Hybrid Coding Gain and Hybrid ARQ Based on Conception of Majority Voting

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Abstract. Based on the conception of majority voting, a hybrid ARQ scheme for binary linear block codes over AWGN channel is presented. At each transmission, the received word is kept in memory buffers if errors are detected. If there are still errors after a number of transmissions, the re-transmission stops to improve the throughput efficiency. From the received words, the decoding process of proposed hybrid ARQ first determines each bit as 1 or 0 based on the majority of the corresponding bits in the received words. The word resulting from voting is then decoded by algebraic decoder as the output code word. Simulation results show that the proposed hybrid ARQ scheme outperforms ARQ scheme in both error-correcting performance and throughput efficiency, particularly under low SNR. Additionally, a criterion in terms of throughput and coding gain is proposed for overall evaluation of hybrid ARQ schemes compared to that of ARQ scheme. Comparisons between plurality-based hybrid ARQ and majority-based hybrid ARQ are also made on the basis of proposed hybrid coding gain.

Keywords: hybrid ARQ, throughput, majority voting, coding gain.

1 Introduction

Basically, communications between people are misunderstood by one another most of the time. Hence, repetitions are required for better understanding. Likewise, asking for re-transmission in digital communication is a way to guarantee receiving the correct information, particularly for something that can not be misinterpreted such as money transfer, launching missile, operating nuclear power. However, time is limited in some cases. Hence, many have proposed different kinds of strategy to improve the throughput efficiency of hybrid ARQ (Auto-Repeat Request) schemes In general, there are two major principles to improve throughput efficiency. The best way is to find a threshold which can discriminate with very high probability whether the received word is in error or not and so to determine if re-transmission is required. However, it is hard to find such a threshold. A stringent threshold always results in an inefficient system. On the other hand, a loose threshold reduces the error-correcting performance. The other way of improving the throughput is collecting useful information at each transmission to make an early stop of re-transmission. Then, throughput efficiency can be improved. So, many studies proposed hybrid ARQ algorithms utilize accumulation of SNR [1,2,3] or increment of parity bits at each re-transmission [4]. Some algorithms base on adaptive code rates [5,6], on time out technique with a threshold of path metrics [7], and on cascaded encoding [8] to improve throughput efficiency. Other algorithms base on criteria such as decoding failure interpreted as a re-transmission [9], lack of orthogonal check sums [10], or bits with low reliabilities for which fail to converge during the iterations [11]. More algorithms exploit the unique structures of particular codes such as low-density parity check codes (LDPC) [12], generalized burst-trapping codes [13], zigzag code [14], and KM code [15]. Recently, one proposed the hybrid ARQ based on the conception of plurality voting [16]. In stead of voting by code words, this paper extends the idea of voting by bits proposed in [17] with more simulation results and comparisons between the two which will be presented in more detail in the subsequent sections.

2 Hybrid ARQ Based on Majority Voting

For convenience, the acronym MVHARQ is used to represent the proposed majority voting hybrid ARQ scheme. Suppose that a code $C(n, k, d_{min})$ is used for error-correcting, where n denoted length of code word, k message length, and d_{min} minimum distance between code words. Consider a code word $X=[x_1\ x_2\ \ldots x_n]$ transmitted using BPSK modulation technique over additive white Gaussian Noise channel. And $Y=X+AWGN=[\ y_1\ y_2\ \ldots y_n]$ denotes the received signal. By hard decision as (1), the received word is then $R=[\ r_1\ r_2\ \ldots r_n]$.

$$r_i = HD(y_i) = \begin{cases} 0, & y_i \le 0 \\ 1, & y_i > 0 \end{cases} \tag{1}$$

The MVHARQ scheme operates as the following and a simplified block diagram of decoding flow as shown in Figure 1 illustrates the decoding steps.

Step1. Check if any error occurred by R mod G over GF(2), where G the generating polynomial.

Step2. If the syndrome is zero, R is the output code word. If not, ask for re-transmission and design a number of threshold η for an upper bound of re-transmission.

Step3. Keep the received words of each transmission in memory buffers. When the number of transmissions reaches the threshold η, re-transmission stops.

Step4. Decode in sense of majority.

Assume that the following words were received, where r_{ij} denoted the jth bit of ith received word and R_M denoted the word resulting from majority voting.

$R_1=[\ r_{11}\ r_{12}\ \ldots r_{1n}]$

$R_2=[\ r_{21}\ r_{22}\ \ldots r_{2n}]$

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$R_\eta=[\ r_{\eta 1}\ r_{\eta 2}\ \ldots r_{\eta n}]$

$R_M=[\ r_{M1}\ r_{M2}\ \ldots r_{Mn}]$, where $r_{Mj}=maj(r_{1j}\ r_{2j} \ldots r_{\eta j})$

Step5. Decode R_M by hard decision decoding (HDD).

$C_M=HDD(R_M)$.

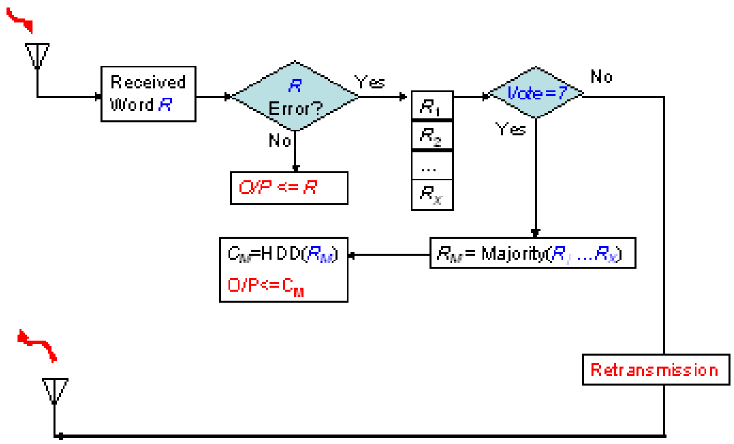

Fig. 1. Simplified decoding flow of MVHARQ

Example: For simplicity, suppose that BCH(15,7,5) is used, re-transmission threshold η is 7, and the following words are received.

R_1=[1 1 0 1 1 1 0 1 0 1 1 0 0 1 0]
R_2=[1 1 0 1 0 1 0 1 0 1 1 0 0 0 0]
R_3=[1 1 0 0 1 0 0 1 0 1 0 1 1 1 1]
R_4=[0 1 0 1 1 1 0 1 0 1 1 1 1 0 1]
R_5=[1 0 1 1 1 0 0 1 1 1 0 1 0 1 0]
R_6=[1 1 0 0 1 1 1 0 0 1 1 1 1 0 1]
R_7=[1 0 1 1 0 1 0 1 0 0 1 0 1 1 0]
Then, the word resulting from majority voting is
R_M=[1 1 0 1 1 1 0 1 0 1 1 1 1 1 0]
R_M is then decoded by a hard decision algebraic decoder and the decoded code word C_M is as follow.
C_M=*HDD*(R_M)= [0 1 1 0 0 0 1 1 0 1 1 1 1 1 0]

3 Simulation Results

The notation HARQxx used in the simulation results represent the performances of convectional hybrid ARQ schemes, where the first number behind the HARQ denoted the capability to detect the number of errors and the second one denoted the capability to correct the number of errors. For instance based on Golay code (23,12,7)[18], the convectional ARQ scheme is equivalent to HARQ60 which can detect 6-error and correct no error. In addition, the number between V and H in the notation MVxHARQ denotes the number of votes. In other words, the number means the maximum number of transmissions. Referring to Figure 2 cited from [17], the proposed MVHARQ schemes outperform the HARQ60 scheme in error-correcting performance by around 0.5 dB of coding gain at block error rate (BER) 10^{-5} based on perfect code Golay(23,12,7). In case of other non-perfect code such as BCH(15,7,5), the proposed MVHARQ schemes still effectively work. As shown in Figure 3 cited from [17], coding gain is improved by around 0.3 dB at BER 10^{-3}.

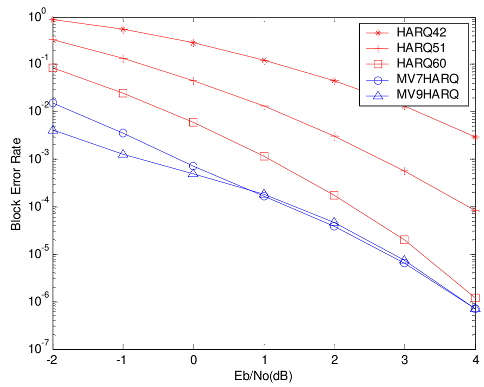

Fig. 2. Error correcting performances bases on selective repeat ARQ using Golay(23,12,7) code, where HARQxx in red are convectional schemes, and MVxHARQ in blue are the proposed schemes with threshold x.

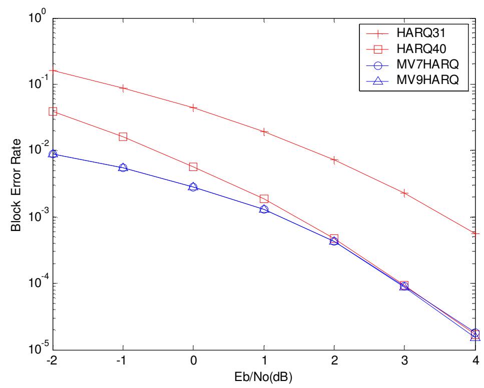

Fig. 3. Error correcting performances bases on selective repeat ARQ using BCH(15,7,5) code, where HARQs in red are convectional schemes, and MVxHARQ in blue are the proposed schemes with threshold x.

Furthermore, the throughput efficiency of proposed MVHARQ is defined as (2), the inverse of 1 plus the number of re-transmissions[16].

$$Throughput \quad (\%) = \frac{1}{1 + \# \, retransmis \; sions} \tag{2}$$

In case of FEC scheme, there is no re-transmission and the throughput becomes

$$Throughput \quad (\%) = \frac{1}{1+0} = 100\% \tag{3}$$

As illustrated in Figure 4 and Figure 5 cited from [17], the throughput efficiency of proposed MVHARQ is improved with respect to that of HARQ60 and HARQ40 schemes based both on Golay (23,12,7) and on BCH(15,7,5) respectively.

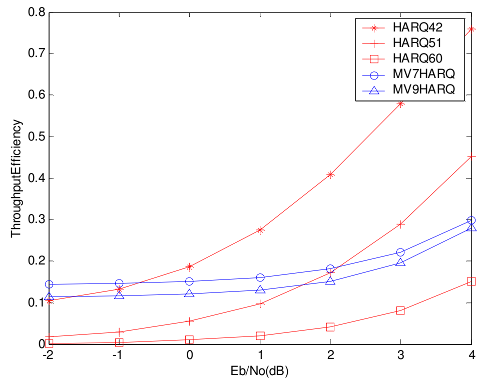

Fig. 4. Throughput efficiency bases on selective repeat ARQ using Golay(23,12,7) code, where all curves are normalized to that of the FEC scheme and corresponding to the associated curves in Fig. 2.

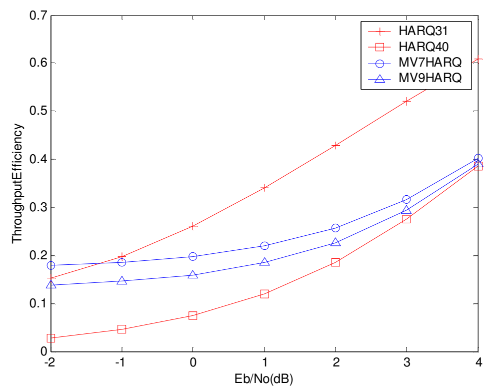

Fig. 5. Throughput efficiency bases on selective repeat ARQ using BCH(15,7,5) code, where all curves are normalized to that of the FEC scheme and corresponding to the associated curves in Fig.3.

4 Overall Evaluation in Terms of HCG

Normally, a hybrid ARQ scheme with higher throughput efficiency results in lower error-correcting whereas demanding error-correcting performance degrades the throughput. Therefore, some hybrid ARQ schemes outperform others in error-correcting performance while some exceed others in throughput efficiency. There is no standard to judge which scheme is better than the other as a whole. In consideration of error-correcting and throughput efficiency, we define an equation as (4) to evaluate the overall performance of a hybrid ARQ scheme and term it as hybrid coding gain (HCG) on basis of ARQ scheme.

$$HCG = \frac{CG_{vsARQ}}{1 - Throughput_{vs\,\mathrm{ARQ}}} \tag{4}$$

In (4), the CG_{vsARQ} is defined as the coding gain (CG) with respect to that of ARQ scheme at a specific BER and the $Throughput_{vsARQ}$ is the throughput improved with respect to that of ARQ at the Eb/No where the curve of MVxHARQ interests with the line of specific BER. Put it another way, Figure 6 pictorially defines the HCG. By example of proposed MV7HARQ using Golay(23,12,7) as shown in Figure 2, the CG_{vsARQ} at BER=10^{-5} is about 3.5dB – 3dB = 0.5dB. Referred to Figure 3, the $Throughput_{vsARQ}$ of MV7HARQ is around 22% - 9% = 13%. Hence, The HCG of MV7HARQ is $\frac{0.5\,dB}{1-13\%} \approx 0.57\,dB$. Likewise, the HCG using BCH code is around $\frac{0dB}{1-1\%} \approx 0dB$ at BER=10^{-5}. In measure of HCG, it concludes that using Golay(23,12,7) attains more profit than using BCH(15,7,5) in the proposed hybrid ARQ scheme. In spite of the decoding complexity, HCG is a useful benchmark to evaluate a system's performances as a whole.

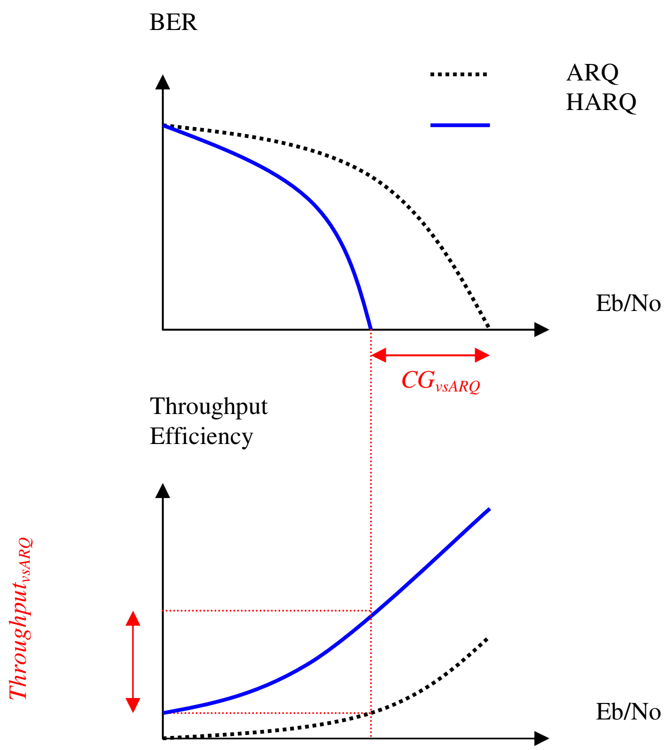

Fig. 6. Definition of CG_{vsARQ} & $Throughput_{vsARQ}$

5 Comparisons between Plurality Based and Majority Based HARQ

In [16], it has proposed the HARQ based on the conception of plurality voting. In that, decoding decision is made on the plurality of code word candidates. Unlike plurality voting, the majority voting HARQ operates on bits of received words. Referring to Figure 7 and Figure 8 with the case of V3-HARQ, the improved throughput is around 20% and the improved error-correcting performance is around 0.5dB coding gain. So, the HCG of HARQ scheme based on plurality voting at BER of 10^{-4} is around 0.625. Likewise, the HARQ scheme based on majority voting at BER of 10^{-4} with the case of MV7 has a little bit differences in error-correcting and throughput by reference of Figure 2 and Figure 4. The HCG is around 0.705. Such outcomes mean that both schemes have almost the same performances.

Table 1 summarizes the strengths and the weakness between the two schemes based on Golay code(23,12,7).

Performances	Plurality-based HARQ	Majority-based HARQ
Throughput	≈20%	≈15%
Error-Correcting	≈0.5dB	≈0.6dB
HCG	≈0.625dB	≈0.705dB

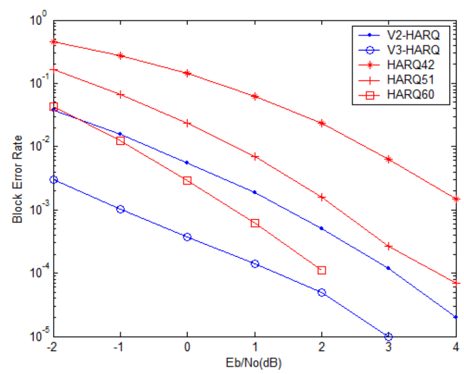

Fig.7 Error correcting performances bases on selective repeat ARQ using Golay(23,12,7) code, where HARQxx in red are convectional schemes, and VxHARQ in blue are the HARQ schemes of plurality voting with vote number x.

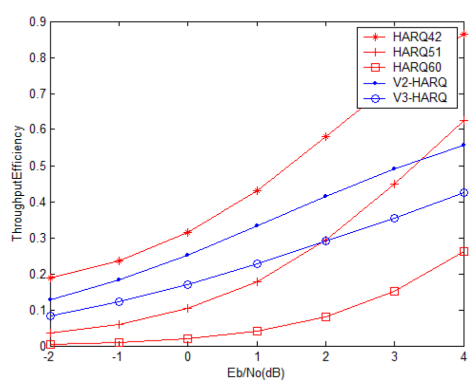

Fig.8 Throughput efficiency bases on selective repeat ARQ using Golay(23,12,7) code, where all curves are normalized to that of the FEC scheme and corresponding to the associated curves in Fig. 7.

6 Conclusion

As demonstrated in the simulation results, the proposed MVHARQ schemes exceed ARQ scheme in both error-correcting performance and throughput efficiency, particularly under negative Eb/No. As a saying is "No pain no gain". The improvements of system performance are at the expense of decoding complexity. In aspect of space complexity, memory buffers are required to keep received words at each transmission for the proposed MVHARQ scheme compared to that of ARQ scheme. Additionally, a hardware unit is necessary to find the majority of received words from bit to bit. Accordingly, time complexity is increased in finding the majority bits. Finally, a HDD is inevitable as well to determine the output code word. Without the concern of decoding complexity, the proposed MVHARQ scheme outperforms the conventional ARQ scheme in standard of HCG. In comparison with the HARQ scheme based on plurality voting proposed in [16], the proposed HARQ scheme based on majority voting from bit to bit has almost the performances in sense of HCG. Also, the HCG can be used to evaluate other hybrid ARQ schemes of different kind and to determine which kind of system is desirable, despite of decoding complexity.

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