ADAPTIVE STABILIZATION AND SYNCHRONIZATION OF LÜ-LIKE ATTRACTOR

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ABSTRACT

This paper derives new results for the adaptive chaos stabilization and synchronization of Lü-like attractor with unknown parameters. The Lü-like attractor is one of the recently discovered 3-scroll chaotic systems, which was proposed by D. Li (2007). First, adaptive control laws are determined to stabilize the Lü-like attractor to its unstable equilibrium at the origin. These adaptive laws are established using Lyapunov stability theory. Then adaptive synchronization laws are determined so as to achieve global chaos synchronization of identical Lü-like attractors with unknown parameters. Numerical simulations are presented to validate and demonstrate the effectiveness of the proposed adaptive control and synchronization schemes for the Lü-like attractor.

Keywords

Adaptive Control, Chaos, Stabilization, Synchronization, Lü-like Attractor, Lyapunov Stability Theory.

1. INTRODUCTION

Chaotic systems are nonlinear dynamical systems which are highly sensitive to initial conditions.

The stabilization of chaotic systems aims to design state feedback control laws that stabilizes the chaotic systems around the unstable equilibrium points. Active control method is deployed when the system parameters are known and adaptive control method is deployed when the system parameters are unknown [1-4].

In 1990, Pecora and Carroll [5] introduced a method to synchronize two identical chaotic systems and showed that it was possible for some chaotic systems to be completely synchronized. From then on, chaos synchronization has been widely explored in a variety of fields including physical systems [6], chemical systems [7], ecological systems [8], secure communications [9-10], etc.

In most of the chaos synchronization papers in the literature, the *master-slave* or *drive-response* formalism has been practiced. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of chaos synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

The first seminal work on chaos synchronization was developed by Pecora and Carroll [5]. This was followed by a variety of impressive methods for the synchronization of chaotic systems such as the OGY method [11], active control method [12-18], adaptive control method [19-24], sampled-data feedback synchronization method [25], time-delay feedback method [26], backstepping method [27], sliding mode control method [28-33], etc.

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In this paper, we discuss the adaptive control and synchronization of Lü-like attractor (Li, [34], 2007) with unknown parameters

This paper is organized as follows. In Section 2, we derive results for the adaptive control of Lü-like attractor with unknown parameters. In Section 3, we derive results for the adaptive synchronization of Lü-like attractors with unknown parameters. Section 4 contains a summary of the main results derived in this paper.

2. ADAPTIVE CONTROL OF LÜ-LIKE ATTRACTOR

2.1 Theoretical Results

The Lü-like attractor (Li, 2007) is a three-scroll chaotic system, which is described by

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + dx_1x_3 \\ \dot{x}_2 &= -x_1x_3 + bx_2 \\ \dot{x}_3 &= -\varepsilon x_1^2 + x_1x_2 + cx_3\end{aligned} \tag{1}$$

where x_i, $(i = 1, 2, 3)$ are the state variables and a, b, c, d, ε are positive constants.

The Lü-like system (1) is chaotic when the parameter values are taken as

$$a = 40, \quad b = 20, \quad c = 5/6, \quad d = 0.5, \quad \varepsilon = 0.65 \tag{2}$$

The state orbits of the Lü-like system (1) are described in Figure 1. The phase portrait shows that the Lü-like attractor (1) is a symmetric, toroidal, 3-scroll chaotic system.

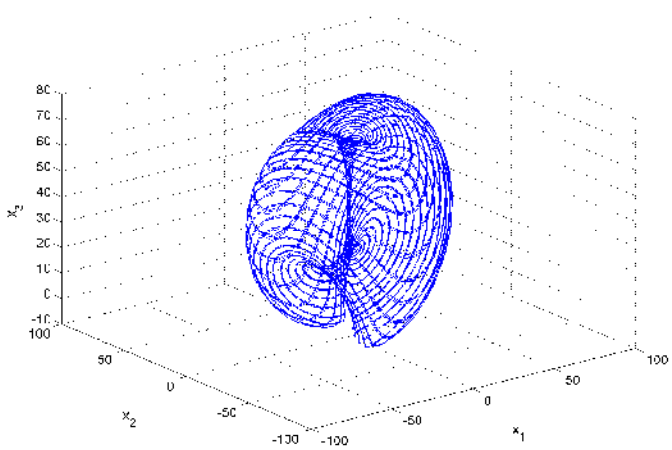

Figure 1. State Orbits of the Lü-like Attractor

When the parameter values are taken as in (2), the system (1) is chaotic and the system linearization matrix at the equilibrium point $E_0 = (0, 0, 0)$ is given by

$$A = \begin{bmatrix} -40 & 40 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 0.8333 \end{bmatrix}$$

which has the eigenvalues

$$\lambda_1 = -40, \quad \lambda_2 = 20 \quad \text{and} \quad \lambda_3 = 0.8333.$$

Since λ_2 and λ_3 are positive eigenvalues, it is immediate from Lyapunov stability theory [35] that the system (1) is unstable at the equilibrium point $E_0 = (0,0,0)$.

In this section, we design adaptive control law for globally stabilizing the Lü-like attractor (1) when the parameter values are unknown.

Thus, we consider the controlled Lü-like attractor given by the dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + dx_1x_3 + u_1 \\ \dot{x}_2 &= -x_1x_3 + bx_2 + u_2 \\ \dot{x}_3 &= -\varepsilon x_1^2 + x_1x_2 + cx_3 + u_3 \end{aligned} \tag{3}$$

where u_1, u_2 and u_3 are feedback controllers to be designed using the states and estimates of the unknown parameters of the system.

In order to show that the controlled system (3) globally converges to the origin asymptotically, we consider the following adaptive control functions

$$\begin{aligned} u_1 &= -\hat{a}(x_2 - x_1) - \hat{d}x_1x_3 - k_1x_1 \\ u_2 &= x_1x_3 - \hat{b}x_2 - k_2x_2 \\ u_3 &= \hat{\varepsilon}x_1^2 - x_1x_2 - \hat{c}x_3 - k_3x_3 \end{aligned} \tag{4}$$

where $\hat{a}, \hat{b}, \hat{c}, \hat{d}$ and $\hat{\varepsilon}$ are estimates of the parameters a, b, c, d and ε, respectively, and $k_i, (i = 1,2,3)$ are positive constants.

Substituting the control law (4) into the Lü-like chaotic system (3), we obtain

$$\begin{aligned} \dot{x}_1 &= (a - \hat{a})(x_2 - x_1) + (d - \hat{d})x_1x_3 - k_1 x_1 \\ \dot{x}_2 &= (b - \hat{b})x_2 - k_2 x_2 \\ \dot{x}_3 &= -(\varepsilon - \hat{\varepsilon})x_1^2 + (c - \hat{c})x_3 - k_3 x_3 \end{aligned} \tag{5}$$

Let us now define the parameter errors as

$$e_a = a - \hat{a}, \quad e_b = b - \hat{b}, \quad e_c = c - \hat{c}, \quad e_d = d - \hat{d}, \quad e_\varepsilon = \varepsilon - \hat{\varepsilon} \tag{6}$$

Using (6), the closed-loop dynamics (5) can be written compactly as

$$\begin{aligned} \dot{x}_1 &= e_a(x_2 - x_1) + e_d x_1x_3 - k_1 x_1 \\ \dot{x}_2 &= e_b x_2 - k_2 x_2 \\ \dot{x}_3 &= -e_\varepsilon x_1^2 + e_c x_3 - k_3 x_3 \end{aligned} \tag{7}$$

For the derivation of the update law for adjusting the parameter estimates $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{\varepsilon},$ the Lyapunov approach is used.

Consider the quadratic Lyapunov function

$$V(x_1, x_2, x_3, e_a, e_b, e_c, e_d, e_\varepsilon) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2 + e_\varepsilon^2\right) \tag{8}$$

which is a positive definite function on R^8.

Note also that

$$\dot{e}_a = -\dot{\hat{a}}, \quad \dot{e}_b = -\dot{\hat{b}}, \quad \dot{e}_c = -\dot{\hat{c}}, \quad \dot{e}_d = -\dot{\hat{d}} \text{ and } \dot{e}_\varepsilon = -\dot{\hat{\varepsilon}}. \tag{9}$$

Differentiating V along the trajectories of (7) and using (9), we obtain

$$\begin{aligned} \dot{V} = & -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 + e_a\left[x_1(x_2 - x_1) - \dot{\hat{a}}\right] + e_b\left[x_2^2 - \dot{\hat{b}}\right] \\ & + e_c\left[x_3^2 - \dot{\hat{c}}\right] + e_d\left[x_1^2 x_3 - \dot{\hat{d}}\right] + e_\varepsilon\left[-x_1^2 x_3 - \dot{\hat{\varepsilon}}\right] \end{aligned} \tag{10}$$

In view of Eq. (10), the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{a}} &= x_1(x_2 - x_1) + k_4 e_a \\ \dot{\hat{b}} &= x_2^2 + k_5 e_b \\ \dot{\hat{c}} &= x_3^2 + k_6 e_c \\ \dot{\hat{d}} &= x_1^2 x_3 + k_7 e_d \\ \dot{\hat{\varepsilon}} &= -x_1^2 x_3 + k_8 e_\varepsilon \end{aligned} \tag{11}$$

where k_4, k_5, k_6, k_7 and k_8 are positive constants.

Substituting (11) into (10), we get

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 - k_4 e_a^2 - k_5 e_b^2 - k_6 e_c^2 - k_7 e_d^2 - k_8 e_\varepsilon^2 \tag{12}$$

which is a negative definite function on R^8.

Thus, by Lyapunov stability theory [35], it follows that the systems (7) and (8) are globally exponentially stable, which shows that the closed-loop system (7) is globally exponentially stable and the parameter estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{d}(t)$ and $\hat{\varepsilon}(t)$ converge exponentially to the actual values of the system parameters for all initial conditions.

Hence, we obtain the following result.

Theorem 1. The Lü-like chaotic system (3) with unknown parameters is globally and exponentially stabilized for all initial conditions $x(0) \in R^3$ by the adaptive control law (4), where the update law for the parameters is given by (11) and k_i, $(i = 1, \ldots, 8)$ are positive constants. ■

2.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the chaotic system (3) with the adaptive control law (4) and the parameter update law (11).

The parameters of the Lü-like chaotic system are selected as

$$a = 40, \quad b = 20, \quad c = 5/6, \quad d = 0.5, \quad \varepsilon = 0.65$$

For the adaptive and update laws, we take

$$k_i = 3, \quad (i = 1, 2, \ldots, 8).$$

Suppose that the initial values of the parameter estimates are

$$\hat{a}(0) = 8, \;\; \hat{b}(0) = 4, \;\; \hat{c}(0) = 7, \;\; \hat{d}(0) = 2, \;\; \hat{\varepsilon}(0) = 10.$$

The initial values of the Lü-like chaotic system are taken as

$$x_1(0) = 20, \quad x_2(0) = 18, \quad x_3(0) = 24.$$

When the adaptive control law (4) and the parameter update law (11) are used, the controlled The Lü-like chaotic system converges to the equilibrium $E_0 = (0, 0, 0)$ exponentially as shown in Figure 2.

The parameter estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{d}(t)$ and $\hat{\varepsilon}(t)$ are shown in Figure 3 from which it is clear that the parameter estimates converge exponentially to the original values of the system parameters.

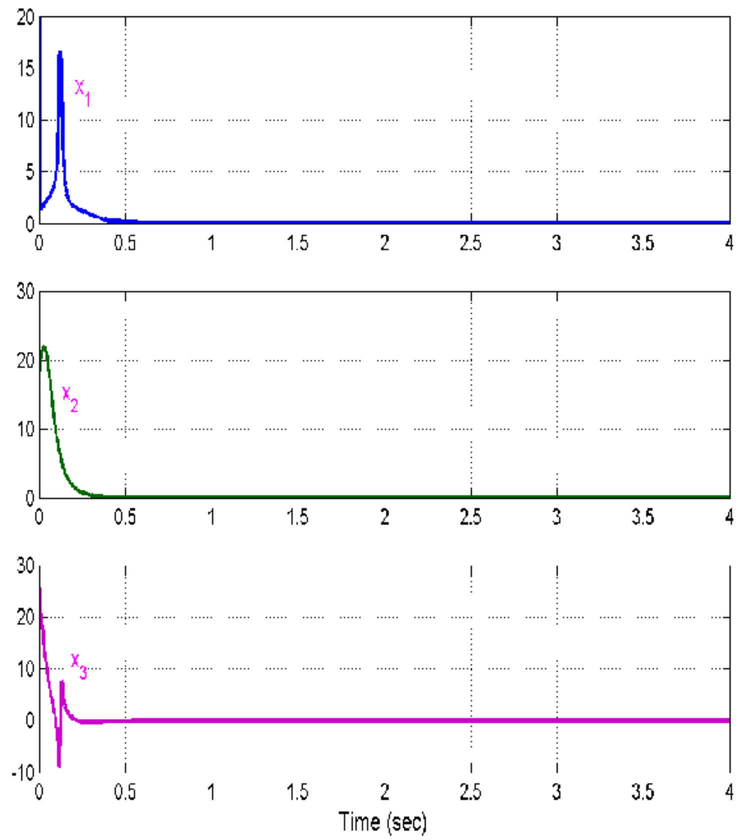

Figure 2. Time Responses of the Controlled Liu-Like Chaotic System

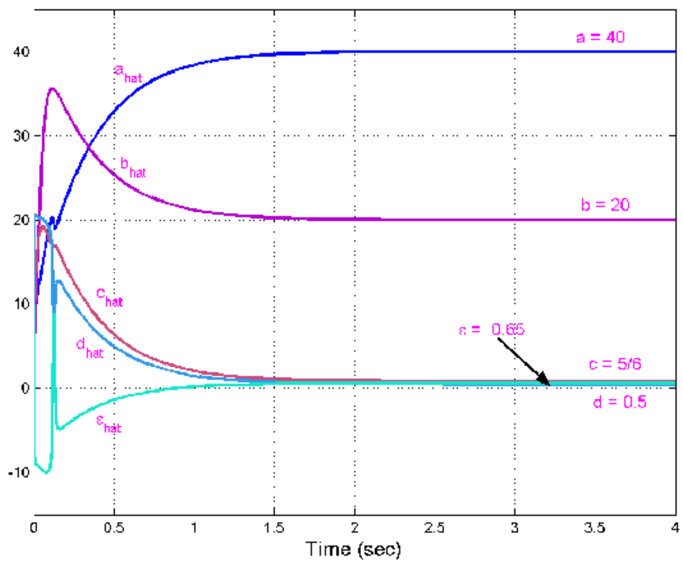

Figure 3. Parameter Estimates $\hat{a}(t),\ \hat{b}(t),\ \hat{c}(t),\ \hat{d}(t),\ \hat{\varepsilon}(t)$

3. ADAPTIVE SYNCHRONIZATION OF IDENTICAL LÜ-LIKE ATTRACTORS

3.1 Theoretical Results

In this section, we discuss the adaptive synchronization of identical Lü-like chaotic systems (Li, 2007) with unknown parameters.

As the master system, we consider the Lü-like system dynamics described by

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + dx_1x_3 \\ \dot{x}_2 &= -x_1x_3 + bx_2 \\ \dot{x}_3 &= -\varepsilon x_1^2 + x_1x_2 + cx_3\end{aligned} \tag{13}$$

where $x_i,\ (i = 1, 2, 3)$ are the state variables and a, b, c, d, ε are unknown system parameters.

The system (13) is chaotic when the parameter values are taken as

$$a = 40,\quad b = 20,\quad c = 5/6,\quad d = 0.5,\quad \varepsilon = 0.65$$

As the slave system, we consider the controlled Lü-like system dynamics described by

$$\begin{aligned}\dot{y}_1 &= a(y_2 - y_1) + dy_1y_3 + u_1 \\ \dot{y}_2 &= -y_1y_3 + by_2 + u_2 \\ \dot{y}_3 &= -\varepsilon y_1^2 + y_1y_2 + cy_3 + u_3\end{aligned} \tag{14}$$

where $y_i,\ (i = 1, 2, 3)$ are the state variables and $u_i,\ (i = 1, 2, 3)$ are the nonlinear controllers to be designed.

The synchronization error is defined by

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3 \end{aligned} \tag{15}$$

Then the error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + d(y_1 y_3 - x_1 x_3) + u_1 \\ \dot{e}_2 &= b e_2 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= c e_3 - \varepsilon(y_1^2 - x_1^2) + y_1 y_2 - x_1 x_2 + u_3 \end{aligned} \tag{16}$$

Let us now define the adaptive control functions $u_1(t), u_2(t), u_3(t)$ as

$$\begin{aligned} u_1 &= -\hat{a}(e_2 - e_1) - \hat{d}(y_1 y_3 - x_1 x_3) - k_1 e_1 \\ u_2 &= -\hat{b} e_2 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\ u_3 &= -\hat{c} e_3 + \hat{\varepsilon}(y_1^2 - x_1^2) - y_1 y_2 + x_1 x_2 - k_3 e_3 \end{aligned} \tag{17}$$

where $\hat{a}, \hat{b}, \hat{c}, \hat{d}$ and $\hat{\varepsilon}$ are estimates of the parameters a, b, c, d and ε, respectively, and $k_i, (i = 1, 2, 3)$ are positive constants.

Substituting the control law (17) into (16), we obtain the error dynamics as

$$\begin{aligned} \dot{e}_1 &= (a - \hat{a})(e_2 - e_1) + (d - \hat{d})(y_1 y_3 - x_1 x_3) - k_1 e_1 \\ \dot{e}_2 &= (b - \hat{b}) e_2 - k_2 e_2 \\ \dot{e}_3 &= (c - \hat{c}) e_3 - (\varepsilon - \hat{\varepsilon})(y_1^2 - x_1^2) - k_3 e_3 \end{aligned} \tag{18}$$

Let us now define the parameter errors as

$$\begin{aligned} e_a &= a - \hat{a} \\ e_b &= b - \hat{b} \\ e_c &= c - \hat{c} \\ e_d &= d - \hat{d} \\ e_\varepsilon &= \varepsilon - \hat{\varepsilon} \end{aligned} \tag{19}$$

Substituting (19) into (18), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= e_a(e_2 - e_1) + e_d(y_1 y_3 - x_1 x_3) - k_1 e_1 \\ \dot{e}_2 &= e_b e_2 - k_2 e_2 \\ \dot{e}_3 &= e_c e_3 - e_\varepsilon(y_1^2 - x_1^2) - k_3 e_3 \end{aligned} \tag{20}$$

For the derivation of the update law for adjusting the estimates of the parameters, the Lyapunov approach is used.

Consider the quadratic Lyapunov function

$$V(e_1, e_2, e_3, e_a, e_b, e_c, e_d, e_\varepsilon) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2 + e_\varepsilon^2\right) \tag{21}$$

which is a positive definite function on R^8.

Note also that

$$\dot{e}_a = -\dot{\hat{a}}, \quad \dot{e}_b = -\dot{\hat{b}}, \quad \dot{e}_c = -\dot{\hat{c}}, \quad \dot{e}_d = -\dot{\hat{d}} \text{ and } \dot{e}_\varepsilon = -\dot{\hat{\varepsilon}}. \tag{22}$$

Differentiating V along the trajectories of (20) and using (22), we obtain

$$\begin{aligned} \dot{V} &= -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a \left[e_1(e_2 - e_1) - \dot{\hat{a}} \right] + e_b \left[e_2^2 - \dot{\hat{b}} \right] \\ &\quad + e_c \left[e_3^2 - \dot{\hat{c}} \right] + e_d \left[e_1(y_1 y_3 - x_1 x_3) - \dot{\hat{d}} \right] + e_\varepsilon \left[-e_3(y_1^2 - x_1^2) - \dot{\hat{\varepsilon}} \right] \end{aligned} \tag{23}$$

In view of Eq. (23), the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{a}} &= e_1(e_2 - e_1) + k_4 e_a \\ \dot{\hat{b}} &= e_2^2 + k_5 e_b \\ \dot{\hat{c}} &= e_3^2 + k_6 e_c \\ \dot{\hat{d}} &= e_1(y_1 y_3 - x_1 x_3) + k_7 e_d \\ \dot{\hat{\varepsilon}} &= -e_3(y_1^2 - x_1^2) + k_8 e_\varepsilon \end{aligned} \tag{24}$$

where k_4, k_5, k_6, k_7 and k_8 are positive constants.

Substituting (24) into (23), we get

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_a^2 - k_5 e_b^2 - k_6 e_c^2 - k_7 e_d^2 - k_8 e_\varepsilon^2 \tag{25}$$

which is a negative definite function on R^8. Thus, by Lyapunov stability theory [35], it is immediate that the synchronization error and the parameter error decay to zero exponentially with time for all initial conditions. Hence, we have proved the following result.

Theorem 2. The identical Lü-like chaotic systems (13) and (14) with unknown parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law (17), where the update law for parameters is given by (24) and $k_i, (i = 1, \ldots, 8)$ are positive constants. ■

3.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the two systems of differential equations (13) and (14) with the adaptive control law (17) and the parameter update law (24).

The parameter values of the Lü-like chaotic systems are taken as

$$a = 40, \quad b = 20, \quad c = 5/6, \quad d = 0.5, \quad \varepsilon = 0.65$$

We take the positive constants $k_i, \ (i = 1, \ldots, 5)$ as $k_i = 3$ for $i = 1, 2, \ldots, 8$.

Suppose that the initial values of the parameter estimates are

$$\hat{a}(0) = 1, \quad \hat{b}(0) = 8, \quad \hat{c}(0) = 2, \quad \hat{d}(0) = 5, \quad \hat{\varepsilon}(0) = 4.$$

We take the initial values of the master system (13) as $x(0) = (5, 2, 9)$ and the initial values of the slave system (14) as $y(0) = (4, 12, 6)$.

Figure 4 shows the adaptive chaos synchronization of the identical Lü-like chaotic systems (13) and (14). Figure 5 shows that the estimated values of the parameters $\hat{a}, \hat{b}, \hat{c}, \hat{d}$ and $\hat{\varepsilon}$ converge exponentially to the actual values of the system parameters.

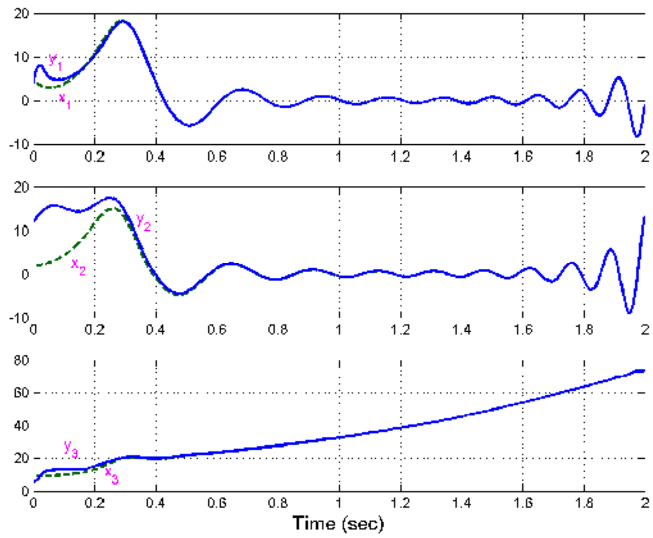

Figure 4. Adaptive Synchronization of Identical Lü-Like Chaotic Systems

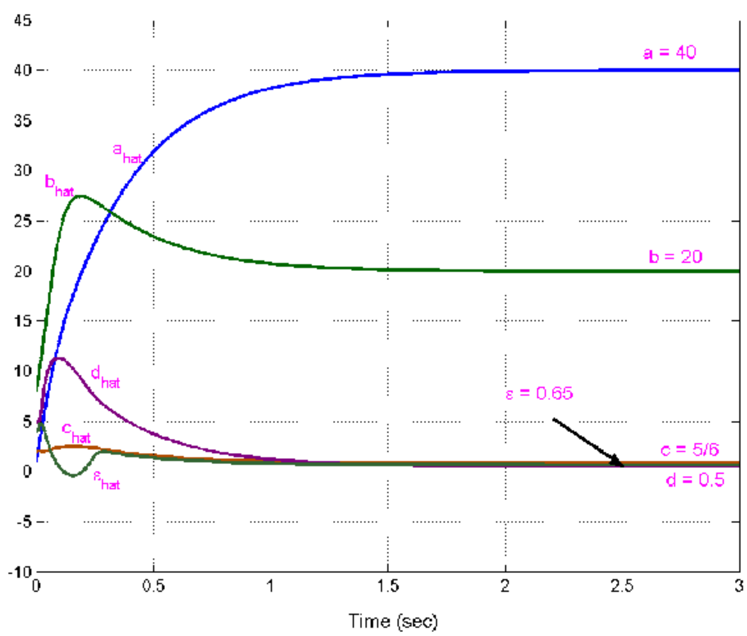

Figure 5. Parameter Estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{d}(t), \hat{\varepsilon}(t)$

4. CONCLUSIONS

In this paper, we applied adaptive control theory for the stabilization and synchronization of the Lü-like chaotic system (Li, 2007) with unknown system parameters. First, we designed adaptive control laws to stabilize the Lü-like chaotic system to its unstable equilibrium point at the origin based on the adaptive control theory and Lyapunov stability theory. Then we derived adaptive synchronization scheme and update law for the estimation of system parameters for identical Lü-like chaotic systems with unknown parameters. Since the Lyapunov exponents are not required for these calculations, the proposed adaptive control method is very effective and convenient to achieve chaos control and synchronization of the Lü-like chaotic system. Numerical simulations are shown to demonstrate the effectiveness of the proposed adaptive stabilization and synchronization schemes.

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