(x,y,z)\,d\,z\,d\,y\,d\,x-2A_{II00}\times\right.$$

$$\times(A_{VV20}-A_{IV11})+A_{IV01}(1+A_{IV01}+A_{II10})\big]\big[2A_{IV00}(1+A_{IV01}+A_{II10})+2A_{IV01}A_{IV00}-2(A_{IV10}+1+A_{VV10})\times$$

$$\times A_{II00}\big]\Big\},\ B_1=2A_{IV00}A_{IV01}(1+A_{IV01}+A_{II10})^2-8\left[A_{IV11}-A_{II20}-\frac{1}{L_xL_yL_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_I(x,y,z)\,d\,z\,d\,y\,d\,x\right]+$$

$$+A_{IV00}A_{IV01}A_{II00}+A_{IV01}^2(A_{IV00}+A_{IV00}A_{IV01}+A_{IV00}A_{II10}-4A_{IV10}A_{II00})-2\big[A_{IV01}(1+A_{IV01}+A_{II10})+$$

$$\left.+\frac{2A_{II00}}{L_xL_yL_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_I(x,y,z)\,d\,z\,d\,y\,d\,x-2A_{II00}(A_{VV20}-A_{IV11})+A_{IV01}(1+A_{IV01}+A_{II10})\right]\big[2A_{IV00}(1+A_{IV01}+$$

$$+A_{II10})-2(A_{IV10}+A_{VV10}+1)A_{II00}+2A_{IV01}A_{IV00}\big],\ B_0=4\left[\frac{1}{L_xL_yL_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_I(x,y,z)\,d\,z\,d\,y\,d\,x-A_{IV11}+\right.$$

$$+A_{II20}\big]A_{II00}A_{IV01}^2+A_{IV01}^2(A_{IV01}+A_{II10}+1)^2-\big[A_{IV01}(1+A_{IV01}+A_{II10})-2A_{II00}(A_{VV20}-A_{IV11})+A_{IV01}\times$$

$$\left.\times(1+A_{IV01}+A_{II10})+\frac{2A_{II00}}{L_xL_yL_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_V(x,y,z)\,d\,z\,d\,y\,d\,x\right]^2,\ y=\sqrt[3]{\sqrt{q^2+p^3}-q}-\sqrt[3]{\sqrt{q^2+p^3}+q}+\frac{B_2}{6},$$

$$q=(2B_1B_3-8B_0)\frac{B_2}{48}+\frac{B_2^3}{216}+\frac{B_0(4B_2-B_3^2)-B_1^2}{8},\ p=\frac{1}{72}\big[3(2B_1B_3-8B_0)-2B_2^2\big],\ A=\sqrt{8y+B_3^2-4B_2},$$

$$\alpha_{2\Phi_I}=A_{II20}-\frac{1}{\Theta L_xL_yL_z}\int\limits_0^{\Theta}(\Theta-t)\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}k_I(x,y,z,T)I(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t+$$

$$+\frac{1}{L_xL_yL_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_{\Phi I}(x,y,z)\,d\,z\,d\,y\,d\,x \quad (14)$$

$$\alpha_{2\Phi_V}=A_{VV20}-\frac{1}{\Theta L_xL_yL_z}\int\limits_0^{\Theta}(\Theta-t)\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}k_V(x,y,z,T)V(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t+$$

$$+\frac{1}{L_xL_yL_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_{\Phi V}(x,y,z)\,d\,z\,d\,y\,d\,x.$$

After the substitution we obtain the equation for parameter α_{2C} for any value of parameter γ. We analyzed distributions of concentrations of dopant and radiation defects in space and time by using the second-order approximations framework the method of averaged of function corrections. The obtained analytical results have been checked by comparison with results of numerical simulation.

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