FURTHER RESULTS ON THE JOINT TIME DELAY AND FREQUENCY ESTIMATION WITHOUT EIGEN-DECOMPOSITION

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ABSTRACT

Joint Time Delay and Frequency Estimation (JTDFE) problem of complex sinusoidal signals received at two separated sensors is an attractive problem that has been considered for several engineering applications. In this paper, a high resolution null (noise) subspace method without eigenvalue decomposition is proposed. The direct data Matrix is replaced by an upper triangular matrix obtained from Rank-Revealing LU (RRLU) factorization. The RRLU provides accurate information about the rank and the numerical null space which make it a valuable tool in numerical linear algebra. The proposed novel method decreases the computational complexity of JTDFE approximately to the half compared with RRQR methods. The proposed method generates estimates of the unknown parameters which are based on the observation and/or covariance matrices. This leads to a significant improvement in the computational load. Computer simulations are included in this paper to demonstrate the proposed method.

KEYWORDS

Time delay, Frequency estimation, Subspace estimation method, Rank revealing, Matrix decomposition, LU factorization, QR factorization, Signal space, Null (noise) space, MUSIC, root-MUSIC.

1. INTRODUCTION

A precise Time Delay Estimation (TDE) between two or more noisy versions of the same signal received at spatially separated sensors is an essential subject that has been used in many applications such that positioning and tracking, speed sensing, direction finding, biomedicine, exploration geophysics, etc [1], [2].Similarly, frequency estimation [3], [4] has been comprehensively addressed in signal processing literature. Recently, these two problems were joined as a Joint Time Delay and Frequency Estimation problem (JTDFE)[5], [6], which appeared in many applications like synchronization in Code Division Multiple Access (CDMA) systems, speech enhancement, and pitch estimation using a microphone array.

A Discrete-Time Fourier Transform (DTFT) based method has been derived for estimating the time difference of arrival between sinusoidal signals received at two separated sensors [7]. A subspace algorithm based on State-Space Realization (SSR) has been proposed for JTDFE [8]. In SSR the frequency estimates is obtained directly from the eigenvalues of the state transition matrix, while the delay is determined using the observation matrix and the estimated frequencies itself. Now, the super-resolution technologies are mainly based on subspace fitting algorithms, such as Multiple Signal Classification (MUSIC) [9], Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [10] and Generalized Eigen values Utilizing Signal Subspace Eigen vectors (GEESE) [11]. It has been proved that the performance of MUSIC and

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ESPRIT is inferior to the Maximum Likelihood (ML) estimation algorithm when they are used in the arrival angle estimation in the antennal array [12]. Besides the subspace fitting algorithms, there is another super-resolution algorithm which is based on the spectral moment estimation [13], and the performance is also superior to the MUSIC and ESPRIT. However, it is only used in the arrival angle estimation [14], and has not been introduced into the application of time-delay estimation. The null space of the JTDEF was extracted by applying the Propagator Method (PM) [15], Rank Revealing QR factorization (RRQR) [16] and an extra step was made in [16] to convert the complex matrix to a real data one via the unitary transformation of a square Toepltiz complex data matrix.

In this paper, the problem of estimating time delay and frequencies of received signal using Rank Revealing LU (RRLU) matrix decomposition [17], [18] in combination with the well-known MUSIC/root-MUSIC algorithm is addressed. It is well-known that the computational load of the LU-based method is significantly lower, as it does not involve eigenvalue decomposition (EVD) or singular value decomposition (SVD) of the cross-spectral matrix (CSM) of received signals.

In [17], [19] two theoretical approximations for computing the numerical rank of a triangular matrix were introduced. This triangular matrix can be obtained by means of the LU factorization. LU factorization implementation includes several improvements over the QR algorithm [20]. Specifically, an incremental condition estimator is employed to reduce the implementation cost. The principle is based on a RRLU factorization [17], [18]-[19] which allows extraction from the CSM, necessary information to estimate the subspace noise.

This paper is structured as follows. In Section 2, the system model and the problem formulation is presented which is similar to the models [6], [15]. The development of the proposed method is presented in Section 3. In Section 4, the performance of the method is illustrated through MATLAB simulations. A comparison with the RRQR and SSR is made. Finally, some concluding remarks follow in Section 5.

2. PROBLEM FORMULATION

Consider the discrete-time sinusoidal signals $x(n)$ and $y(n)$which are two sensors measurements that satisfying

$$\begin{aligned} x(n) &= s(n) + u(n) \\ y(n) &= s(n-D) + z(n),\ n = 0,1,\ldots,N-1 \end{aligned} \tag{1}$$

Where

$$s(n) = \sum_{i=1}^{P} a_i e^{j\omega_i n} \tag{2}$$

The source signal $s(n)$is demonstrated by a sum of P complex sinusoids where the amplitudes (a_i) are unknown and complex-valued constants. The normalized radian frequencies (ω_i) are different for every *i* and has been arranged in ascending order as $\omega_1 < \omega_2 \ldots < \omega_P$without loss of generality. To simplify the problem, it is assumed that, the number of sources *P* either known or pre estimated [21]. The two terms $u(n)$and $z(n)$are representing the two zero mean, additive white complex Gaussian noise processes independent of each other. Also, parameters *N* represent the number of samples collected at each channel. The variable D is the delay between the received copies of the signals$s(n)$ at the two separated sensors, which is unknown and is to be estimated. The received data at the first and the second sensor with *N* available samples will be collected in the vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ respectively, so the problem becomes a problem of estimation of both the frequencies and the time delay from two N-points vectors given by:

$$\boldsymbol{x} = [x(0), x(1), \dots \dots \dots x(N-1)]^T \tag{3}$$

$$\boldsymbol{y} = [y(0), y(1), \dots \dots \dots y(N-1)]^T \tag{4}$$

3. DEVELOPMENT OF PROPOSED METHOD

The development of the proposed method is divided into two parts. In the first part, the frequencies are estimated using the received data at the first sensor and by applying the RRLU method with the root-MUSIC [22] algorithm. In the second part, the received data at the both sensors and the estimated frequencies in the first part are used to extract the time delay information by calculating P eigenvalues using RRLU one more time.

3.1Frequency Estimation

Using the received data at the first sensor given by (3), a Hankel Matrix of size $L \times (N-L+1)$ is constructed as:

$$\boldsymbol{X} = \begin{bmatrix} x(0) & x(1) & \dots & x(N-L) \\ x(1) & x(2) & \dots & x(N-L+1) \\ \vdots & \vdots & \ddots & \vdots \\ x(L-1) & x(L) & \dots & x(N-1) \end{bmatrix} \tag{5}$$

The parameter L which controls the number of rows and columns in the matrix $\boldsymbol{X}$ should satisfy:

$$P + 1 \leq L \leq N - P - 1 \tag{6}$$

In other words the row rank and the column rank should be at least $P+1$. The matrix $\boldsymbol{X}$ can be rewritten as

$$\boldsymbol{X} = [\boldsymbol{x_0} \quad \boldsymbol{x_1} \quad \boldsymbol{x_2} \quad \cdots \quad \boldsymbol{x_{N-L}}] \tag{7}$$

Where the i^{th} column of $\boldsymbol{X}$ is given by:

$$\boldsymbol{x_i} = [x(i), x(i), \dots \dots \dots x(L+i-1)]^T \tag{8}$$

and it can be written as:

$$\boldsymbol{x_i} = \boldsymbol{A_L}(\boldsymbol{\omega})(\boldsymbol{\varphi}(\boldsymbol{\omega}))^i \boldsymbol{a} + \boldsymbol{u}_i \tag{9}$$

where $i = 0,1, \dots \dots N-L$

$$\boldsymbol{A_L}(\boldsymbol{\omega}) = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{j\omega_1} & e^{j\omega_2} & \dots & e^{j\omega_P} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(L-1)\omega_1} & e^{j(L-1)\omega_2} & \dots & e^{j(L-1)\omega_P} \end{bmatrix} \tag{10}$$

$$\boldsymbol{\varphi}(\boldsymbol{\omega}) = diag\left(e^{j\omega_1} \ e^{j\omega_2} \dots \dots \ e^{j\omega_P} \right) \tag{11}$$

$$\boldsymbol{a} = [a_1, a_2, \dots \dots a_P]^T \tag{12}$$

$$\boldsymbol{u}_i = [u(i)\, u(i+1) \dots \dots u(i+L-1)]^T \tag{13}$$

The received data matrix can be expressed as

$$\boldsymbol{X} = \left[\boldsymbol{A_L(\omega)a} \quad \boldsymbol{A_L(\omega)\varphi(\omega)a} \dots \dots \boldsymbol{A_L(\omega)}\big(\boldsymbol{\varphi(\omega)}\big)^{\boldsymbol{N-L}}\boldsymbol{a}\right] + [\boldsymbol{u_0 u_1} \dots \dots\ \boldsymbol{u_{N-L}}]$$

or simply:

$$\boldsymbol{X} = \boldsymbol{A_L(\omega)}\left[\boldsymbol{I} \quad \boldsymbol{\varphi(\omega)}\big(\boldsymbol{\varphi(\omega)}\big)^{\boldsymbol{2}} \dots \big(\boldsymbol{\varphi(\omega)}\big)^{\boldsymbol{N-L}}\right]\boldsymbol{a} + [\boldsymbol{u_0 u_1} \dots \dots\ \boldsymbol{u_{N-L}}]$$

Matrix $\boldsymbol{X}$ can be expressed as product of a lower triangular matrix $\boldsymbol{L}$ (with 1's on the diagonal) of dimension $L \times L$, and an upper triangular matrix $\boldsymbol{U}$ of dimension $L \times (N-L+1)$ using LU decomposition as shown

$$\boldsymbol{X} = \boldsymbol{LU} = \begin{bmatrix} \boldsymbol{L_{11}} & \boldsymbol{0} \\ \boldsymbol{L_{21}} & \boldsymbol{L_{22}} \end{bmatrix} . \begin{bmatrix} \boldsymbol{U_{11}} & \boldsymbol{U_{12}} \\ \boldsymbol{0} & \boldsymbol{U_{22}} \end{bmatrix} \tag{14}$$

Here the sub matrix $\boldsymbol{U_{11}}$ is an upper triangular matrix of size$(P \times P)$ and the sub matrix $\boldsymbol{U_{12}}$ is of size$\big(P \times (N-L+1)\big)$. Since the norm of $\boldsymbol{U_{22}}$ is small norm, the basis of the noise space can be easily extracted using the sub matrix $\widetilde{\boldsymbol{U}} = [\boldsymbol{U_{11}} \quad \boldsymbol{U_{12}}]$.

$$\boldsymbol{X} \approx \tilde{\boldsymbol{L}}\widetilde{\boldsymbol{U}} = \begin{bmatrix} \boldsymbol{L_{11}} \\ \boldsymbol{L_{21}} \end{bmatrix} . [\boldsymbol{U_{11}} \quad \boldsymbol{U_{12}}] \tag{15}$$

Here the matrix $\widetilde{\boldsymbol{U}}$ is defined as the signal space of $\boldsymbol{X}$. Obviously, any vector belongs to null space $\boldsymbol{G}$ should satisfy

$$\boldsymbol{X.G} \approx \tilde{\boldsymbol{L}}\widetilde{\boldsymbol{U}}.\boldsymbol{G} = \begin{bmatrix} \boldsymbol{L_{11}} \\ \boldsymbol{L_{21}} \end{bmatrix} . [\boldsymbol{U_{11}} \quad \boldsymbol{U_{12}}] \begin{bmatrix} \boldsymbol{g_1} \\ \boldsymbol{g_2} \end{bmatrix} = \boldsymbol{0}$$

or simply:

$$[\boldsymbol{U_{11}} \quad \boldsymbol{U_{12}}] \begin{bmatrix} \boldsymbol{g_1} \\ \boldsymbol{g_2} \end{bmatrix} = \boldsymbol{0}$$

$$\boldsymbol{U_{11}g_1} + \boldsymbol{U_{12}g_2} = \boldsymbol{0}$$

Since $\boldsymbol{U_{11}}$ is a nonsingular matrix, $\boldsymbol{g_1}$ can be written in terms of $\boldsymbol{g_2}$ as

$$\boldsymbol{g_1} = -\boldsymbol{U_{11}^{-1}U_{12}g_2}$$

Rewrite matrix $\boldsymbol{G}$ as

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{g_1} \\ \boldsymbol{g_2} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{U_{11}^{-1}U_{12}} \\ \boldsymbol{I}_{(L-P)} \end{bmatrix} \boldsymbol{g_2} = \boldsymbol{Hg_2} \tag{16}$$

So, $\widetilde{\boldsymbol{U}}.\boldsymbol{H} = \boldsymbol{0}$. It can be observed here is that the columns of the matrix that represent the null space $\boldsymbol{H}$ are not orthonormal. To fulfill orthonormality, the orthogonal projection onto this subspace has been used in order to improve the performance by making the columns of the null space of $\boldsymbol{H}$ orthonormal.

$$\boldsymbol{Q} = \boldsymbol{H}(\boldsymbol{H}^{\boldsymbol{H}}\boldsymbol{H})^{-\boldsymbol{1}}\boldsymbol{H}^{\boldsymbol{H}} \tag{17}$$

Apply MUSIC like search algorithm [22] to estimate the frequencies using the following estimation function

$$\hat{P}_{MU}(e^{j\omega}) = \frac{1}{\boldsymbol{A}_{\boldsymbol{L}}(\boldsymbol{\omega})^{\boldsymbol{H}}\boldsymbol{Q}\boldsymbol{A}_{\boldsymbol{L}}(\boldsymbol{\omega})} \tag{18}$$

A root-MUSIC may be used instead of searching for the peaks of the estimation function in (18). The frequency estimates may be taken to be the angles of the *p* roots of the polynomial *D*(z) that are closest to the unit circle

$$D(z) = \sum_{\boldsymbol{i}=\boldsymbol{0}}^{\boldsymbol{L}-\boldsymbol{1}} \boldsymbol{V}_{\boldsymbol{i}}(z)\boldsymbol{V}_{\boldsymbol{i}}^{*}(1/z^{*}) \tag{19}$$

where $\boldsymbol{V}_{\boldsymbol{i}}(z)$ is the z-transform of the i^{th} column of the projection matrix $\boldsymbol{Q}$ [22].

3.2 Time Delay Estimation

In this section, a new technique to estimate the time delay using Hankel complex data matrices from both sensors is suggested. Estimated frequencies in (18) or (19) from part 3.1 are used to estimate the time delay information. To proceed in time delay estimation, let $\left(\frac{N}{2} \times \frac{N}{2}\right)$ Hankel matrix constructed from (4) as:

$$\boldsymbol{Y} = \begin{bmatrix} y(0) & y(1) & \dots & y(N/2-1) \\ y(1) & y(2) & \dots & y(N/2) \\ \vdots & \vdots & \ddots & \vdots \\ y(N/2-1) & y(N/2) & \dots & y(N-1) \end{bmatrix} \tag{20}$$

The matrix$\boldsymbol{X}$can be rewritten as

$$\boldsymbol{Y} = [\boldsymbol{y}_{\boldsymbol{0}} \quad \boldsymbol{y}_{\boldsymbol{1}} \quad \boldsymbol{y}_{\boldsymbol{2}} \quad \cdots \quad \boldsymbol{y}_{\boldsymbol{N}-\boldsymbol{L}}] \tag{21}$$

where the i^{th} column of $\boldsymbol{Y}$ is given by:

$$\boldsymbol{y}_{\boldsymbol{i}} = [y(i),\ y(i),\ \dots\dots\dots y(L+i-1)]^{T} \tag{22}$$

and it can be written as:

$$\boldsymbol{y}_{\boldsymbol{i}} = \boldsymbol{A}_{\boldsymbol{L}}(\boldsymbol{\omega})\boldsymbol{\Omega}(\boldsymbol{\omega},\boldsymbol{D})(\boldsymbol{\varphi}(\boldsymbol{\omega}))^{i}\boldsymbol{a} + \boldsymbol{z}_{i} \tag{23}$$

where

$$\boldsymbol{\Omega}(\boldsymbol{\omega}, \boldsymbol{D}) = diag\left(e^{-jD\omega_1} \ e^{-jD\omega_2} \ \dots \dots \ e^{-jD\omega_P} \right)$$

$$\boldsymbol{z}_i = \left[z(i) \ z(i+1) \ \dots \dots z\left(i + \frac{N}{2} - 1\right) \right]^T, \qquad i = 0,1, \dots (\frac{N}{2} - 1)$$

The received data matrix can be formulated as

$$\boldsymbol{Y} = \left[\boldsymbol{A\Omega a} \ \boldsymbol{A\Omega \varphi a} \dots \boldsymbol{A\Omega \varphi}^{(\frac{N}{2}-1)} \boldsymbol{a} \right] + [\boldsymbol{z}_0 \boldsymbol{z}_1 \dots \boldsymbol{z}_{(\frac{N}{2}-1)}] \tag{24}$$

Or simply:

$$\boldsymbol{Y} = \boldsymbol{A}_L(\boldsymbol{\omega})\boldsymbol{\Omega} \left[\boldsymbol{I} \quad \boldsymbol{\varphi}(\boldsymbol{\omega}) (\boldsymbol{\varphi}(\boldsymbol{\omega}))^{\mathbf{2}} \dots (\boldsymbol{\varphi}(\boldsymbol{\omega}))^{\boldsymbol{N-L}} \right] \boldsymbol{a} + [\boldsymbol{z}_0 \boldsymbol{z}_1 \dots \boldsymbol{z}_{(\frac{N}{2}-1)}] \tag{25}$$

Data matrix $\boldsymbol{\Gamma}$ can be created by combining the matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$ from (3) and (18) as:

$$\boldsymbol{\Gamma} = [\boldsymbol{X} \quad \boldsymbol{Y}] \tag{26}$$

Applying LU algorithm to (26)

$$\boldsymbol{\Gamma} = \boldsymbol{LU} = \begin{bmatrix} \boldsymbol{L}_{11} & \mathbf{0} \\ \boldsymbol{L}_{21} & \boldsymbol{L}_{22} \end{bmatrix} . \begin{bmatrix} \boldsymbol{U}_{11} & \boldsymbol{U}_{12} \\ \mathbf{0} & \boldsymbol{U}_{22} \end{bmatrix} \tag{27}$$

Here the sub matrix$\boldsymbol{U}_{\mathbf{11}}$is an upper triangular matrix of size $(P \times P)$ and the sub matrix $\boldsymbol{U}_{\mathbf{12}}$ is of size $\left(P \times (N - L + 1)\right)$. Since $\boldsymbol{U}_{\mathbf{22}}$ has small norm one can easily extract the basis of the noise space form matrix $\widetilde{\boldsymbol{U}} = [\boldsymbol{U}_{\mathbf{11}} \quad \boldsymbol{U}_{\mathbf{12}}]$.

$$\boldsymbol{\Gamma} \approx \tilde{\boldsymbol{L}}\widetilde{\boldsymbol{U}} = \begin{bmatrix} \boldsymbol{L}_{\mathbf{11}} \\ \boldsymbol{L}_{\mathbf{21}} \end{bmatrix} . [\boldsymbol{U}_{\mathbf{11}} \quad \boldsymbol{U}_{\mathbf{12}}] \tag{28}$$

Here the matrix $\widetilde{\boldsymbol{U}}$ is defined as the signal space of $\boldsymbol{X}$. As matrix $\boldsymbol{\Gamma}$ of rank P, the matrices $\boldsymbol{U}_{\mathbf{11}}$ and $\boldsymbol{U}_{\mathbf{12}}$ are of size $P \times L$. From (23) the P singular values of the matrix $\widehat{\boldsymbol{\Omega}}$ are corresponding to the P diagonal elements of $\boldsymbol{\Omega}$. Therefore, the time delay estimation can be found as:

$$\widehat{D} = \frac{trace\left(diag\left(\angle\left((\boldsymbol{U}_{11}^{\Gamma})^{\dagger} . \boldsymbol{U}_{12}^{\Gamma} \right) \right) \right)}{\sum_{i=1}^{P} \widehat{\omega}_{\boldsymbol{i}}} \tag{29}$$

4. SIMULATION RESULTS

In this section, the performance of the proposed method is compared with state-space realization method in [5]. In the First experiment two-sinusoidal signals with amplitudes $a_1 = a_2 = 1/\sqrt{2}$, $\omega_1 = 0.3\pi$ rad/s and $\omega_2 = 0.6\pi$ rad/s are considered. The simulation has been done under AWGN environment with different SNRs and 500 independent Monte-Carlo realizations. The number of signal samples was 200 and while L was 100. The MSE of the frequencies is defined as

$$FMSE_{dB} = 10log_{10}\left(\frac{1}{N_t P}\sum_{i=1}^{N_t}\sum_{j=1}^{P}\left(\omega_j - \widehat{\omega}_j\right)^2\right) \quad (30)$$

where $\widehat{\omega}_j$ is the estimate of ω_j, and N_t is the number of Monte-Carlo trials. The MSE of the frequencies estimate is compared with the State-Space Realization (SSR) method in [5].Figure 1 plots the MSE of the frequencies versus SNR. The achieved performance has been significantly improved, especially at SNR ≥ -3 dB compared with SSR method. Figure 2 plots the MSE of the time delays which is defined as:

$$DMSE_{dB} = 10log_{10}\left(\frac{1}{N_t}\sum_{i=1}^{N_t}\left(D - \widetilde{D}\right)^2\right) \quad (31)$$

In the second experiment, only one change has made for the first experiment; the amplitudes are assumed to be complex with $a_1 = (1+i)/\sqrt{2}$ and $a_2 = (1-i)/\sqrt{2}$. Figure 3shows that the behavior of the frequency MSE is similar to Figure 1. An intermediate performance of RRLU between SSR and RRQR is appeared. Figure 4plots the MSE of the time delay versus SNR for the same case.

A third expermement is done by using only 40 signal samples. The amplitudes is set to ($a_1 = a_2 = 1/\sqrt{2}$). The frequenciesis assumed to be ($\omega_1 = 0.3\pi$ rad/s and $\omega_2 = 0.6\pi$ rad/s). Figure 5 showes the MSE of the frequency estimation. For SNR >10dB RRLU showes an intermediate performance between both SSR and RRQR. While Figure 6 shows the MSE of the time delay estimation and for the range of SNR>10dB the three method are mergeing to the same performance. In many applications and published papers the SNR range is tested from 0dB to 20dB [23]. In this paper, the simulation results have tested a wider range from -10dB to 30dB, to check the convergence of the proposed method.

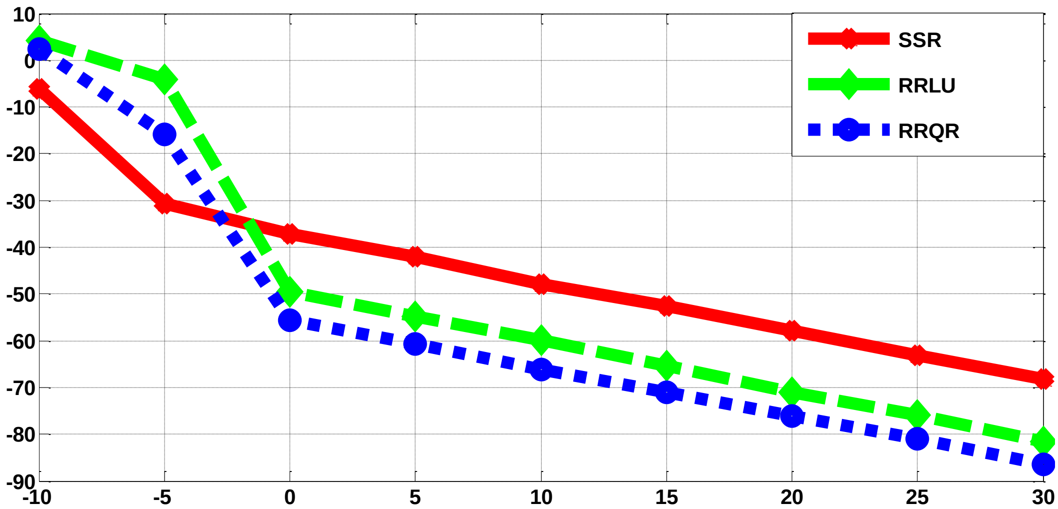

Figure 1. MSE of frequency estimation versus SNR ($a_1 = a_2 = 1/\sqrt{2}$, $\omega_1 = 0.3\pi$ rad/s and $\omega_2 = 0.6\pi$ rad/s, N=200)

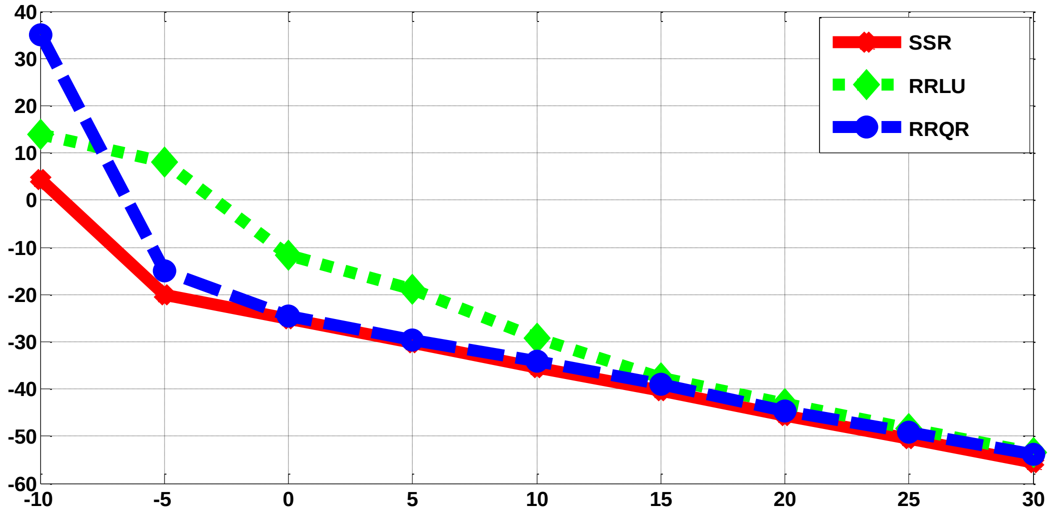

Figure 2. MSE of Delay estimation versus SNR (a_1= a_2=$1/\sqrt{2}$, $\omega_1 = 0.3\pi$ rad/s and $\omega_2 = 0.6\pi$ rad/s, N=200)

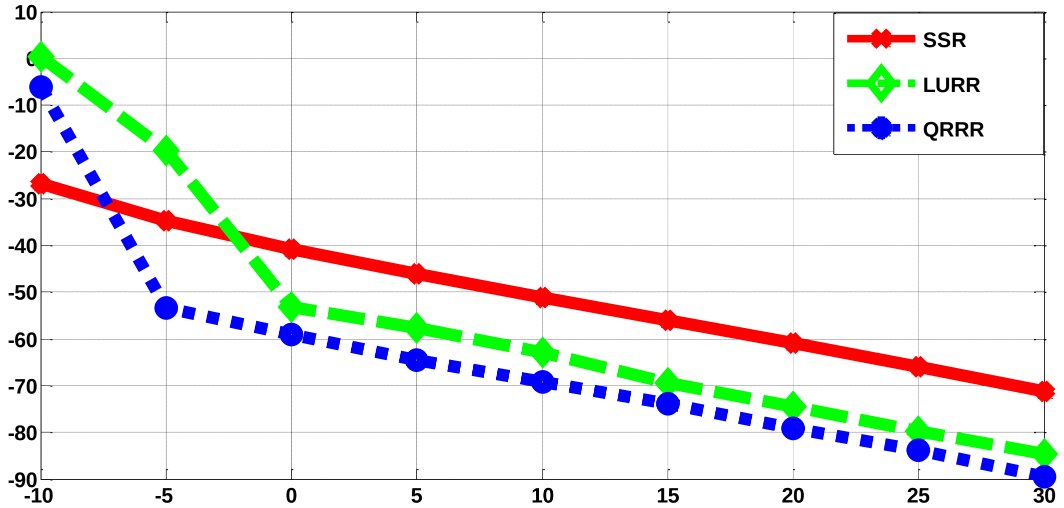

Figure 3. MSE of frequency estimation versus SNR (a_1= $(1+i)/\sqrt{2}$ and a_2= $(1-i)/\sqrt{2}$, $\omega_1 = 0.3\pi$ rad/s and $\omega_2 = 0.6\pi$ rad/s, N=200)

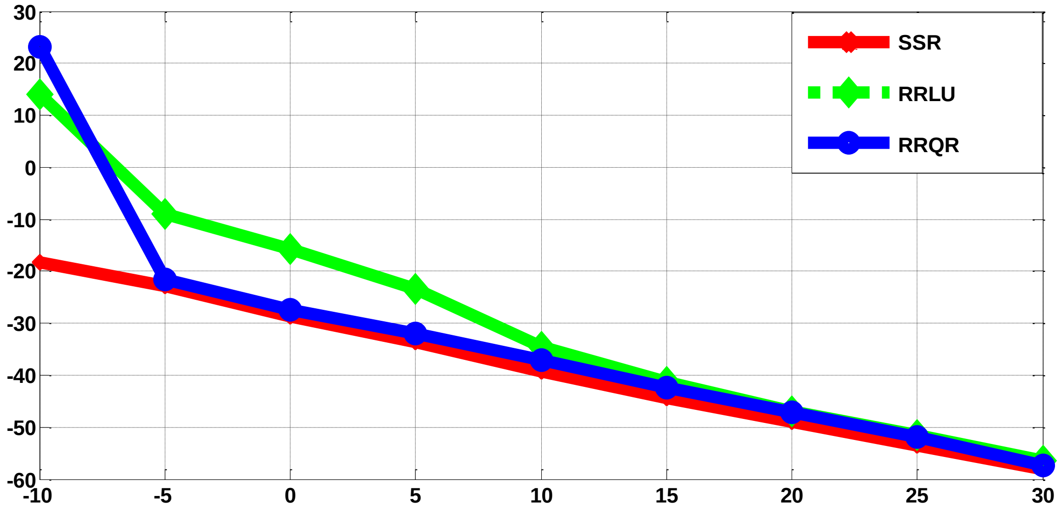

Figure 4. MSE of Delay estimation versus SNR (a_1= (1+i)/$\sqrt{2}$ and a_2= (1-i)/$\sqrt{2}$.,$\omega_1 = 0.3\pi$ rad/s and $\omega_2 = 0.6\pi$ rad/s, N=200)

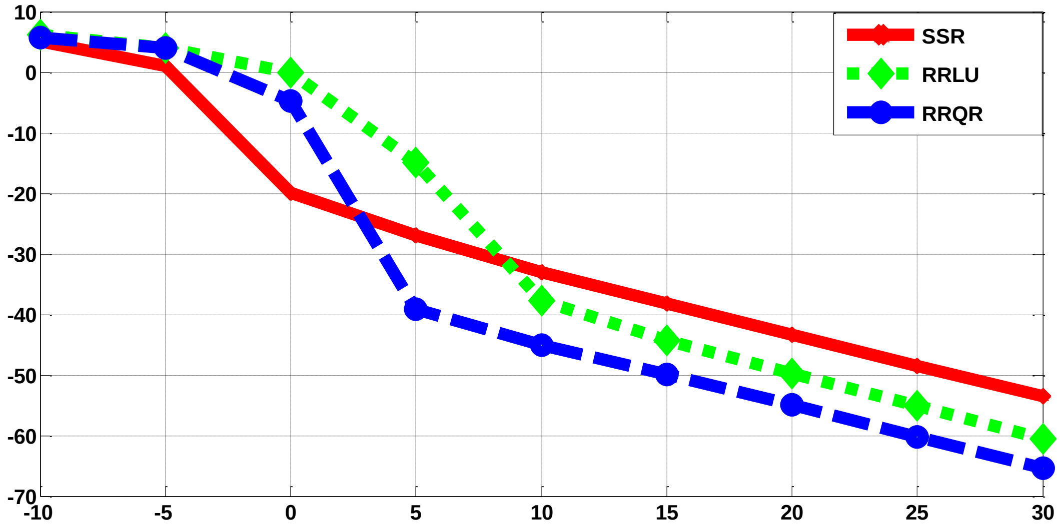

Figure 5. MSE of frequency estimation versus SNR (a_1= a_2=1/$\sqrt{2}$, $\omega_1 = 0.3\pi$ rad/s and $\omega_2 = 0.6\pi$ rad/s, N=40)

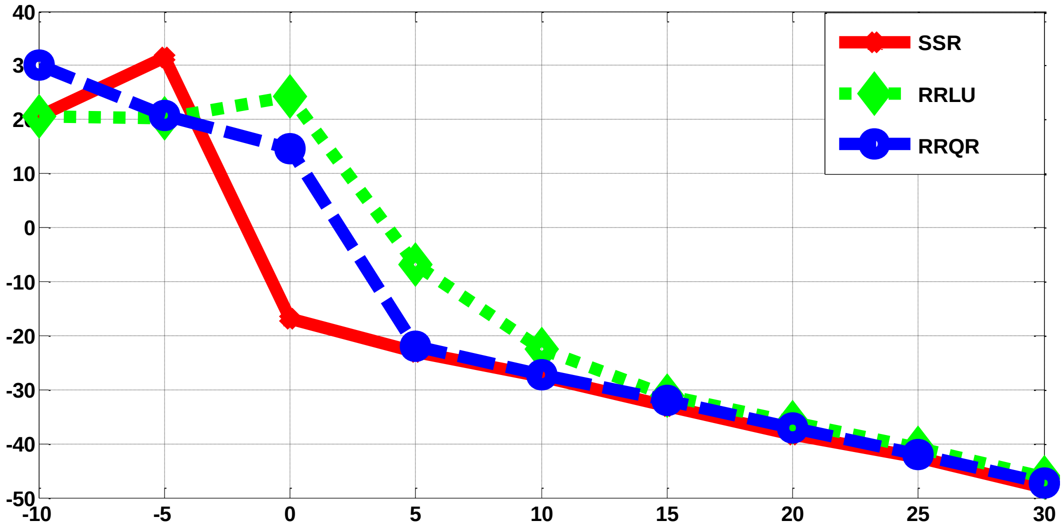

Figure 6. MSE of time delay estimation versus SNR ($a_1= a_2=1/\sqrt{2}$, $\omega_1 = 0.3\pi$ rad/s and $\omega_2 = 0.6\pi$ rad/s, N=40)

In the forth experment the amplitudes are assumed complex and set to ($a_1= (1+i)/\sqrt{2}$ *and* $a_2= (1-i)/\sqrt{2}$) , and that is the only change that made compared with thired experement. For SNR>5dB as showen in both Figures 7 and Figure 8 the performance of both frequency and time delay estimators in the three method are very close to each other, keeping in minde the superiorty of RRLU as it is less complexity.

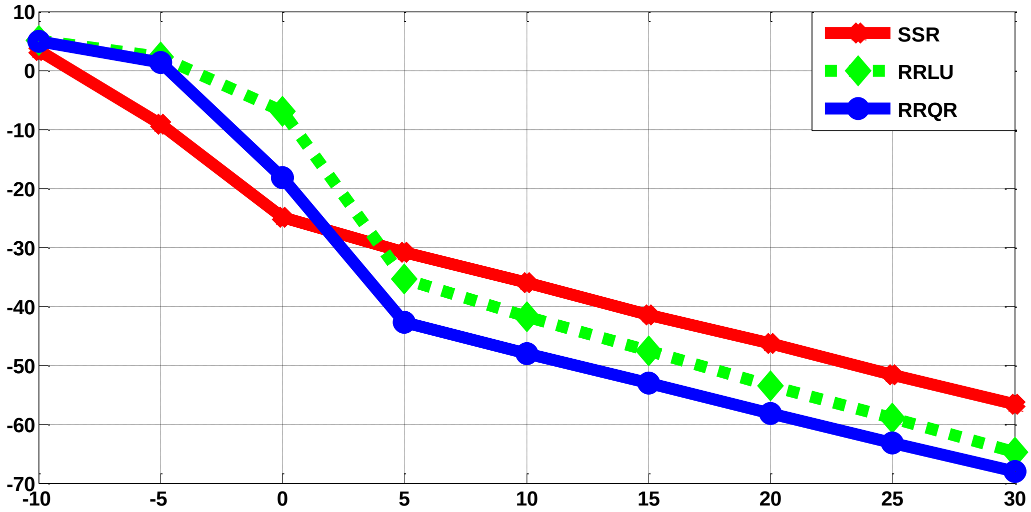

Figure 7. MSE of frequency estimation versus SNR ($a_1= (1+i)/\sqrt{2}$ *and* $a_2= (1-i)/\sqrt{2}$., $\omega_1 = 0.3\pi$ rad/s and $\omega_2 = 0.6\pi$ rad/s, N=40)

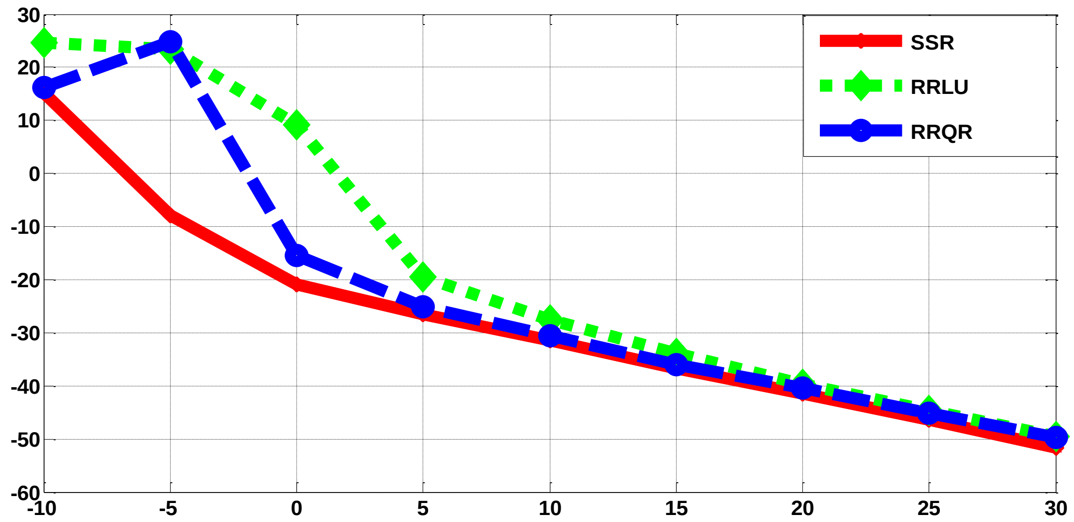

Figure 8. MSE of Delay estimation versus SNR $(a_1=(1+i)/\sqrt{2}$ *and* $a_2=(1-i)/\sqrt{2}$., $\omega_1=0.3\pi$ rad/s and $\omega_2=0.6\pi$ rad/s, N=40)

The performanceof the frequency estimator as a function of number of snapshots is illustrated in Figure 9 with contant SNR of 0 and 10 dB. The number of snapshots is varied from 40 to 400. It is obvious that a beter performance can be obtained as the number of snapshots is increased. Figure 10 plots the performance of the time delay estimator.

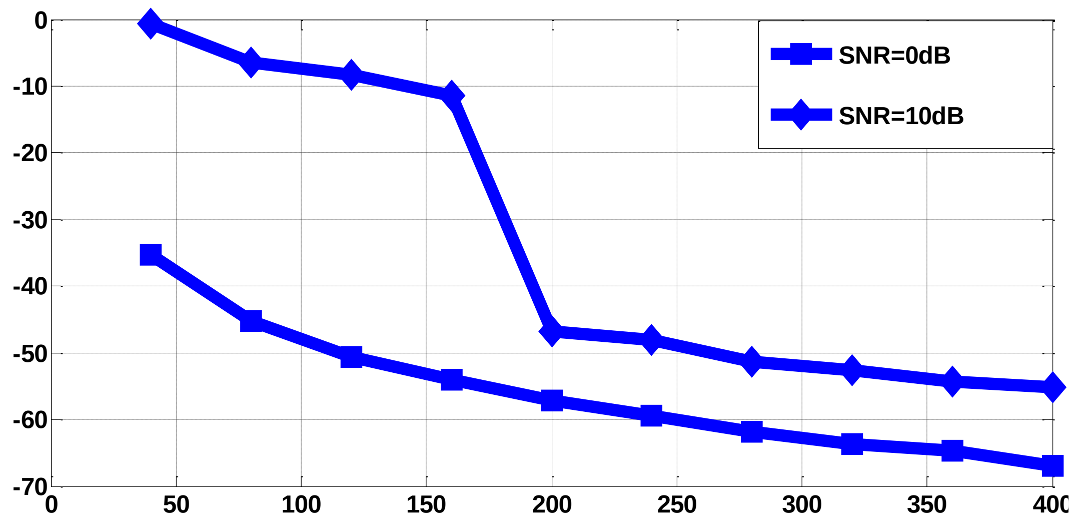

Figure 9. MSE of frequency estimation versus Number of signal samples ($a_1=a_2=1/\sqrt{2}$, $\omega_1=0.3\pi$ rad/s and $\omega_2=0.6\pi$ rad/s)

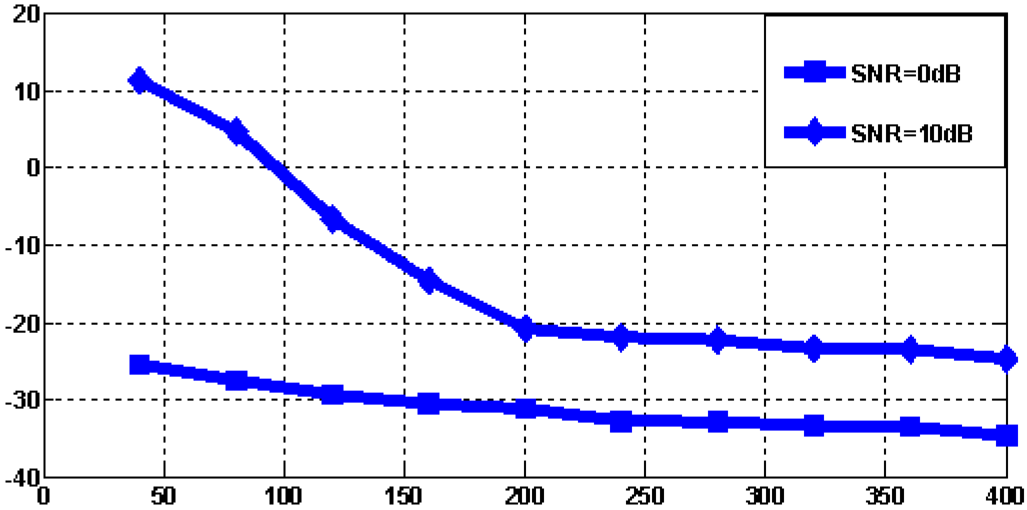

Figure10. MSE of time delay estimation versus Number of signal samples (a_1= a_2=$1/\sqrt{2}$, $\omega_1 = 0.3\pi$ rad/s and $\omega_2 = 0.6\pi$ rad/s)

The last experiment is designed to test the performance of the frequency estimator as a function of the frequency spacing. The first frequency is set to $\omega_1 = 0.3\pi$ rad/s, the second frequency is assumed to be from $\omega_1 + 0.01$to $\omega_1 + 0.04$. From figure 11 one can conclude that if the spacing is greater than 0.15 rad/s the RRLU start showing better performance compared with the SSR.

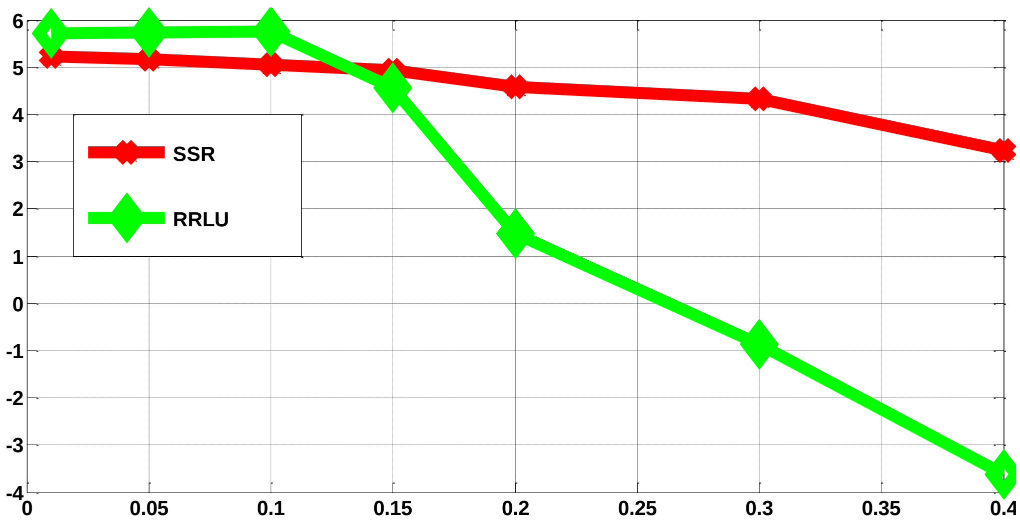

Figure 11. MSE of frequency estimation versus spacing. SNR (a_1= a_2=$1/\sqrt{2}$, $\omega_1 = 0.3\pi$ rad/s and $\omega_2 = spacing + \omega_1$ rad/s, N=40, SNR=0dB)

5. CONCLUSION

In this paper a new technique is proposed for Joint Time Delay and Frequencies Estimation of sinusoidal signals received at two separated sensors by applying the RRLU based method. The frequencies of complex sinusoids are estimated using the received data matrix. The developed frequency estimator shows outstanding performance compared with the State-Space Realization. The RRLU time delay estimator remarkably improves the performance of the MSE compared with SSR estimator and it is very similar to the RRQR estimator.

REFERENCES

[1] G. C. Carter, *Coherence and Time Delay Estimation: An Applied Tutorial for Research, Development, Test and Evaluation Engineers*. Piscataway, NJ: IEEE Press, 1993.

[2] H. C. So, "A comparative study of two discrete-time phase delay estimators," IEEE Trans. Instrum. Meas., vol. 54, no. 6, pp. 2501–2504, Dec. 2005.

[3] S. M. Kay, *Modern Spectral Estimation: Theory and Applications.* Englewood Cliffs, NJ: Prentice-Hall, 1988.

[4] P. Stoica, "List of references on spectral line analysis," *Signal Process.*, vol. 31, no. 3, pp. 329–340, Apr. 1993.

[5] M .Qasaymeh , G. Hiren, N. Tayem and M. E. Sawan "Joint time delay and frequency estimation without Eigen-decomposition" IEEE Signal Processing Letters, vol. 16, pp. 422–425, May 2009.

[6] G. Liao, H. C. So, and P. C. Ching, "Joint time delay and frequency estimation of multiple sinusoids," in *Proc. IEEE Int. Conf. Acoust. Speech, Signal Processing*, Salt Lake, UT, May 2001, pp. 3121–3124.

[7] H. C. So, "Delay estimation for sinusoidal signals," *Proc. Inst. Elect. Eng.—Radar, Sonar, Navigat.*, vol. 148, no. 6, pp. 318–324, Dec. 2001.

[8] Y. Wu, H C So and P.C. Ching, "Joint Time Delay and Frequency Estimation via State-Space Realization," *IEEE Signal Processing Letters*, vol. 10, pp. 339–342, November 2003.

[9] R. O. Schmidt, "Multiple emitter location and signal parameter estimation" IEEE Trans. Acoust., Speech, Signal Processing, vol. 37, pp 720-740, May 1989

[10] C.L. Bastard, V. Baltazart and Y.D. Wang, "Modified ESPRIT (M-ESPRIT) Algorithm for time delay estimation in both any noise and any radar pulse context by a GPR Radar," Signal Processing, vol.90, pp.173-179, 2010.

[11] Y.H. Kim, "Multipath parameter estimation for radio propagation channel measurements with a vector network analyzer," IEEE Transactions on Vehicular Technology, vol.59, no.1, pp.48-52, 2010.

[12] I. Ziskind and M. WAX, "Maximum likelihood localization of multiple sources by alternating projection," IEEE Transactions on Acoustics, Speech, and Signal Processing, vol.36, no.10, pp.1553-1560, 1988.

[13] A. Monakov and G.A. Varfolomeev, "Resolution of signal sources via spectral moment estimation," IEEE Transactions on Aerospace and Electronic Systems, vol.42, no.3, pp.770-777, 2006.

[14] A. Monakov, "Spectral moment estimation of an extended target in ULA," IEEE Transactions on Aerospace and Electronic Systems, vol.44, no.1, pp.360-366, 2008.

[15] M. Qasaymeh, Gami Hiren, Tayem Nizar, Ravi Pendse, M. E Sawan, "Propagator Method for Joint Time Delay and Frequency Estimation", 42nd IEEE conference on Signal, System and computers, Asilomar, CA, USA, Oct. 2008.

[16] M. Qasaymeh, G. Hiren, T. Nizar, "Rank Revealing QR Factorization for Jointly Time Delay and Frequency Estimation", *Vehicular Technology Conference, 2009. VTC Spring April, 2009. IEEE 69thBarcelona, Spain*,page(s): 1 – 4.

[17] C.-T. Pan, 2000, "On the existence and Computation of rank-revealing LU factorizations": Linear Algebra Appl., Vol .316, pp. 199-222.

[18] L. Miranian, M. Gu, 2003, "Strong rank revealing LU factorizations": Linear Algebra Appl., Vol. 367, pp. 1-16.

[19] Tsung-Min Hwang et al., "Improved Bound for Rank Revealing LU factorizations", Linear Algebra Appl. 261 (1997) 173-186.
[20] Tony F. Chan, "Rank Revealing QR factorizations", Linear Algebra Appl. 88/89 (1987) 67-82.
[21] H. T. Wu, J. F. Yang, and F. K. Chen, "Source number estimators using transformed gerschgorin radii," IEEE Trans. Signal Processing, vol. 43, pp. 1325–1333, June 1995.
[22] Hayes, "Statistical Digital Signal Processing" Wiley 1996.

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