Global Stabilization of a Class of Nonlinear System Based on Reduced-order State Feedback Control

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ABSTRACT

The problem of global stabilization for a class of nonlinear system is considered in this paper.The sufficient condition of the global stabilization of this class of system is obtained by deducing thestabilization of itself from the stabilization of its subsystems. This paper will come up with a designmethod of state feedback control law to make this class of nonlinear system stable, and indicate the efficiency of the conclusion of this paper via a series of examples and simulations at the end. Theresults presented in this paper improve and generalize the corresponding results of recent works.

KEYWORDS

Nonlinear systems; Reduced-order control; Global stabilization.

1. Introduction

Stabilization is a seriously important topic in control system designing [1]-[13]. In recent years the stabilization of nonlinear systems has won extensive attention from researchers, and some achievements has been made. The problem is simplified into the question of stabilization of low order systems, the sufficient condition of the system is gained, and that the state feedback control law is designed by combining centre manifold theory with part feedback linearization (e.g., [14]).

The design method to construct global stabilization feedback control law is founded and several sufficient condition of global stabilization for the system is gained by applying linearization method to a class of nonlinear systems (e.g., [15, 16]). The sufficient condition of local stabilization and global stabilization for the system is gained by researching the stabilization question of minimum phase nonlinear system (e.g., [16]). A new method to design control laws is given, and it has been proved that the corresponding close-loop system can be globally stable under appropriate condition by researching the stabilization question of a class of affine nonlinear

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system with standard form (e.g., [17]). The sufficient condition for the global stabilization of several smooth feedback is given by researching the global stabilization of cascade systems constructed with linear systems which can be made stable and asymptotically stable nonlinear systems (e.g., [18]). The stabilizationquestion of a class of triangle system is researched with smooth output feedback (e.g., [19]). The global stabilization question of cascade system made up of two nonlinear systems is researched via constructing Lyapunov function (e.g., [20]). The sufficient condition for the system to be stable is gained, and a state feedback control law is designed out, through researching the stabilization question of a class of non-minimum phase-nonlinear system with its reduced-order control system(e.g., [21]). The sufficient condition for the system to be stable is gained, and that a state feedback control law is designed out through through researching the stabilization question of a class of non-minimum phase-nonlinear system based on drive control (e.g., [22]).

This paper will research the global stabilization question of a class of nonlinear system on the base of analysis, (e.g. See Ref. [8]-[12]), and deduce the stabilization of the original system from the stabilization of its subsystem-reduced-order control system. According to the features of this class of nonlinear system, this paper will come up with a design method for the corresponding state feedback control law, and will prove that close-loop system is globally asymptotically stable under appropriate condition with Lyapunov second method. This paper will indicate the efficiency of the conclusion of this paper via a series of examples and simulations at the end. In this paper, if not specially illustrated, k · k refers to Euclid norm, k · kF refers to Frobenius norm, | · | refers to scalar function or absolute value of function.

2. System analysis

Consider the following nonlinear system

$$\begin{cases} \dot{x} = f(x) + 2h(x,y)y, \\ \dot{y} = u + g(x,y)y. \end{cases} \tag{1}$$

where x ∈Rm and y ∈Rn−m are state vectors, u ∈Rn−m is a input vector. f(x) is a vector

function of x, h(x, y) is a vector function of (x, y), g(x, y) is a scalar function of (x, y), and f(x), g(x, y) and h(x, y) are at least C1 functions about x or (x, y), respectively.

Assumption 1. The subsystem of system (1)

$$\dot{x} = f(x). \tag{2}$$

is globally stable.

Remark 1. This paper uses the static stability of system (1) to discuss its dynamic stability. The purpose of *Assumption 1* is to ensure the static stability of system (1).

In order to obtain main result of this paper, we should do further supposition.

Assumption 2. The subsystem (2) of system (1) is globally stable and there is a *Lyapunov* function v(x) which satisfies

$$\begin{aligned}\dot{v}(x)\mid_{(2)} &= \frac{\partial v(x)}{\partial x}f(x) \\ &= [\frac{\partial v(x)}{\partial x_1}\ \frac{\partial v(x)}{\partial x_2}\ \cdots\ \frac{\partial v(x)}{\partial x_m}]\begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_m(x)\end{bmatrix} \\ &< -1, \forall \parallel x \parallel \geq M.\end{aligned}$$

where M is a positive constant.

Before proving main theorems, present the following analysis and *Lemma 1* at first.
Consider the following nonlinear system

$$\begin{cases} \dot{x} = f(x, y), \\ \dot{y} = -yg(x, y). \end{cases} \tag{3}$$

where x ∈Rm and y ∈Rn−m are state vectors, f(x, y) and g(x, y) are vector function of (x, y), and f(x, y) and g(x, y) are at least C1 functions about(x, y).

Lemma 1. (e.g. see Ref. [23]) If the subsystem of system (3)

$$\dot{x} = f(x, 0). \tag{4}$$

is globally stable, and there is a continuous differentiable scalar functions v(x) such that limkxk→0 v(x) =+∞, and the derivative of along the trajectory of system (4) at k x k> 0 such that v(x) ≤ 0, then system (3) is globally stable.

Remark 2. *Lemma 1* obtained is rather significant for the proving of the main theorems in this paper, the main idea of which is that the stabilization of the original system can be proved when reduced-order subsystem such that static stabilization and some essential assumed conditions. Before the theorem of this paper given, we must give the following *Lemma 2* at first, which is is easy to understand.

Lemma 2. (e.g. see Ref. [26], [27]) Assume that subsystem (2) of nonlinear system (1) is globally asymptotically stable, then from *Converse-Lyapunov theorem*, we know that there must be a positive function v(x) which satisfies

$$\begin{aligned}\dot{v}(x)\mid_{(2)} &= \frac{\partial v(x)}{\partial x}f(x) \\ &= [\frac{\partial v(x)}{\partial x_1}\ \frac{\partial v(x)}{\partial x_2}\ \cdots\ \frac{\partial v(x)}{\partial x_m}]\left[\ f_1(x), f_2(x), \cdots, f_m(x)\ \right]^T \\ &\leq 0, \forall x \in R^m - \{0, 0, \cdots, 0\}.\end{aligned} \tag{5}$$

Without loss of generality. We can suppose

$$\dot{v}(x) \leq -\varepsilon < 0, \forall x \in R^m - \{0, 0, \cdots, 0\}.$$

where " is a very small positive.

3. Main results

In the process of researching nonlinear systems, it's always desired that the states of the system can reach stable values, so that as many interferences as possible could be avoided. Therefore, making systems stable has a strong practical applicability. Because of that this question has a certain actual application background, the research in this paper has some practical meaning.

3.1 Reduced-order state feedback controller design

Theorem 1. Assume that subsystem (2) of nonlinear system (1) satisfies *Assumption 2* and there is a scalar function $K(x, y)$ for system (1) which satisfies:

$$K(x,y) > \| [\frac{\partial v(x)}{\partial x_1} \frac{\partial v(x)}{\partial x_2} \cdots \frac{\partial v(x)}{\partial x_m}] \|_2^2 \cdot \| h(x,y) \|_F^2 . \tag{6}$$

Then there is a state feedback control low

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{n-m} \end{bmatrix} = \begin{bmatrix} -y_1 K(x,y) \\ -y_2 K(x,y) \\ \vdots \\ -y_{n-m} K(x,y) \end{bmatrix} - \begin{bmatrix} y_1 g(x,y) \\ y_2 g(x,y) \\ \vdots \\ y_{n-m} g(x,y) \end{bmatrix} \tag{7}$$

such that the system (1) globally asymptotically stable, where the elements of $K(x, y)$ are at least C_1 functions about (x, y).

Proof. After substitution the state feedback control low (7) into system (1) we see that

$$\begin{cases} \dot{x} = f(x) + 2h(x,y)y, \\ \dot{y} = -yK(x,y). \end{cases} \tag{8}$$

According to *Lemma 2*, we can construct a *Lyapunov function*

$$V(x,y) = v(x) + \frac{1}{2} y^T y.$$

then its derivative along the trajectory of system (1) is

$$\begin{aligned}\dot{V}(x,y)\mid_{(1)} &= [\frac{\partial v(x)}{\partial x_1}\ \frac{\partial v(x)}{\partial x_2} \cdots \frac{\partial v(x)}{\partial x_m}]\begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_m(x) \end{bmatrix} \\ &\quad +2[\frac{\partial v(x)}{\partial x_1}\ \frac{\partial v(x)}{\partial x_2} \cdots \frac{\partial v(x)}{\partial x_m}]\cdot h(x,y)y + \| y \|^2 \cdot [-K(x,y)] \\ &\leq 0 + \| [\frac{\partial v(x)}{\partial x_1}\ \frac{\partial v(x)}{\partial x_2} \cdots \frac{\partial v(x)}{\partial x_m}] \| \cdot \| h(x,y) \|_F \cdot \| y \| + \| y \|^2 \cdot [-K(x,y)].\end{aligned}$$

Combined with condition (6) we easy to know

$$\dot{V}(x) < 0, \forall x \in R^m - \{0, 0, \cdots, 0\}.$$

Then we can conclude from *Lemma 1* and analysis, e.g. see Ref. 8-12, that system (1) is globally asymptotically stable.

3.2 Further result in mathematics

The condition of subsystem (2) respect to system (1) is weakens and the further investigation is given.

Theorem 2. Assume that subsystem (2) of nonlinear system (1) is satisfies *Assumption 1*. Then there must be a positive M, when $K(x, y) + g(x, y) < -M$, there is a state feedback control low

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{n-m} \end{bmatrix} = \begin{bmatrix} y_1 K(x,y) \\ y_2 K(x,y) \\ \vdots \\ y_{n-m} K(x,y) \end{bmatrix} \tag{9}$$

such that the system (1) globally asymptotically stable, where the elements of $K(x, y)$ are at least C_1 functions about (x, y).

Proof. After substitution the state feedback control low (9) into system (1) we see that

$$\begin{cases} \dot{x} = f(x) + 2h(x,y)y, \\ \dot{y} = yK(x,y) + g(x,y)y. \end{cases} \tag{10}$$

According to *Lemma 2*, we can construct a *Lyapunov function*

$$V(x, y) = v(x) + \frac{1}{2}y^T y.$$

then its derivative along the trajectory of system (1) is

$$\begin{aligned}
\dot{V}(x,y)\mid_{(1)} &= [\frac{\partial v(x)}{\partial x_1}\ \frac{\partial v(x)}{\partial x_2}\ \cdots\ \frac{\partial v(x)}{\partial x_m}] \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_m(x) \end{bmatrix} \\
&\quad -\varepsilon + 2[\frac{\partial v(x)}{\partial x_1}\ \frac{\partial v(x)}{\partial x_2}\ \cdots\ \frac{\partial v(x)}{\partial x_m}] \cdot h(x,y)y + \parallel y \parallel^2 \cdot [K(x,y) + g(x,y)] \\
&\leq 0 + \parallel [\frac{\partial v(x)}{\partial x_1}\ \frac{\partial v(x)}{\partial x_2}\ \cdots\ \frac{\partial v(x)}{\partial x_m}] \parallel \cdot \parallel h(x,y) \parallel_F \cdot \parallel y \parallel \\
&\quad + \parallel y \parallel^2 \cdot [K(x,y) + g(x,y)].
\end{aligned}$$

Combined with condition $K(x, y) + g(x, y) < -M$ we easy to know that so long as

$$M > \frac{\parallel [\frac{\partial v(x)}{\partial x_1}\ \frac{\partial v(x)}{\partial x_2}\ \cdots\ \frac{\partial v(x)}{\partial x_m}] \parallel_2^2 \cdot \parallel h(x,y) \parallel_F^2}{\varepsilon}.$$

there must be

$$\dot{V}(x) < 0, \forall x \in R^m - \{0, 0, \cdots, 0\}.$$

Then we can conclude from *Lemma 1* and analysis, e.g. see Ref. 8-12, that system (1) is globally asymptotically stable.

4. System simulations

In this section, in order to show that the approach of this paper to this sore of control system is effective and convenient, we give some illustrative examples and simulation results.

Example 1. Consider the following second-order nonlinear system

$$\begin{cases} \dot{x} = -x^3 + 2y^2, \\ \dot{y} = u + 3xy^2 + y^2. \end{cases} \tag{11}$$

Solution. Base on the theorem above, for the subsystem of system (11), we can construct a *Lyapunov function* which is given by

$$v(x) = x^2.$$

It is easy to know that the subsystem of system (11)

$$\dot{x} = -x^3.$$

is globally asymptotically stable.

Base on the condition (6) of *Theorem 1*, let

$$K = 4x^2y^2.$$

Then, there is a state feedback control law

$$u = -4x^2y^3 - 3xy^2 - y^2. \tag{12}$$

such that the system (11) is globally asymptotically stabilization.

Simulation. Without loss of generality. Let the initial values of simulation

$$\begin{cases} x(0) = -4, \\ y(0) = +3. \end{cases}$$

and the input control signal

$$u = \begin{cases} 0.5, & \text{for } t \geq 0, \\ 0, & \text{for } t < 0. \end{cases}$$

The sample time is 0.1s, and the simulation time are 50s seconds.

The dynamic response of the system (11) with the state feedback control law (12) is show in Figure 1.

From the Figure 1, we can know that the controlled closed-loop system of the system (11) is asymptotically stable.

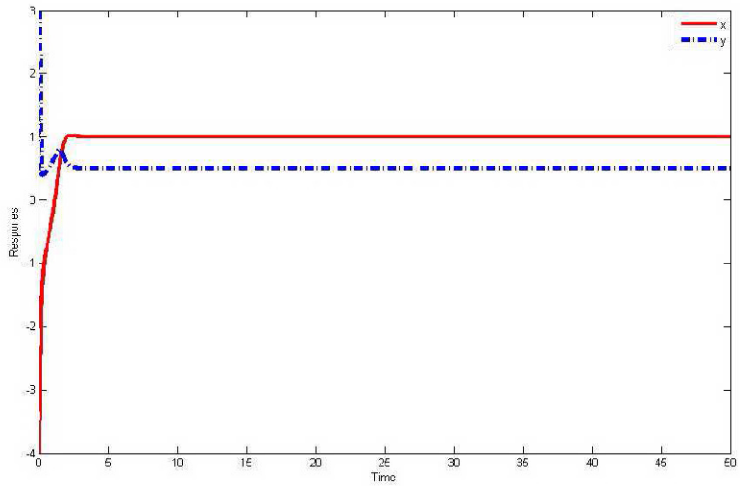

Figure 1. Dynamic response of the system (11) with the state feedback control law (12).

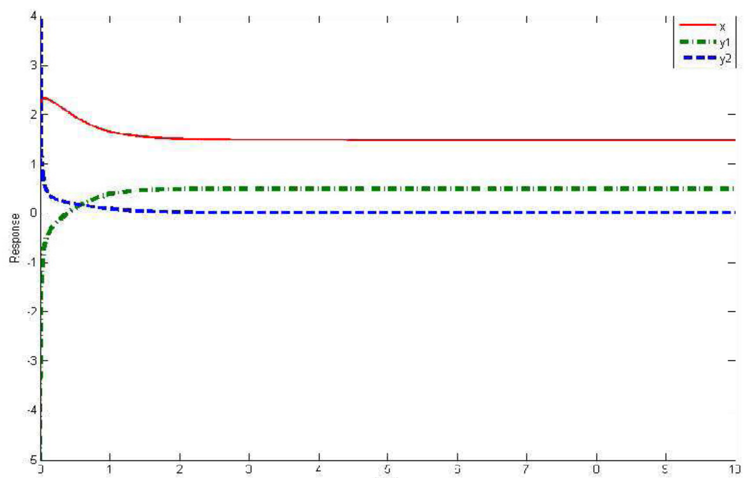

Figure 2. Dynamic response of the system (13) with the state feedback control law (14).

Example 2. Consider the following third-order nonlinear system

$$\begin{cases} \dot{x} = -x^3 + 2y^2, \\ \dot{y}_1 = u_1 + xy_1^2 + xy_1y_2 + xy_1y_2^3, \\ \dot{y}_2 = u_2 + xy_1y_2 + xy_2^2 + xy_2^4. \end{cases} \tag{13}$$

Solution: Base on the theorem above, for the subsystem of system (13), we can construct a *Lyapunov function* which is given by

$$v(x) = \frac{1}{2}x^2.$$

It is easy to know that the subsystem of system (13)

$$\dot{x} = -x.$$

is globally asymptotically stable.

Base on the condition (6) of *Theorem 1*, let

$$K = x^2(2y_1^2 + 2y_2^2) + 1.$$

Then, there is a state feedback control law

$$\begin{cases} u_1 = -2x^2y_1^3 - 2x^2y_1y_2^2 - y_1 - xy_1^2 - xy_1y_2 - xy_1y_2^3, \\ u_2 = -2x^2y_1^2y_2 - 2x^2y_2^2 - y_2 - xy_1y_2 - xy_2^2 - xy_2^4. \end{cases} \quad (14)$$

such that the system (13) is globally asymptotically stabilization.

Simulation. Without loss of generality. Let the initial values of simulation

$$\begin{cases} x(0) = 2, \\ y_1(0) = -5, \\ y_2(0) = 4. \end{cases}$$

and the input control signal

$$u = \begin{cases} 1, & \text{for } t \geq 0, \\ 0, & \text{for } t < 0. \end{cases}$$

The sample time is 0.1s, and the simulation time are 10s seconds.
The dynamic response of the system (13) with the state feedback control law (14) is show in Figure 2.

From the Figure 2, we can know that the controlled closed-loop system of the system (13) is asymptotically stable [28].

5. Conclusion

Based on some work of pioneers, the global stabilization problem for a class of nonlinear system is investigated in this paper, and that the stabilization of the original system is obtioned from the stabilization of its reduced-order subsystem. A design method for the corresponding state feedback control law is given according to the features of this class of nonlinear system, and that the closeloop system is globally asymptotically stable under appropriate condition is proved with Lyapunov second method. This paper has indicated the efficiency of the conclusion of this paper via a series of examples and simulations at the end.

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References

[1] P.He, S. H. Ma, and T. Fan, "Finite-time mixed outer synchronization of complex networks with coupling time-varying delay," Chaos, vol. 22, no. 4, pp. 043151, Dec. 2012.

[2 H.Niu, S. H. Ma, T. Fan, C. Chen, P. He. "Linear state feedback stabilization of unified hyperchaotic systems," World Journal of Modelling and Simulation, vol. 10, no. 1, pp. 34–48, Feb. 2014.

[3] P.He, Q. Zhang, C. Jing, C. Chen, T. Fan. "Robust exponential synchronization for neutral complex networks with discrete and distributed time-varying delays: A descriptor model transformation method," Optimal Control, Applications and Methods, 2013. [online] http://dx.doi.org/10.1002/oca.2094.

[4] P.He, C. Jing, T. Fan, C Chen. "Robust decentralized adaptive synchronization of general complex networks with coupling delayed and uncertainties," Complexity, 2013.[online] http://dx.doi.org/10.1002/cplx.21472.

[5] P.He, C. Jing, C. Chen, T. Fan, H. Nik. "Synchronization of general complex networks via adaptive control schemes," PRAMANA-Journal of Physics, 2014. In press.

[6] P.He, C. Jing, T. Fan, C. Chen. "Outer synchronization of complex networks with multiple coupling time-varying delays," International Journal of Control and Automation, vol. 6, no. 4, pp. 197–216, Sep. 2013.

[7] P.He, C. Jing, T. Fan, C. Chen. "Robust adaptive synchronization of complex networks with multiple coupling time-varying delays," International Journal of Automation and Control, vol.7, no. 4, pp. 223–248, Dec. 2013.

[8] P.He, and G. Q. Tan, “H∞ observer design for a class of time-delay nonlinear systems with external disturbance,” International Journal of Applied Mathmatics and Mechanics, vol. 8, no. 16, pp. 67-78, June 2012.
[9] P.He, X. H. Ren, and F. H. Li, “Robust stabilization of uncertain unified chaotic systems,” Journal of Central China Normal University (Natural Sciences), vol. 46, no. 1,pp. 35-39, Feb.2012.
[10] P.He, and F. Tan, “Linear state feedback stabilization for controlled chaotic systems,” International Journal of Nonlinear Science, vol. 12, no. 3, pp. 373-384,Dec. 2011.
[11] C.Chen, T. Fan, B. Wang, H. Nik, P. He. ”Feedback linearization synchronization of unified chaotic systems,” Journal of Applied Nonlinear Dynamics, 2014. Inpress.
[12] P.He, H. Y. Lan, and G. Q. Tan, “Delay-independent stabilization of linear systems with multiple time-delays,” World Academy of Science, Engineering and Technology, vol. 51, no. 3,pp. 1007-1011, Mar. 2011.
[13] P.He, “Partial stabilization of a class of nonlinear systems via center manifold theory,” World Academy of Science, Engineering and Technology, vol. 51, no. 3, pp. 1012-1024, Mar. 2011.
[14] Marino R., Feedback stabilization of single-input nonlinear system, Systems and Control Letters. 10:201–206, 1988.
[15] BaumannW., RughW., Feedback control of nonlinear systems, IEEE Transactions on Automatic Control. 31:44–47, 1986.
[16] Byres C. I., Isidori A., Asymptistic stabilization of minimum phase nonlinear systems, IEEE Transactions on Automatic Control. 36:1122–1237, 1991.
[17] Seibert P., Snarez R., Global stabilization of a certain class of nonlinear systems, Systems and Control Letters. 16:17–23, 1991.
[18] Saberi A., Kokotovic P., Suaamann H. J., Global stabilization of partially linear composite systems, SIAM Control. 28:1491–1503, 1990.
[19] Qian C., Lin W., Output feedback stabization of planar systems without controllableobservable linearization, IEEE Transactions on Automatic Control. 47:2068–2073, 2002.
[20] Mazenic F., Sepulchre R., Jankovic M., Lyapunov function for stable cascades and applications to global stabilization, IEEE Transactions on Automatic Control. 44:1795–1800, 1999.
[21] Cheng D. Z., Hu Q. X., Qin H. S., Feedback diagonal canonical form and its application to stabilization of nonlinear systems, Science in China Ser. F: Information Sciences. 48:201–210,2005.
[22] Juan C., Manuel A., Passivity based stabilization of non-minimum phase nonlinear systems,Kybernetika. 45:417–426, 2009.
[23] Outbid R., Jghima H., Comments on the stabilization of nonlinear systems by adding an integrator,IEEE Transactions on automatic control. 41:1804–1807, 1996.
[24] Cheng D. Z., Guo Y. Q., Stabilization of nonlinear systems via the center manifold approach,Systems and Control Letters. 57:511–518, 2008.
[25] LinW., Global asymptotic stabilization of general nonlinear systems with stable free dynamicsvia passivity and bounded feedback, Automatic. 32:915–924, 2009.
[26] Khalil H. K., Weber M. E., Nonlinear systems, Second etition, Prentice Hall, Inc. New Jersey,1996.
[27] Feng Y. J., Dong Y. L., Global stabilization for a class of nonlinear system, Journal of tianjinpolytechnic university. 26:69–72, 2007. (in Chinese)
[28] Fan T., Chen C. Z., Ren X. H., He P., Adaptive synchronization of delayed Chen chaotic system, Discontinuity, Nonlinearity and Complexity. 2014, in press.

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